
# General equilibrium as competitive equilibrium: The significance of Walras' achievement from a Cournotian viewpoint

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# General equilibrium as competitive equilibrium:

# The significance of Walras' achievement from a Cournotian viewpoint

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**Abstract**

Cournot's *Recherches* contains a sketch of the general equilibrium research programme, as well as the model of an exchange economy as a system verifying Walras' law. General equilibrium analysis had nonetheless to wait for Walras to occupy the centre of theoretical economics, since it was dismissed by Cournot for its lack of simplicity and robustness. We suggest these motives to depend upon his view of competition as a strategic interaction between producers. Correlatively, the way to the Walras' construct appears to pass through his own view of competition, with economic agents giving up by pure conduct their potential market power.

**Keywords:** Cournot; Walras; general equilibrium; competition.

## 1. Introduction

General equilibrium theory is unquestionably and legitimately associated with the work of Walras (1874). Yet, we find in Cournot (1838) not just a set of sparse materials (concepts, theories, technical tools) to be inherited and exploited by Walras, as usually admitted, but also a sketch of the *general* (as opposed to *partial*) equilibrium research programme (in the beginning of ch. XI), as well as a formal general equilibrium system representing exchange relations between several places (in ch. III), prefiguring Walras' general equilibrium model of a pure exchange economy (see Plantz, 1964). In spite of the latter construct, the research programme involving general equilibrium was readily given up by Cournot as surpassing "the forces of mathematical analysis and of our practical methods of calculation" (Cournot, 1838, p. 146). In this paper we will be interested, not so much in questions of priority or paternity (see Dos Santos Ferreira, 2007), but rather in the obstacles that, beyond pure complexity and technical difficulty, may have led Cournot to put aside a general equilibrium approach, together with Walras' strategy to circumvent them.

We believe that the obstacles imagined by Cournot in the way to general equilibrium are in part connected with the *direct*, strategic, interdependence of individual decisions across markets, which is responsible for both the complexity of calculations and the lack of robustness of results (see d'Aspremont *et al.*, 1997, for a technical

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discussion). Walras' strategy begins precisely with the crucial simplifying step that consists in allowing exclusively for relations between individuals that are mediated by "the market". Market power, the ability to independently influence market prices, appears in this context as just a market imperfection that we can easily neglect. To be explicit, Walras shifts Cournot's "indefinite" competition from its status of limit case in the *Recherches* to the status of general case in the *Éléments* (Walras, 1874, p. 665).

This shift was historically a welcome step, since it opened the way to the construction of a general equilibrium theory at the core of theoretical economics: who would care of building and praising the theory of a mere limit case? However, while turning the perfectly competitive equilibrium into the standard of general equilibrium, Walras also strips Cournot's producers, or rather his own entrepreneurs, not just of their market power, but also of their leading role in the working of the economy: as he himself puts it, "one can even disregard the intervention of entrepreneurs at the state [of equilibrium]" (Walras, 1874, p. 284). The active role is now assigned to consumers, a point caricaturally illustrated by modern macroeconomic general equilibrium models where a representative consumer, directly constrained by the technology, takes all the relevant consumption, production and investment decisions. From a Cournotian perspective, Walras' theoretical achievement thus appears as a switch in economic visions, from an economy of leading profit maximizing producers in face of an indistinct mass of consumers to an economy of utility maximizing consumers assisted by insignificant entrepreneurs.

We will try to substantiate these points in the following, by detailing Cournot's contribution to the foundations of neoclassical economics, with an emphasis on its general equilibrium overtones (section 2), and then by discussing Walras' reconstruction of those foundations with a view to building a general equilibrium system (section 3). We conclude on the two contrasting visions in section 4.

## 2. Cournotian foundations of neoclassical economics

In spite of growing understandable resistance to the usage of the term "neoclassical economics" (Colander, 2000), we shall for convenience use the term "neoclassical" to qualify a theoretical approach which is characterized in the first place by *methodological individualism* or, more precisely, by the *individual rationality* hypothesis. This hypothesis implies the *marginalist* principle, which is nothing but the expression of first order conditions for maximization of some objective function. The second major neoclassical attribute is the hypothesis of perfect *coordination* within well organized markets, opening the way to an analysis restricted to *equilibrium* situations, preferably without specific reference to *general* equilibrium, in order to extend to Marshallian economics the neoclassical qualification. Whether the emphasis on *subjective utility* should be viewed as yet another neoclassical attribute is more debatable, at least in a modern perspective, but that emphasis has certainly

played a major role in the way the generation of Walras, Jevons, Menger and Marshall has perceived its own contribution to economic thought. So has it in the view of Veblen, who coined the term "neoclassical economics" (Aspromourgos, 1986). Independently of any discussion about the necessity and/or sufficiency of this combination of supposedly neoclassical attributes, there is no doubt that most (not to say all) historians ascribe to the aforementioned generation the paternity of neoclassical economics as thus characterized. Nevertheless, except for the subjective utility component, all the preceding attributes are explicitly formulated, one generation before, by Cournot (1838). We will consider them successively in the following subsections.

### *2.1 The law of demand: How far into methodological individualism?*

In the beginning of chapter IV of the *Recherches*, devoted to the "law of demand", Cournot states as his "sole axiom" (or his "sole hypothesis") "that everyone seeks to get the highest possible value out of his property or of his work" (p. 46). This is nothing else than an explicit (although concise) formulation of the *individual rationality hypothesis*. Applied to producers, it implies that each one of them acts so as to maximize the profit (or the value) of his firm. An immediate example of such application is given in the following chapter, where monopoly is analysed. Given the law of demand $D = F(p)$ relating to any price $p$ the quantity $D$ that can be sold at that price, the maximization of the producer's profit $pD - \varphi(D)$ (where $\varphi$ denotes the cost function) requires, as the first order necessary condition for a differentiable function $F$, the equation: $D + p[dD/dp] = [d\varphi(D)/dD]\ [dD/dp]$ (a reformulation of eq. (2) in Cournot, 1838, p. 63). This is the equality of what we now call marginal revenue and marginal cost (both expressed here in terms of the decision variable $p$) – an instance of the *marginalist principle*. This principle is then extended in further chapters, with the appropriate adaptations, to competing suppliers of perfectly substitutable goods (ch. VII and ch. VIII) and to "concurring" suppliers of perfectly complementary goods (ch. IX).

However, the expression of the marginalist principle in the context of consumers' decisions, namely the equation of marginal utilities weighted by price reciprocals, will have to wait for the next generation. This is not the consequence of Cournot's denial of individual consumers' rationality, but rather of his dismissal of the notion of *utility*, of which "there is no fixed measure", so that judgments on utility are but "matters of appraisal, not questions that can be solved by calculation or logic argument" (Cournot, 1838, p. 6, n. 1). Consumers' behaviour is hence immediately described by Cournot at an aggregate level and without explicit microeconomic foundations, by a law of demand $F$ which results from many "moral causes that can be neither enumerated nor measured" (*ibid*, p. 50), and is consequently susceptible only of statistical approximation.

There is however more than a simple retreat in Cournot's attitude. Indeed, aggregation may by itself be responsible for some properties (for instance, continuity) of the law of demand, that an individual demand function would not necessarily display: "the larger the market size, the more varied the combinations of needs,

riches or even fancies among the consumers, the closer the function $F(p)$ will get to vary continuously with $p$" (Cournot, 1838, p. 53). This conjecture is now well known to be true: aggregation has smoothing effects on demand under dispersion of individual characteristics (Sondermann, 1975). Consumer heterogeneity can even be the source of the primary property of the law of demand *assumed* by Cournot, namely that $F$ is a decreasing function. This remarkable result or, more generally, the result that for two different price *vectors* $p$ and $p'$ the scalar product $[F(p) - F(p')]\,[p - p']$ is negative, was first proved in the seminal Hildenbrand (1983) article on the "law of demand", initiating a research program which attempts to obtain demand structure through aggregation (see Hildenbrand and Kneip, 2005, and references therein for more recent results). We will recall this program when discussing Walras' microfoundations of the theory of consumer's behaviour.

To conclude on this first point, there is no doubt that Cournot adhered to methodological individualism: after all, his explicit formulation of the individual rationality hypothesis appears in the beginning of the chapter devoted to the law of demand. But there is no doubt either that this very chapter is the expression of a tempered methodological individualism, since observable aggregate consumers' behaviour, admittedly supposed to result from individual rational actions, nevertheless eludes the construction of a formal microeconomic theory, accompanied by an explicit aggregation procedure.

### *2.2 The law of one price: Perfect and perfectly communicating markets*

After individual rationality, the other fundamental neoclassical hypothesis is the one of perfect coordination within well organized markets, the parts of which "are united by free trade relations, so that the prices are easily and promptly levelled" in each market (Cournot, 1838, p. 55 n.1). As acknowledged by Marshall (1890, p. 324), this statement is an early expression of the *law of indifference* (Jevons, 1871, pp. 90-93): "*in the same open market, at any one moment, there cannot be two prices for the same kind of article*" (*ibid,* p. 91). A point to be stressed about this law of one price is that, according to the neoclassical approach, it prevails in perfect markets even *outside equilibrium*. This is true for instance in the course of the Walrasian *tâtonnement* (cf. Walras, 1874, p. 189), but also when Cournotian producers of the same good deviate while understanding that "the price is necessarily the same" for each competitor (Cournot, 1838, p. 88) – an ingredient of Cournot's analysis of homogeneous oligopoly that was neglected by Bertrand (1883) when he erroneously supposed that each Cournotian duopolist strives to undercut his rival.

The law of one price is not confined to isolated markets for homogeneous goods. It applies to any system of perfectly communicating markets, as a condition requiring that all arbitrage opportunities be constantly exhausted. In chapter III of the *Recherches*, Cournot examines the relations between $r$ exchange places $i = 1,\ldots,r$, introducing the exchange rates $c_{ik}$ from place $i$ to place $k$ as unknowns, and the debts $m_{ik}$ of place $i$ (in its own currency) to place $k$ as data. As $c_{ik} = 1/c_{ki}$, the $r^2$ exchange rates are readily reduced to $r(r-1)/2$. Then, given that

the inequality $c_{ik} \neq c_{ij}\, c_{jk}$ would open an arbitrage opportunity, "it suffices to know the exchange coefficients from one place to all the other to be able to infer all the exchange rates between the latter", further reducing to $r - 1$ the number of unknowns (Cournot, 1838, p. 32). In other words, the no arbitrage condition reduces the $(r \times r)$-matrix of relative prices to one vector $(1, c_{12}, \ldots, c_{1r})$ of prices referring to an arbitrarily chosen place (here, place 1) or, as Walras will later say (in a transposition of the same analysis to a system of commodity markets), of prices expressed in terms of an arbitrary *numéraire* (Walras, 1874, p. 171).

The reduction to $r - 1$ of the number of unknowns is of course important in assessing whether they may be determined within the system of equations which expresses the balance of the accounts of all $r$ places (or, equivalently, the equality of supply and demand for all $r$ currencies), namely (eq. (*b*), p. 33, with simplified notations):

$$\sum_{k \neq i} m_{ik} = \sum_{k \neq i} m_{ki} c_{ki}, \quad i = 1, \ldots, r.$$

This system seems to be overdetermined but, by multiplying both sides of each equation $i$ by $c_{i1}$ and using the no arbitrage condition, Cournot transforms the preceding system into (eq. (*d*), p. 34):

$$\sum_{k \neq i} m_{ik} c_{i1} = \sum_{k \neq i} m_{ki} c_{k1}, \quad i = 1, \ldots, r.$$

He then concludes (*ibid*, p.34): "If we add together all but the first of these equations, and if we erase on each side the balancing terms, we fall back on the first equation. Hence, we have a number of distinct equations just equal to the one of independent variables." This argument, closely followed by Walras (1874, pp. 171-173), implicitly introduces what will become known as *Walras' law*.

### *2.3 The equilibrium approach: From monopoly to the limit case of indefinite competition*

Coordination in perfect and, more generally, perfectly communicating markets does not only ensure the fulfilment of the law of one price. It leads naturally to an equilibrium approach: the theory will admit but profiles of individually rational actions that are mutually compatible. In chapter III of the *Recherches*, balanced accounts (implying the equality of currency supplies and demands) are already an equilibrium condition if one sticks to "the assumption that no money is actually transported from one place to another" (Cournot, 1838, p. 32). When Walras takes up the same system of exchange relations, he enunciates three equilibrium conditions, namely the maximization of consumers' utilities (ignored by Cournot), the absence of arbitrage opportunities (discussed in the previous subsection), and precisely the condition "that everyone should receive in proportion of what he gives or give in proportion of what he receives, each commodity having a sole price in terms of *numéraire*, that which makes total effective demand equal to total effective supply" (Walras, 1874, p. 187).

Equilibrium further requires for each individual a correct perception of his/her environment, otherwise individual rationality will not be preserved *ex post*. In the beginning of his analysis of monopoly, Cournot (1838, p. 61) admits the existence of a process of trial and error (*tâtonnement*) converging to the profit maximizing price, a process that can only make sense (in the case of a rational producer) if the information on the law of demand is imperfect. The optimal decision may then appear as the equilibrium of a dynamic learning process.

Cournot's decisive contribution in this context is however the construction of a concept of equilibrium applying to strategic interaction between $n$ producers (with $n$ such that $1 < n < \infty$). This concept appears as a first instance of the solution concept for non-cooperative games introduced more than one century later by Nash (1950). Formally, an equilibrium situation is described by a profile of producers' strategies – *quantities* (or rather quantity targets) when producers compete for the sale of a homogeneous output (ch. VII), *prices* when they concur in the supply of complementary outputs (ch. IX) – such that no producer can increase his/her profit by choosing a different strategy. Individual rationality is thus an essential ingredient of the equilibrium concept, but so is coordination, translating into mutual consistency in the sense that every producer has a correct perception of all competitors' actions at equilibrium. Again, equilibrium is viewed by Cournot as the possible outcome of a dynamic learning process, now not only on the law of demand but also on the competitors' eventual strategies (Cournot, 1838, pp. 90-91).

We shall not develop the details of Cournot's contribution to oligopoly theory. The relevant point for our purpose is that, by increasing the number $n$ of producers of a homogeneous good from 1 (the limit case of monopoly, with no competition) to infinity (the other limit case, of *indefinite* competition), Cournot obtains a progressive decline and eventual exhaustion of the capacity of each producer to independently manipulate the market price: "The effects of competition have reached their limit, when each of the partial productions $D_k$ is *inappreciable*, not only in relation to total production $D = F(p)$, but also in relation to the derivative $F'(p)$, so that the partial production $D_k$ could be subtracted from $D$ without determining any appreciable variation in the price of the commodity" (Cournot, 1838, p. 101). As this limit is approached, producers' outputs become negligible with respect to market size, and their marginal costs tend at equilibrium to catch up the market price. We shall in the following keep in mind the status of *limit case* characterizing indefinite competition in Cournot's work, even if he devotes the whole chapter VIII to this case, which "introduces a great simplification in the calculations" (*ibid*, p. 101) and often turns out to be a good approximation.

### *2.4 Drawing and withdrawing the general equilibrium approach*

Cournot's contribution to economic theory is usually confined to partial equilibrium analysis. This is indeed what most of the *Recherches* is about. As a matter of fact, Cournot cannot be seen in this respect as an epigone of Monsieur Jourdain, doing partial equilibrium analysis without knowing it, since he starts chapter XI by accurately

defining the approach: "We have so far examined how the law of demand for every commodity in particular, combined with the conditions of production of that commodity, determined its price and regulated its producers' incomes: we looked as given and invariable the prices of other commodities and the incomes of other producers" (Cournot, 1838, p. 146). But he immediately remarks that "in reality, the economic system is a whole all parts of which hold together and react upon one another." And he pursues: "An increase in the income of producers of commodity A will influence the demand for commodities B, C, etc., the incomes of the producers of these commodities, which will bring about as a knock-on effect a change in the demand for commodity A. It thus seems that, in the complete and rigorous solution of problems relative to some parts of the economic system, one cannot dispense with embracing the whole system" (*loc cit*). Clearly, Cournot is aiming at a complementary general equilibrium approach, not necessarily to analyse the whole economy for its own sake, but just to obtain a thorough picture of anyone of its parts.

Curiously, after drawing this sketch of general equilibrium, Cournot backs down, as he fears that "[embracing the whole system] would surpass the forces of mathematical analysis and of our practical methods of calculation, even if one could assign numerical values to all the constants" (*loc cit*). And, in order "to elude the difficulty", he finally opts for contenting himself with "some order of approximation", having just in view "average results", observed after compensation of opposite effects (*ibid*, pp. 146-147 and 151). There is however some difficulty in understanding Cournot's position, since most of the *Recherches* concerns purely theoretical issues and does not depend upon "our practical methods of calculation". Also, "the fact that [Cournot] developed a mathematical system to demonstrate the determination of the rate of exchange between countries, which could easily have been employed with but slight modification to formulate a general equilibrium model, compounds the difficulty of explaining his apparent inability to complete his own models" (Plantz, 1964, p. 199).

We may however try an explanation. The mathematical system mentioned by Plantz, which is that, already evoked, of chapter III, was designed to analyse exchange relations, in an economy without production. Cournot is however principally concerned with interactions between producers. Of course, should he feel satisfied with the convenient approximation of indefinite competition, formulating a general equilibrium model would not be beyond his forces. But indefinite competition prevails only in the limit, when producers' actions, if independently considered, become insignificant. This is certainly not the world Cournot had in mind.

The contents of chapter IX give some support to our tentative explanation. Cournot considers a system of $n$ monopolistic markets which concur to supply perfectly complementary inputs to a downstream market where producers use $m_i$ units of input $i$ to obtain one unit of their "composite" output. Notice that the coefficients $m_1,\ldots,m_n$ are nothing but Walras' production (or *manufacturing*) coefficients (Walras, 1874, pp. 304-5), or Leontief input-output coefficients. By implicitly assuming indefinite competition in the downward market, the

price $p$ of the composite commodity is taken as equal to the sum $m_1p_1 + \ldots + m_np_n$ (eq. (a), for $n = 2$, in Cournot, 1838, p. 113). Denoting by $F$ the law of demand for the composite commodity and by $F_i$ the law of *final* demand for the $i$-th elementary commodity, Cournot uses the law of *total* demand for this commodity (*ibid*, p. 122)

$$D_i = F_i(p_i) + m_i F\left(m_i p_i + \sum_{k \neq i} m_k p_k\right)$$

to express monopolist $i$'s profit $p_iD_i - \varphi_i(D_i)$ to be maximized, as in chapter IV. The significant point is that the monopolist is assumed here to perform this maximization not by just taking the price $p$ of the composite commodity as given, but by taking into account the impact of any manipulation of his price $p_i$ (through the variation of $p$) on the demand for his output originating in the downward market. Observe that Cournot is thus invalidating his own statement of chapter XI that "the law of demand for every commodity in particular" has hitherto been examined by looking "as given and invariable the prices of other commodities."

Monopolist $i$ is assumed to anticipate the variation of $p$ associated with his price manipulation by taking as given the input prices $p_1,\ldots,p_{i-1},p_{i+1},\ldots,p_n$ set by the other monopolists. This assumption leads directly to an equilibrium concept which is a further instance (in addition to that of ch. VII) of the Nash solution concept. But the point we want to stress is that the model of chapter IX is also a further instance (in addition to that of ch. III) of a system of communicating markets. This system is now open (so that Walras' law is not verified anymore), but it displays a more complex, strategic, form of interaction across markets. The price to pay for the complexity is that, after formulating the first order conditions for producers' profit maximization, namely, for $i$=1,…,$n$,

$$F_i(p_i) + m_i F\left(\sum_{k=1}^{n} m_k p_k\right) + \left[F_i'(p_i) + m_i^2 F'\left(\sum_{k=1}^{n} m_k p_k\right)\right]\left[p_i - \varphi_i'(D_i)\right] = 0\,,$$

Cournot must admit that "they look too complicated for general consequences to be easily deduced from them" – a judgment addressing not only the potential difficulty of calculation (cf. "easily"), but also the robustness of potential results (cf. "general").

He then decides to proceed "to a case which is much more important in applications, and which can be easily treated with all the desirable generality, that where each of the concurring commodities is produced under indefinite competition", instead of monopoly (Cournot, 1838, p. 123). The first order condition for producer $i$'s profit maximization then imposes the equality of price and marginal cost $p_i = \varphi_i'(D_i)$ which, by inverting the function $\varphi_i'$, leads to the supply function $\Omega_i(p_i)$. The equilibrium conditions can now be expressed, more simply, in terms of the equality of supply and demand (*ibid*, p. 124):

$$\Omega_i(p_i) = F_i(p_i) + m_i F\left(\sum_{k=1}^{n} m_k p_k\right), \quad i = 1,\ldots,n\,.$$

With this simplification, made possible by the assumption of indefinite competition, Cournot opens in fact the way to general equilibrium analysis – a way he seemed reluctant to follow, but which was eventually taken by Walras.

## 3. Walras' reconstruction of Cournotian foundations

In the preface to the 4th edition of the *Éléments*, Walras writes that he owes to his father, Auguste Walras, "the fundamental principles of his economic doctrine", and to Augustin Cournot "the employment of calculus for the exposition of that doctrine" (Walras, 1874, p. 5). Walras' estimate of his intellectual indebtedness is certainly unfair to Cournot, whose legacy, as suggested in the previous section, concerns a large set of building blocks to be partly used in the construction of the Walrasian theoretical edifice. Nevertheless, in order to achieve that construction and before engaging in the patient assemblage of all those blocks, Walras had both to take care of the important missing block of consumer theory – not for its own sake, but as the means to analyse market phenomena (Jaffé, 1972) – and to knock down the obstacle perceived by Cournot in the way of general equilibrium theory. There is no doubt about the success of Walras' enterprise, but these two founding steps were not completely innocuous and were going to have lasting consequences on the dominant vision of the working of a market economy. We shall consider successively these points in the four following subsections.

### *3.1 Microtheoretical foundations of demand*

Cournot had dismissed the notion of utility, with no "fixed measure" and hence irrelevant for economic calculation. In so doing, he had bypassed the analysis of the individual consumer's decisions, and started directly from the *aggregate* law of demand $F(p)$. Walras begins on the contrary the 6th lesson of the *Éléments* by assuming "empirical" *individual* demand functions $f_{hi}(p_h)$ for each commodity $h = 1,2$ and each consumer $i$, then using the *explicit* aggregation procedure: $F_h(p_h) = \Sigma_i f_{hi}(p_h)$ (Walras, 1874, p. 85, with modified index notations). The price $p_h$ is in fact the relative price of commodity $h$ in terms of the other commodity, so that, in the generalized analysis of the 12th lesson, for $h = 1,\ldots,m$, the argument of $f_{hi}$ is a vector of $m - 1$ coordinates. Dependence of the demand for commodity $h$ upon the prices of all the other commodities expressed in terms of the *numéraire* is now explicit: by contrast with Cournot's law of demand, Walras' demand function is from the start a genuine piece of *general* equilibrium analysis.

The next step in the quest for microtheoretical foundations consists in rationalizing the individual demand functions (8th lesson). Walras calls on two types of utility: *extensive* utility, to explain the quantity of each commodity that is demanded at zero price, and *intensive* utility, to explain the slope of the demand curve. As concerns the former, utility may be identified with a quantity, and hence appears immediately as "a measurable magnitude" (Walras, 1874, p. 104), contrary to what Cournot had pretended. As concerns the latter, Walras admits

that it expresses a complex relation involving the consumer's attitude toward different commodities as well as his/her initial endowments. Hence, in order to proceed, he simply *supposes* that "there is a standard of measurement of needs intensity or of intensive utility, common not only to similar units of the same kind of wealth but also to different units of diverse kinds of wealth" (*ibid*, p. 107). He then assumes a decreasing function $\varphi_{hi}$ associating with each quantity of commodity $h$ potentially consumed by consumer $i$ "the intensity of the last need satisfied by that quantity" (*ibid*, p. 109), in other words its marginal utility, which he calls *rareté*, in tribute to his father's approach to value theory. By integrating this function, Walras obtains the *effective utility* got by consumer $i$ from the consumption of commodity $h$, ready to be added up across commodities.

Maximization of the resulting total effective utility through exchange supposes that the ratio of marginal utility to price is equalized for all commodities. Denoting by $q_{hi}$ and $z_{hi}$ the endowment and the net trade of commodity $h$ by consumer $i$, and choosing commodity 1 as the *numéraire*, Walras (1874, p. 179, with slightly modified notations) thus obtains, in addition to the budget equation

$$z_{1i} + \sum_{h=2}^{m} p_h z_{hi} = 0 ,$$

the $m - 1$ utility maximization conditions

$$\varphi_{hi}(q_{hi} + z_{hi}) = p_h \varphi_{1i}(q_{1i} + z_{1i}), \quad h = 2,...,m .$$

Solution in the $m$ net trades $z_{1i},\ldots,z_{mi}$ of this system of $m$ equations gives (*loc cit*):

$$z_{hi} = f_{hi}(p_2,...,p_m), \quad h = 1,...,m ,$$

namely the equations of consumer $i$'s net demand functions for all $m$ commodities (dependence upon endowments $q_{1i},\ldots,q_{mi}$ having been left implicit). It then suffices to add up all the individual net demand functions, and to make them equal to zero at equilibrium:

$$\sum_i f_{hi}(p_2,...,p_m) \equiv F_h(p_2,...,p_m) = 0, \quad h = 1,...,m .$$

Notice that, by aggregating the individual budget equations, it clearly appears that equilibrium in $m - 1$ markets implies equilibrium in the $m$-th market (*Walras' law*), so that the last equation system is not overdetermined.

By just accepting to *assume* the measurability of utility, Walras has thus been able to extend the application of the axiom of individual rationality from producers to consumers, and to establish microtheoretical foundations to aggregate demand. Of course, Walras' analysis is yet confined to the case of an additively separable utility function, and is yet far from the idea that this function is but a representation of the individual consumer's preferences, in other words that utility is only a derived concept – an idea the story of which begins with Pareto. Still, Walras' step is important, because it opens the way to welfare analysis, and because it tends to symmetrise the modelling of the economy, extending to consumers the status of rational deciders monopolized by Cournot's

producers. As a consequence, Walras' approach clearly adheres more strictly to methodological individualism than Cournot's law of demand.

However, Cournot's position has been unexpectedly vindicated in some sense. We now know from the Sonnenschein–Mantel–Debreu theorem that individual rationality confers but little structure to the aggregate demand function. In particular, we do not necessarily obtain for this function the properties that ensure uniqueness and *tâtonnement* stability of general equilibrium. Some of these properties (those generalizing the monotonicity of Cournot's law of demand) are not necessarily verified even at the individual level, and others are lost through aggregation. Conversely, as already mentioned when we have discussed Cournot's law of demand, aggregation may confer structure to the demand function, provided we accept to formulate appropriate assumptions on the distribution of relevant individual characteristics, lending to it a large enough spread. In other words, we are in some sense back to Cournot.

### *3.2 Converting indefinite competition into the general case*

We suggested that the complexity and virtual lack of robustness of a general equilibrium analysis with strategic interaction between producers might have been one of the reasons for Cournot to put aside a general equilibrium program. We also observed that he took indefinite competition to be a case "which can be easily treated with all the desirable generality", introducing in the calculation a great simplification and being relevant in many important applications (cf. the beginning of ch. VIII in Cournot, 1838). This research strategy was not fully explored by Cournot, but was taken up by Walras to build his general equilibrium theory. Walras did however not content himself with introducing indefinite competition as a simple approximation, no matter how good; on the contrary, he converted *absolute free competition* – the corresponding concept in his own framework – into the general case, a step which allowed him to claim for generality of his theoretical construct.

Of the 42 lessons of the *Éléments*, the first four are devoted to the object and divisions of political economy, the next 36 developing an analysis which refers "to the one and only hypothesis of absolute free competition, applying to exchange, production and capitalisation" (Walras, 1874, p. 655). The two last lessons consider restrictions to *laissez faire* in the production and circulation of wealth, namely tariffs and monopolies, as well as the intervention of public authorities addressing its distribution, in particular in the form of taxes. Such restrictions are all responsible for "exceptions" to the general case, that is, for "perturbations" of the mechanism of free competition (*ibid*, p. 656). As concerns monopoly, Walras starts with the presentation of Cournot's theory, and then comments that, contrary to his predecessor who went "from the case of a single monopolist to the case of two monopolists, and finally from monopoly to indefinite competition", he preferred "to start from indefinite competition, *which is the general case*, to arrive at monopoly, which is a particular case" (Walras, 1874, p. 665; our emphasis). Such methodological choice enabled him "to relate to the rational and rigorous exchange [and]

production equations the empirical and approximate equation of demand as a function of price" (*ibid*), in other words, to replace Cournot's statistical approach to the law of demand by a theoretical, rationally founded, approach.

Walras' statement is problematic, since it makes no sense to consider indefinite competition as the general case, at least if we stick to Cournot's understanding of *indefinite* as qualifying the *limit* attained by competition when each competitor $k$'s production $D_k$ becomes negligible with respect to market demand $D$ (formally, when $D_k/D$ tends to zero). Consider by analogy the case of the special theory of relativity, generalizing the Galilean relativity principle (cf. Einstein, 1916/1920, § XI). Under uniform translation at velocity $v$ of a coordinate system with respect to another, which of the two coordinate transformations, the Galilei and the Lorentz, should be seen as the general one, knowing that the former obtains as the limit of the latter when velocity $v$ becomes negligible relative to the velocity of light $c$ (formally, when the ratio $v/c$ tends to zero)? The answer is obvious.

We conclude that "indefinite" must have another meaning in Walras' statement. It might be the case that it qualifies the number of firms, a *variable* to be determined through *free entry and exit* (Dockès and Potier, 2005). Indeed, Walras points out that, if there were no monopoly, the profit of the firm would keep attracting competitors until it eventually vanished (cf. Walras, 1874, p. 662). Alternatively, it might be the case that Walras is just using in this passage Cournot's term, even if it is now qualifying the normal case. Let us however ignore semantic considerations and concentrate on substance. The point is that free competition, a regime where firms cannot, or simply do not, take advantage of price manipulability to increase their profits above zero is now going to occupy the central – not to say the almost exclusive – place in economic theorizing. In the preface to the 4th edition of the *Éléments*, Walras declares that "*pure political economy* is essentially the theory of the determination of prices under a hypothetical regime of absolute free competition" (Walras, 1874, p. 11). He then explains in a footnote that *free competition* first applies to sellers of services undercutting each other and to buyers of products outbidding one another, and second to entrepreneurs, as a means (among others) to equalize prices with production costs (*ibid*, p.11 n. 1). But undercutting and outbidding are just the constituent actions of the *tâtonnement* process in the course of which the agents attempt to discover (and attain) equilibrium prices, equating effective demand and supply as expressed *at those prices* (cf. Walras, 1874, pp. 70-72). This means that demanders' and suppliers' decisions have already been made at this stage for each possible configuration of supposedly *non-manipulable* prices.

A point that should be emphasized is that price non-manipulability is now a definitional although largely implicit property of Walras' absolute free competition, rather than the consequence of agents' insignificance, as in Cournot. Once monopoly is put aside, the number of firms and hence their market shares are irrelevant as regards the nature of competition, as viewed by Walras. As a matter of fact, even a single firm may be enough to

implement the production equilibrium that corresponds to the normal state of free competition between a large number of firms, provided that firm adopts "competitive" behavioural rules (cf. Walras, 1874, p. 284). This way of taking competition, at least in its absolute (or *perfect*) form, just "in the sense that each producer and consumer regards the prices paid and received as independent of his own choices" (Arrow and Debreu, 1954, p. 265), has in fact become standard in much of modern general equilibrium theory. In the Arrow-Debreu model there are *m* consumers and *n* firms, but *m* and *n* may be both equal to one, as in Walras' example. However, the literature on large economies, beginning essentially with Aumann's seminal paper on markets with a continuum of traders, operated a come back to Cournot's view of indefinite competition as a limit case: "The essential idea of [the notion of *perfect competition*] is that the economy under consideration has a 'very large' number of participants, and that the influence of each individual participant is 'negligible,' [so that] *a mathematical model appropriate to the intuitive notion of perfect competition must contain infinitely many participants*" (Aumann, 1964, p. 39; author's emphasis).

Walras' position and its modern counterpart in the Arrow-Debreu approach are to some extent problematic. Indeed, "to regard prices paid and received as independent of one's own choices" misrepresents one's environment in markets where one side (if not both) has only few participants. Such misrepresentation cannot fit the hypothesis of full individual rationality, which is one of the founding neoclassical hypotheses. In other words, at the very moment he makes an important step to consecrate this hypothesis by building a theory of the individual consumer, Walras moves back to some form of bounded rationality by his view of competition. In fact, as we shall show in the next subsections, Walras' retreat from a strict application of the rationality hypothesis is still more serious than what has just been suggested.

### *3.3* ***Confining free competition within the tâtonnement process***

Competition appears for the first time in the *Recherches* at the beginning of chapter VII, as an interaction between two *producers* of the same good, striving to maximize his profit "each one on his own", that is, without colluding with the rival. In other words, for Cournot, competition is characterized by individual rationality together with non-cooperation. These two characteristics apply to a hypothetical trial and error process, along which each producer adjusts in turn his estimate of the other's eventual choice, and then chooses the quantity that maximizes his profit (individual rationality), when taking as fixed his new estimate of the competitor's quantity (non-cooperation). Of course, the two characteristics of competition also apply to the outcome of that iterative process – the *equilibrium* – and so whatever the number of competitors, even when this number tends to infinity, that is, when competition becomes indefinite.

As well known, this form of strategic interaction is absent from the *Éléments*. Moreover, Cournot's *indefinite competition* should not be identified with Walras' *absolute free competition*, even if price non-manipulability

prevails in both as regards supply and demand decisions. Indeed, competition (appearing in the 5th lesson of the *Éléments*) denotes now a completely different kind of interaction, between consumers on the long side of the market: either suppliers undercutting each other under excess supply or demanders outbidding one another under excess demand. Thus defined, competition can only persist during the *tâtonnement* process, before any transaction can take place and as a symptom of disequilibrium, since at equilibrium there is by definition neither excess supply nor excess demand.

Walras' view of competition takes us far away not only from Cournot's but also from the standard neoclassical view identifying perfect competition with price-taking behaviour of all market participants, and of course from the conventional reference to the supposed Walrasian auctioneer. All Walrasian agents may indeed be price makers in the course of the *tâtonnement* process, so that we should not apply to Walras the well known Arrow's critique: "each individual participant in the economy is supposed to take prices as given and determine his choices as to purchases and sales accordingly; there is no one left over whose job it is to make a decision on price" (Arrow, 1959, p. 43). It is however true that in Walras' analysis as "in the usual formulation of the theory of the perfectly competitive economy […] there is no place for a *rational* decision with respect to prices as there is with respect to quantities" (Arrow, 1959, p. 42; our emphasis). On the one hand, Walrasian consumers rationally decide on the quantities to trade under the conjecture that prices are fixed, on the other hand they are called upon to adjust prices whenever their quantity decisions are mutually incompatible, so that their hypothetical incapacity to manipulate prices is denied in practice. Besides, price adjustment decisions are made in order to alleviate a shortage or a slump, but they are not meant to optimize, taking advantage of the experienced price manipulability. Consequently, full individual rationality is preserved only at equilibrium, when prices cease to need adjustment and competition comes to an end.

The former discussion ignored producers, and we might be tempted to just extend to them what we have said of consumers: Walrasian entrepreneurs would on the one hand decide on quantities in order to maximize their profits under given prices, and would on the other hand adjust prices when finding themselves on the long side of an unbalanced market. However, such extension would not fit Walras' approach, in which the symmetry between consumers and entrepreneurs is far from complete, as we will see in the next subsection.

### *3.4 From competition between profit maximizing producers to profit dissipating competition*

Cournotian producers are profit maximizers. What about Walrasian entrepreneurs? In the 41st lesson of the *Éléments*, Walras opposes the case of a monopolistic industry, where the ruling condition on producers' side is "that the selling price be higher than the cost and that producers make *the highest possible profit*" and the case of an indefinitely competitive industry, where the corresponding condition is "that each product have a sole price in the market, which should be equal to the cost price, so that *producers make neither profit nor loss*" (Walras, 1874,

pp. 662-663; our emphasis). The first component of this competitive equilibrium condition, stating the law of one price, appears as a feature opposing indeed the two regimes, because Walras is about to examine the case of *discriminating* monopoly two paragraphs below. Its second component is the zero profit condition, but without the requirement that zero be "the highest possible profit" for each entrepreneur. By contrast, consumers are explicitly assumed to obtain under the above conditions the maximal satisfaction of their needs. In the 21st and 22nd lessons, Walras formulates the same characterization of equilibrium in the context of production, involving *utility* maximization, the law of one price – the one that equalizes effective supply and effective demand in each market – and, more specifically in the context of production, equality of selling and cost prices (cf. Walras, 1874, pp. 329-330 and 333-334). In order to clarify the meaning of *free* competition, he states that it comprehends the freedom to undercut when supply exceeds demand and to outbid when demand exceeds supply (common to the context of pure exchange), as well as "the freedom left to entrepreneurs to develop their production in case of profit and to cut it in case of loss" (*loc cit*, p. 333; cf. also p. 330). Thus, in an economy with production, in addition to the *tâtonnement* in prices, in which entrepreneurs now take part along with consumers, there is a *tâtonnement* in quantities driven by the sole entrepreneurs (cf. Walras, 1874, pp. 282-284).

During the *tâtonnement* process, but also when equilibrium prevails, it is by no means obvious that Walrasian entrepreneurs are supposed to maximize profits under free competition. At least, such objective is not explicitly attributed to them. Part of the criticisms addressed to the condition imposing the equality of price and *average* cost by the contemporaries of Walras (cf. Walker, 1986, s. 4) were based on its supposed inconsistency with the condition for profit maximization, namely the equality of price (assumed non-manipulable, hence equal to marginal revenue) and *marginal* cost. There is of course a simple line of defence: average cost, whenever *constant*, is equal to marginal cost, so that there is no inconsistency in this case, which happens to be the one explicitly considered in the *Éléments*. Our position is however more radical: Walras "did not use the term 'marginal cost' nor the concept of marginal as distinct from average cost" (Walker, *op cit*, p. 10), not because he "did not, in the formal structure of his model, concern himself with the conditions of short-run equilibrium in production" (*ibid*), but because, outside the "particular" monopolistic case, he was not assuming profit maximization at all (although he did assume cost minimization: Walras, 1874, p. 305). Instead of profit maximization, what appears again and again in the *Éléments* as a characteristic of free competition is the behavioural rule designed to entail neither gain nor loss and commanding "an increase in the quantity of products whose selling price is higher than their cost, and a decrease in the quantity of those whose cost is higher than their price" (Walras, 1874, p. 330). Notice that these quantity adjustments may result from the entry of firms attracted by the anticipation of a gain and the exit of firms discouraged by a loss, but not exclusively. They are also carried out by incumbent firms responding to non-zero profits (cf. Walras, 1874, pp. 283-284).

How rational is such conduct? Even if we admit that under "absolute free competition" marginal revenue reduces to a constant price, producing more in response to a positive profit is not profit enhancing, as soon as the marginal cost exceeds the price. It is true that returns to scale are systematically assumed constant along the *Éléments*, but even in this case a strict application of profit maximization would determine the *immediate* choice of an infinite or a zero scale as soon as the price is respectively higher or lower than the unit cost. The behaviour of Walrasian entrepreneurs thus appears not as individually rational but rather as *satisficing* (Simon, 1987), obeying to a rule of thumb which corresponds to the quantity component of the *tâtonnement* procedure.

Now, even if this rule of thumb ends up ensuring profit maximization at equilibrium, at least under constant returns to scale, competition in Walras' approach does no longer involve profit maximizing firms, as it did in Cournot's. As far as production is concerned, it appears essentially as a process of profit dissipation. Furthermore, its ultimate outcome is in fact the fading out of entrepreneurs, left at a state of equilibrium with a purely catalytic role: "one can even make abstraction at that state of the intervention of entrepreneurs, and consider not only that productive services exchange for products and products for productive services, but also that productive services ultimately exchange for one another" (Walras, 1874, p. 284).

## 4. Concluding remarks: Two contrasting visions of the economy

By bringing together Cournot's *Recherches* and Walras' *Éléments*, the first trait we observe is the continuity of the overall methodological and theoretical approaches, with much of the conceptual framework of the latter borrowed from the former. Both obey to a tempered methodological individualism, even if the reason for introducing a qualification of the methodological individualism in the *Recherches* is not the same as for the *Éléments*. Cournot states the axiom of individual rationality, implying profit maximization by the firms, but he does not explicitly apply it to consumers, whose behaviour is only described in the aggregate. By contrast, Walras develops a theory of the individual consumer's behaviour, conferring microeconomic foundations to aggregate demand as well as to aggregate supply of productive services, but the rationality he attributes to the entrepreneur is only bounded, expressing itself in a simple behavioural rule that might be assigned to an automaton: "expand if you make a profit, contract if you make a loss".

Putting Walras' achievement in a Cournotian perspective is however enlightening not so much because of the continuity of the two methodological and theoretical approaches, but rather because of the contrast between the two economic *visions* (in Schumpeter's sense). Cournot describes an economy led by the actions of rational producers facing a mass of consumers. Being not too numerous in general, Cournot's producers have the power to manipulate market prices, a power which they exert to their own advantage, taking into account competitors' strategies, together with the anticipated consumers' response. By contrast, Walras describes an economy

dominated by individual consumers' decisions, where *entrepreneurs* (replacing Cournots' producers) just play for free the passive role of intermediaries between consumers, who act as demanders of produced goods and as suppliers of productive services. Perfect competition, taken by Cournot as *indefinite*, that is, as a limit case where market power completely vanishes, is made by Walras into the general case. However, Walras' *absolute free competition* concerns principally not the interaction between entrepreneurs in or out of equilibrium, but the interaction between traders on the long side of unbalanced markets, so that it appears as essentially a disequilibrium phenomenon.

Of the two contrasting visions, the latter is certainly the prevalent one in later general equilibrium theory (including much of its modern macroeconomic applications), both as concerns the alleged normality of perfect competition (although not in its disequilibrium interpretation) and as concerns the primary role of the consumption sector. General equilibrium has tended to be *identified* with competitive equilibrium in most of the literature, with a notoriously absent market power, whether economies are assumed to be "large" or not. Firms do maximize profits, which may well be positive at equilibrium (in short run and under decreasing returns to scale), but their contribution to the working of the economy is essentially reduced to the information contained in their production sets and to the distribution of profits to shareholders. The economy is viewed as a huge market anonymously and efficiently reallocating among consumers, according to their preferences, their privately owned resources, whether these resources are immediately available – in the pure exchange case – or whether they have first to be transformed within a production sector. To put it briefly, the economy thus portrayed is one where the consumer is king and where the entrepreneur is just a discreet servant of the market. The opposite vision, patent in Cournot's work, of an economy where initiative rhymes with enterprise, and where firms exert market power, interacting strategically rather than just through impersonal market relations, has been rather a minority view in general equilibrium theory and, in spite of Keynes, in modern macroeconomics.

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