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CAN AGGREGATION ACROSS GOODS BE ACHIEVED BY NEGLECTING THE PROBLEM? PROPERTY INHERITANCE AND AGGREGATION BIAS*

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This article studies the problem of composite commodity in two different frameworks. In one case, aggregation across goods is analyzed for elementary goods that satisfy an optimality condition. The unrestricted case is also examined. The notion of an approximate aggregate representation is formalized and shown to be always possible. Can thereby aggregation issues simply be neglected in economic contributions? I show that the standard economic properties of initial functions are not necessarily inherited by the approximate aggregate. The severity of this problem and the size of the aggregation bias across inputs are investigated empirically.

1. INTRODUCTION

The microeconomic theory of producer and consumer behavior derives structural relationships on the basis of elementary goods and prices. The concept of elementary goods is, however, difficult to handle both because of the huge amount of such goods and because of the difficulty in defining what an elementary good even represents (in the Arrow–Debreu framework, time and localization lead to distinguish physically identical goods). Thus, composite commodities are considered instead in empirical analyses. A direct consequence is the loss of some information relative to the original problem. A more bothersome consequence is that the initial structural framework must be adapted to cope with composite commodities and to retain coherency with the original theory referring to elementary goods. Two principal approaches have been proposed.

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Exact aggregation across goods, developed by, for example, Leontief (1947) and Blackorby et al. (1978), requires that the aggregate representation be valid for any value of the elementary variables. The exact aggregation approach states which conditions on the preferences and the technologies of the individual units have to hold for an exact representation, on the basis of aggregates, to be possible. The conclusions of this approach are rather pessimistic since the required conditions seem rather implausible and are most often rejected in empirical investigations (see, e.g., Blackorby et al., 1986; Diewert and Wales, 1995). Of course, the generality of the requirement of exact aggregation is directly at the origin of the restrictive conditions under which an aggregate description of the initial relationships exists.

In order to avoid these implausible restrictions on admissible preferences (technologies), some authors have followed a practice suggested by Hicks (1936) and Leontief (1936), which consists of restricting the *distribution* of the elementary prices. Often, it is the strict proportionality of all elementary prices of a given subset that is assumed for achieving aggregation. This alternative approach is problematic for two main reasons: first, when decision units behave rationally, there exist links between restrictions on the preferences and restrictions on the distribution of elementary goods and prices; second, strict proportionality of prices is usually rejected when tested. Lewbel (1993, 1996) weakens this assumption and shows that aggregation across goods is achieved when prices are approximately proportional, an assumption that still appears as quite restrictive.

The purpose of this article is to show that an aggregate representation can be achieved under broader conditions than those presented in this stream of the literature. Whereas the exact aggregation approach raises doubts on the very existence of an aggregate representation, I show that it is always possible, by simple transformations of the microeconomic model, to obtain relationships depending on aggregate variables and on the way the elementary variables are distributed. The necessary and sufficient conditions for (i) no aggregation bias and (ii) the inheritance of microeconomic properties in the aggregate are presented. It is shown that these conditions are weaker than those required by exact aggregation and approximate price proportionality. However, the required conditions are still restrictive and may not hold in general. Neglecting the distribution of elementary variables when modeling individual relationships then has the same implication as omitting some relevant explanatory variables, as already underlined by Theil (1954). For this reason, the validity of both the approximate representation and the microeconomic regularity properties remains an empirical question.

Some 40 years ago, Boot and de Wit (1960) presented an inaugural empirical contribution to the aggregation problem. The authors mainly identify and determine empirically the aggregation bias occurring in relationships aggregated across individuals. In the empirical part of this article, aggregation across different intermediate inputs is considered in the context of production analysis. The aggregation bias on scale and substitution elasticities is computed and the validity of microeconomic properties in the aggregate framework is investigated.

In Section 2, the framework and the notations are outlined. The relations between exact aggregated models and those obtained when aggregation is neglected are presented in Section 3, in a context where optimality relationships are not considered. In Section 4, I study the implication of neglecting aggregation when optimality conditions drive the allocation of some goods. In Section 5, the properties of optimized and indirect objective functions are presented. An empirical investigation is presented in Sections 6–8.

2. FROM ELEMENTARY TO AGGREGATE GOODS

Aggregation theory involves several concepts: elementary and aggregate goods, and microeconomic and aggregate theory. The meaning given to these concepts varies across contributions, so that it appears useful to state some generally accepted points before describing some approaches to the aggregation problem.

2.1. *The Microeconomic Framework.* Italic letters are chosen to denote elementary goods and microeconomic relationships while bold letters are used for aggregate goods and relationships. Let $x \in \mathbb{R}^{S_x}$ and $z \in \mathbb{R}^{S_z}$ be the vectors of elementary goods, where S_x and S_z denote the dimensions of the vectors x and z. By convention, any positive components of x and z correspond to a net supply and negative ones to a net demand. Furthermore, x is a vector of choice variables and z a vector of fixed goods.

Let $f: \mathbb{R}^{S_x} \times \mathbb{R}^{S_z} \to \mathbb{R}$ be a continuously differentiable transformation (or utility) function of an optimizing agent. In accordance with all these conventions, the transformation (or utility) function f must be strictly decreasing in x and z to match the standard economic theory. Besides, f is also strictly quasi-concave in x. In the present case, the functional form of f is assumed to be known and parameterized by the vector of technological characteristics $\alpha \in \mathbb{R}^{S_\alpha}$, parameters whose estimation and interpretation represent the purposes of empirical studies. In this article, aggregation of the vector of elementary parameters α will be considered along with the aggregation of elementary goods.

Contrary to the transformation function f, the precise functional form of the profits $\pi(p, x)$ is known and given by $p'x$, where p denotes the exogenous price vector. Two mirrored optimization problems (see Newman, 1982) are discussed below. The first one is given by

$$\max_x \{p'x: f(x, z; \alpha) \geq 0\} \tag{1}$$

The mirrored problem is obtained by inverting the role of the former objective and constraint:

$$\max_x \{f(x, z; \alpha): p'x \geq b\} \tag{2}$$

where $b \in \mathbb{R}$ corresponds to the exogenous part of the profit (budget). This last optimization problem is more usual in consumer analysis, whereas the former is common in production economics.

For both problems, the microeconomic optimality conditions (for an interior solution) can be written as $-\lambda\partial f/\partial x = p$, which should be completed by either $f(x, z; \alpha) = 0$ for (1) or $p'x = b$ for (2); the parameter $\lambda \geq 0$ denotes the Lagrange multiplier. Without loss of generality, the analysis below can therefore be restricted to problem (1).

2.2. *Elementary Goods and Aggregates.* For simplification, the aggregation approach considered in this article is based on real valued aggregators for describing a set of quantities and prices. The elementary goods z are not subject to aggregation. Let $\mathscr{J} = \{\mathscr{J}_1, \mathscr{J}_2, \ldots, \mathscr{J}_J\}$ be a partition of the set of integers indexing the elementary goods into J subsets. The vector of goods x is decomposed according to this partition

$$x = (x_1', \ldots, x_j', \ldots, x_J')' = (x_{11}, \ldots, x_{1S_{x_1}}, \ldots, x_{J1}, \ldots, x_{JS_{x_J}})'$$

where S_{x_j} denotes the dimensions of the vectors x_j. For each subset, the aggregates $\mathbf{x}_j$ are defined as $\mathbf{x}_j = a_{x_j}(x_j; \gamma_{x_j})$, for $j = 1, \ldots, J$; the aggregate prices for the goods belonging to $\mathscr{J}_j$ are given by $\mathbf{p}_j = a_{p_j}(p_j; \gamma_{p_j})$. The aggregator functions a_{x_j} and a_{p_j} are real valued functions, increasing in x_j and p_j, respectively, and twice continuously differentiable. As it will be seen below, the specification of the aggregators and the parameters γ_{x_j} differs in the literature.

In empirical work, the aggregates correspond frequently to sums, that is, $\mathbf{x}_j \equiv \sum_{h \in \mathscr{J}_j} x_{jh}$, or to weighted sums, that is, $\mathbf{x}_j \equiv \sum_{h \in \mathscr{J}_j} \gamma_{jh} x_{jh}$ with $\gamma_{jh} = p_{jh}/\mathbf{p}_j$ or $\gamma_{jh} = p_{jh}^0 / \left(\sum_{h \in \mathscr{J}_j} p_{jh}^0 x_{jh}^0\right)$, and where $\mathbf{p}_j$ is an aggregate price for group $\mathscr{J}_j$. When aggregate prices $\mathbf{p}_j$ are not available at the level of individual units, they are often replaced by a price level $\mathbf{P}_j$ aggregated across several decision units. Vectors of aggregates are denoted by $\mathbf{x}$ and $\mathbf{p}$.

2.3. *The Aggregation Problems.* Two kinds of aggregation problems are discussed in this article. The first one studies the conditions under which $f(x, z; \alpha)$ and $p'x$ can be described on the basis of aggregate goods and prices $\mathbf{x}$ and $\mathbf{p}$, but without considering that some goods may be optimally allocated. The second kind of problems considered studies whether the elementary optimization problem can be equivalently expressed in terms of aggregates.

Out of the optimum, the aggregation across elementary goods usually requires that the following criteria be fulfilled:

$$f(x, z; \alpha) = \mathbf{f}(\mathbf{x}, z; \boldsymbol{\alpha}) \tag{3}$$

and that

$$p'x = \mathbf{p}'\mathbf{x} \tag{4}$$

Since the right-hand sides of Equations (3) and (4) depend on aggregates, this must also be the case for the left-hand sides. This can only be achieved if the elementary variables x and p depend on the aggregates. This dependence must not be strictly deterministic but may be stochastic (and be defined for instance by $\mathbf{f}(\mathbf{x}, z; \boldsymbol{\alpha}) = \mathrm{E}[f(x, z; \alpha) \mid \mathbf{x}] + \epsilon$, where E denotes the expectation operator relative to the distribution of $x \mid \mathbf{x}$). The available tools for achieving the aggregate representations are (i) the specification of the microeconomic and aggregate functions f and $\mathbf{f}$

and (ii) the definition of the aggregate variables and parameters $\boldsymbol{\alpha}$ for given microeconomic relationships. In this article, the forms of the aggregate goods are mainly given by those usually computed by statistical offices.

The second kind of aggregation problems considered here studies whether the elementary optimization problem can be equivalently expressed in terms of aggregates as

$$\max_{\mathbf{x}}\{\mathbf{p}'\mathbf{x}: \mathbf{f}(\mathbf{x}, z; \boldsymbol{\alpha}) \geq 0\} = \max_{x}\{p'x: f(x, z; \alpha) \geq 0\} \tag{5}$$
$$\Leftrightarrow \boldsymbol{\pi}^*(\mathbf{p}, z; \boldsymbol{\alpha}) = \pi^*(p, z; \alpha)$$

Again, the specification of the microeconomic relationships f and $p'x$ and the definition of the parameters $\boldsymbol{\alpha}$ are the available tools for achieving the above aggregation criteria.

For solving these problems, the exact aggregation approach studies which conditions on the microeconomic relationships f and $p'x$ make the above representations possible (see Blackorby et al., 1978). When aggregation is neglected, the analysis begins with a function $\mathbf{f}(\mathbf{x}, z; \boldsymbol{\alpha})$ and assumes that the different aggregation criteria described above are fulfilled. In this case, for given microeconomic relationships f and given aggregates $\mathbf{p}$ and $\mathbf{x}$, the only remaining possibility for fulfilling (3)–(5) is to allow the aggregate parameters $\boldsymbol{\alpha}$ to become functions of the elementary goods x and z.

3. NEGLECTING AGGREGATION IN f AND π OUT OF OPTIMALITY

Up to this point, the aggregate parameter vector $\boldsymbol{\alpha}$ has not been specified. Therefore, it can take any elementary goods as components, and an aggregate representation of the problem is always possible through a convenient reparameterization of the initial problem. By using, for example, a transformation suggested by Lewbel (1992), I rewrite:[2]

$$f(x, z; \alpha) = f\left(x_1 \frac{\mathbf{x}_1}{\mathbf{x}_1}, \ldots, x_J \frac{\mathbf{x}_J}{\mathbf{x}_J}, z; \alpha\right)$$

Let us define vectors of the shares of the elementary goods and prices in the aggregate as $\rho_{x_j} = \left(x_{j1}/\mathbf{x}_j, \ldots, x_{jS_{s_j}}/\mathbf{x}_j\right)'$ and $\rho_{p_j} = \left(p_{j1}/\mathbf{p}_j, \ldots, p_{jS_{x_j}}/\mathbf{p}_j\right)'$, and denote with ρ_x and ρ_p the corresponding vectors with, respectively, ρ_{x_j} and ρ_{p_j} as subvectors. Further, form the vectors $\tilde{\mathbf{x}}$ and $\tilde{\mathbf{p}}$ according to the following pattern for $\tilde{\mathbf{x}}$:

$$\tilde{\mathbf{x}} = (\underbrace{\mathbf{x}_1, \ldots, \mathbf{x}_1}_{S_{x_1}\text{ terms}}, \mathbf{x}_2, \ldots, \underbrace{\mathbf{x}_j, \ldots, \mathbf{x}_j}_{S_{x_j}\text{ terms}}, \ldots, \mathbf{x}_J)'$$

Then, the elementary relationships can be written on the basis of available information on the aggregates:

$$f(x, z; \alpha) = f(\tilde{\mathbf{x}} * \rho_x, z; \alpha) = \mathbf{f}^a(\mathbf{x}, z; \alpha, \rho_x) \tag{6}$$

[2] In fact, Lewbel considers aggregation across decision units.

where the sign $*$ represents the Hadamard product.[3] The superscript a of $\mathbf{f}^a(\mathbf{x}, z; \alpha, \rho_x)$ is introduced for denoting the aggregated transformation (or utility) function, which is parameterized by the technological parameters α and by the distributions ρ_x of elementary goods. When data on ρ_x are available, comparative statics can be carried out on the basis of function $\mathbf{f}^a$, both with respect to $\mathbf{x}$ for given ρ_x, or with respect to ρ_x for given levels of $\mathbf{x}$. When only data on $\mathbf{x}$ are used for the estimation of $\mathbf{f}^a(\mathbf{x}, z; \alpha, \rho_x)$, the aggregate parameters $\boldsymbol{\alpha}_j$ are implicitly defined as a function of α and ρ_x:

$$f(x, z; \alpha) = \mathbf{f}(\mathbf{x}, z; a_\alpha(\alpha; \rho_x)) = \mathbf{f}(\mathbf{x}, z; \boldsymbol{\alpha}) \tag{7}$$

Remark that the above representation, although based on no restrictions on the microeconomic function f, is exact, that is to say, is true for any value of the elementary goods x and z. However, when the aggregate parameter vector $\boldsymbol{\alpha}$ is assumed to be constant (in empirical investigations $\boldsymbol{\alpha}$ is estimated by a constant parameter vector $\hat{\boldsymbol{\alpha}}$), the aggregate representation will only be *approximate*. Therefore, the empirical method for parameter determination will in the last instance proceed to aggregation by expressing the initial information in a more condensed manner.

Thus, it follows from (7) that approaches neglecting aggregation and starting directly with a function $\mathbf{f}(\mathbf{x}, z; \boldsymbol{\alpha})$ also admit some foundations in the theory of aggregation. The aggregation criterion (7) is the opposite extreme of the weak separability approach: now aggregate goods depend solely on elementary goods, but the aggregate parameters entailed in vector $\boldsymbol{\alpha}$ are complex functions of the elementary parameters and of the shares of elementary goods. Such a transformation is clearly possible for any functional form f. In order to better underline the implications of such a transformation, let us consider two examples.

EXAMPLE 1. A Cobb–Douglas production function can be approximately aggregated through the following transformations:

$$\begin{aligned}
f(x, z; \alpha) \geq 0 &\Leftrightarrow \prod_{j=1}^{J} \prod_{h=1}^{S_{x_j}} (-x_{jh})^{\alpha_{jh}} - z \geq 0 \\
&\Leftrightarrow \prod_{j=1}^{J} \prod_{h=1}^{S_{x_j}} \left(-\frac{x_{jh}}{\mathbf{x}_j}\mathbf{x}_j\right)^{\alpha_{jh}} - z \geq 0 \\
&\Leftrightarrow \prod_{j=1}^{J} (-\mathbf{x}_j)^{\boldsymbol{\alpha}_j} \prod_{h=1}^{S_{x_j}} \left(\frac{x_{jh}}{\mathbf{x}_j}\right)^{\alpha_{jh}} - z \geq 0 \\
&\Leftrightarrow \boldsymbol{\alpha}_0 \prod_{j=1}^{J} (-\mathbf{x}_j)^{\boldsymbol{\alpha}_j} - z \geq 0 \\
&\Leftrightarrow \mathbf{f}(\mathbf{x}, z; \boldsymbol{\alpha}) \geq 0
\end{aligned}$$

[3] The Hadamard product between two vectors u and v of the same dimension $u * v$ gives a vector of the same dimension again and with $u_i v_i$ as components.

where $x_{jh} \leq 0$ is the input and $z \geq 0$ is a scalar output. In this example, the aggregate parameters are defined as $\boldsymbol{\alpha}_0 = \prod_h \left(x_{jh}/\mathbf{x}_j\right)^{\alpha_{jh}}$, $\boldsymbol{\alpha}_j = \sum_h \alpha_{jh}$, for $j \neq 0$ and $\boldsymbol{\alpha} = (\boldsymbol{\alpha}_0, \boldsymbol{\alpha}_1, \ldots, \boldsymbol{\alpha}_J)$.

Two remarks are worth being made. First, the aggregate inputs $\mathbf{x}_j$ can, among other possibilities, be chosen as those published by statistical offices. The exact aggregation approach would require aggregate inputs to be defined as

$$\mathbf{x}_j \equiv -\prod_{h=1}^{S_{x_j}} \left(-x_{jh}\right)^{\alpha_{jh}/\boldsymbol{\alpha}_j} \tag{8}$$

In this case the term $\prod_{h=1}^{S_{x_j}} \left(x_{jh}/\mathbf{x}_j\right)^{\alpha_{jh}}$ equals one and the aggregate function $\mathbf{f}$ does not depend on ρ_x. However, aggregates defined as in (8) may not be very easy to obtain from statistical offices. Second, the aggregate parameters have different meanings from disaggregate parameters. Even when each production unit has the same technology (i.e., α_{jh} is identical for each production unit, $\forall j, h$), the aggregate parameter $\boldsymbol{\alpha}_0$ can differ across production units because of different combinations of elementary goods used. Then, differences in $\boldsymbol{\alpha}_0$ do not convey differences in technologies alone. Studies aimed at measuring technical inefficiencies by analyzing how $\boldsymbol{\alpha}_0$ varies across production units are also implicitly testing the significance of aggregation over goods (see, e.g., Cornwell and Schmidt, 1996 for a survey of this literature). Under Lewbel's (1992) mean scaling assumption, $\boldsymbol{\alpha}_0$ is independent of $\mathbf{x}$. This assumption can in some cases be tested without any data on elementary goods: with panel data, a Hausman-type test could be used for this aim. If the independence assumption is not satisfied, the choice of an adequate method for the estimation of $\boldsymbol{\alpha}$ is required. Thus, econometric tools can be useful to handle some aggregation problems.

EXAMPLE 2.[4] The Cobb–Douglas production function already satisfies the requirement of exact aggregation for any partition of the goods and may therefore not appear persuasive. The production function

$$f(x, z; \alpha) = (-x_{11})^{\alpha_{11}} + (-x_{12})^{\alpha_{12}}(-x_{21})^{\alpha_{21}} - z \geq 0$$

which is not weakly separable in the partition $\mathcal{J} \equiv \{\{11, 12\}, \{21\}\}$, can, however, be written as

$$\mathbf{f}(\mathbf{x}, z; \boldsymbol{\alpha}) = \boldsymbol{\alpha}_1(-\mathbf{x}_1)^{\alpha_{11}} + \boldsymbol{\alpha}_2(-\mathbf{x}_1)^{\alpha_{12}}(-\mathbf{x}_2)^{\alpha_{21}} - z \geq 0$$

where $\mathbf{x}_2 \equiv x_{21}$, $\boldsymbol{\alpha}_1 \equiv (x_{11}/\mathbf{x}_1)^{\alpha_{11}}$, $\boldsymbol{\alpha}_2 \equiv (x_{12}/\mathbf{x}_1)^{\alpha_{12}}$ and $\boldsymbol{\alpha} \equiv (\boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \alpha_{11}, \alpha_{12}, \alpha_{21})$. Here, the gain of going from three inputs to two inputs may be offset by the cost of going from three parameters to five.

These examples illustrate that there is an identification problem precluding the interpretation of the parameters $\boldsymbol{\alpha}$ as reflecting strictly individual parameters. Thus, a similar difficulty to the one arising in the context of aggregation across agents (see, e.g., Stoker, 1984) also characterizes aggregation across goods. The marginal impact of the aggregate variable can be decomposed into a direct and an indirect

[4] I am indebted to an anonymous referee for this example.

shift that cannot be identified independently when the function $\mathbf{f}(\mathbf{x}, z; \boldsymbol{\alpha})$ is retained. Indeed, since the aggregate representation only makes sense when elementary goods depend on the aggregate, ρ_x will also be shifted when $\mathbf{x}_j$ varies (hence the notation $\bar{\rho}_x$):

$$\frac{\partial \mathbf{f}}{\partial \mathbf{x}_j} = \frac{\mathrm{d}\mathbf{f}^a}{\mathrm{d}\mathbf{x}_j} = \frac{\partial \mathbf{f}^a}{\partial \mathbf{x}_j} + \frac{\partial \mathbf{f}^a}{\partial \rho'_x}\frac{\partial \bar{\rho}_x}{\partial \mathbf{x}_j} \tag{9}$$

where the last term represents the shift in the distribution of the elementary goods $\partial \bar{\rho}_x / \partial \mathbf{x}_j$ consecutively to a variation in the aggregate. When information on elementary goods (and thus on ρ_x) is available, comparative static can be carried out for given ρ_x, and thus $\partial \mathbf{f}^a / \partial \mathbf{x}_j$ can be identified. However, when $\mathbf{f}(\mathbf{x}, z; \boldsymbol{\alpha})$ is retained, this is no longer the case.

Why is the distributional impact $\partial \mathbf{f}^a / \partial \rho'_x [\partial \bar{\rho}_x / \partial \mathbf{x}_j]$ on $\partial \mathbf{f} / \partial \mathbf{x}_j$ in (9) bothersome? For answering this question, let us remark that the distributional impact vanishes in two extreme cases. The assumption of weak separability would imply that $\partial \mathbf{f}^a / \partial \rho_x = 0$ for any value of x.[5] Strict (or approximate) proportionality of all elementary goods of a given partition with the corresponding aggregate would imply $\partial \bar{\rho}_x / \partial \mathbf{x}_j = 0$. Since the distributional impact in (9) disappears when aggregation across goods is possible, it can be seen as an aggregation bias. Besides, the expression $\partial \mathbf{f}^a / \partial \rho'_x [\partial \bar{\rho}_x / \partial \mathbf{x}_j]$ also entails shifts of *noncorresponding* variables. A marginal variation of $\mathbf{x}_j$ does not only affect the allocation of the goods x_j forming $\mathbf{x}_j$ but may also affect the way other goods x_h, $h \neq j$ are allocated. This point also applies to the marginal impact of z_k:

$$\frac{\partial \mathbf{f}}{\partial z_k} = \frac{\mathrm{d}\mathbf{f}^a}{\mathrm{d}z_k} = \frac{\partial \mathbf{f}^a}{\partial z_k} + \frac{\partial \mathbf{f}^a}{\partial \rho'_x}\frac{\partial \bar{\rho}_x}{\partial z_k}$$

Since this noncorresponding impact is not appealing from an economic point of view, I follow Boot and de Wit (1960) and define it as an aggregation bias.

A similar transformation can also be applied to derive aggregate profits from $p'x$:

$$p'x = \left(\tilde{\mathbf{p}} * \rho_p\right)'(\tilde{\mathbf{x}} * \rho_x) \equiv \boldsymbol{\pi}^a\left(\mathbf{p}, \mathbf{x}; \rho_x, \rho_p\right). \tag{10}$$

A more stringent aggregative criterion than (10) is often required: the product of the aggregate price and quantity should equal the costs (or revenues) of all elementary goods; that is, $p'_j x_j = \mathbf{p}_j \mathbf{x}_j$. If price aggregators for the subset $\mathscr{J}_j$ are allowed to depend on elementary quantities of this subset (in which case, a_{p_j} has also x_j among its components), this last equality can always be achieved by defining $\mathbf{p}_j$ as the *unit value* of the composite good $\mathbf{x}_j$; that is, $\mathbf{p}_j \equiv \sum_h p_{jh} x_{jh} / \mathbf{x}_j$.[6] In this article, aggregate prices are defined as unit prices in order to fulfill (4), but the analysis is easily extended to the less appealing aggregative criteria (10).

[5] Under weak separability, $f(x, z; \alpha)$ can be written as $\mathbf{f}^{ws}(\mathbf{x}, z, \bar{\alpha})$ where $\bar{\alpha}$ is a subvector of α. This aggregate function $\mathbf{f}^{ws}$ is not parameterized by the distribution of elementary goods in the aggregate.

[6] In this case, the parameters γ_{p_j} in $a_{p_j}(p_j; \gamma_{p_j})$ should be identified with $x_j / \mathbf{x}_j$. Reciprocally, aggregate quantities could be defined as $\mathbf{x}_j = \sum_h p_{jh} x_{jh} / \mathbf{p}_j$.

Thus, profits can also be represented in an aggregate way but will be subject to similar inconveniences as those occurring in the transformation function. Indeed, the marginal profit of good $\mathbf{x}_j$ is not equal to its unit price in general. When $\boldsymbol{\pi}^a = \mathbf{p}'\mathbf{x}$, for example,

$$\frac{d\boldsymbol{\pi}^a}{d\mathbf{x}_j} = \mathbf{p}_j + \frac{d\mathbf{p}'}{d\mathbf{x}_j}\mathbf{x}$$

To summarize, in this section it has been seen that an aggregate representation of the initial relationships is possible without restricting microeconomic behavior, by adequately defining aggregate quantities, prices, and parameters. As discussed above, many inconveniences may be related to this practice. In the next section I will analyze what happens when some goods are optimally allocated.

4. FIRST-ORDER OPTIMALITY CONDITIONS WHEN AGGREGATION IS NEGLECTED

Both aggregate functions $\mathbf{f}^a$ and $\boldsymbol{\pi}^a$ depend on the distribution of elementary goods and prices. In such a context, it seems interesting to see under which conditions the regularity properties satisfied by the microeconomic functions are inherited in the aggregate. Can the microeconomic optimization framework be adopted for aggregate functions? Which properties do the aggregate optimized relationships satisfy? In the case where the aggregative criteria (3) and (4) are considered, $\mathbf{f}(\mathbf{x}, z; \boldsymbol{\alpha})$ fulfills the same properties as $\mathbf{f}^a(\bar{\mathbf{x}}, z; \alpha, \bar{\rho}_x)$. For this reason, the properties of $\mathbf{f}^a$ are studied now, in a context where some goods are optimally adjusted.

Using transformations (6) and (10), the initial maximization problem can be expressed as

$$\pi^*(p, z; \alpha) = \max_x \{p'x\colon\ f(x, z; \alpha) \geq 0\} \tag{11}$$

$$= \max_{\bar{\mathbf{x}}} \left\{ \max_{\rho_x} \left\{ \sum_{j=1}^{J} p_j' \rho_{xj} \mathbf{x}_j\colon\ \mathbf{f}^a(\bar{\mathbf{x}}, z; \alpha, \rho_x) \geq 0 \wedge a_{x_j}(x_j) = \bar{\mathbf{x}}_j,\ j = 1, \ldots, J \right\} \right\} \tag{12}$$

$$= \max_{\bar{\mathbf{x}}} \left\{ \sum_{j=1}^{J} p_j' \bar{\rho}_{xj} \bar{\mathbf{x}}_j\colon\ \mathbf{f}^a(\bar{\mathbf{x}}, z; \alpha, \bar{\rho}_x) \geq 0 \right\} \tag{13}$$

Thus, the initial optimization problem can be written equivalently in the form of a two-stage optimization problem. In the second stage, the optimal levels of the elementary goods are chosen for the given value $\bar{\mathbf{x}}_j$ of the aggregates; in the first stage, the optimal levels of the aggregate goods (denoted by $\mathbf{x}_j^a$) are chosen given the aggregate prices and the conditional levels of the elementary goods chosen in the second stage.[7] The solution of the second stage gives the optimal choices of the

[7] Although the inside optimization problem is solved first in (12), it is called the second stage of optimization, in order to be consistent with the literature on two-stage budgeting.

shares of elementary goods in the aggregates as $\bar{\rho}_x(p, z, \bar{\mathbf{x}}; \alpha)$ and depends explicitly on the level $\bar{\mathbf{x}}$ of the aggregate goods.[8] The corresponding optimal levels for the restricted elementary goods are given by $\bar{x}_{jh}(p, z, \bar{\mathbf{x}}; \alpha)$.

Thus, changes in the aggregate $\bar{\mathbf{x}}_j$ have a direct impact on the aggregate objective and an indirect impact through changes in the way elementary goods and prices are distributed. Indeed, the first-order conditions of the above optimization problem (13) are given by

$$\sum_{h \in \mathscr{J}_j} p_{jh} \frac{x_{jh}}{\bar{\mathbf{x}}_j} + \sum_{h=1}^{J} p'_h \frac{\partial \bar{\rho}_{xh}}{\partial \bar{\mathbf{x}}_j} \bar{\mathbf{x}}_j = -\lambda \frac{\partial \mathbf{f}^a}{\partial \bar{\mathbf{x}}_j} - \lambda \frac{\partial \mathbf{f}^a}{\partial \rho'_x} \frac{\partial \bar{\rho}_x}{\partial \bar{\mathbf{x}}_j} \tag{14}$$

where λ denotes the Lagrange multiplier.

When only aggregate data on expenditures, prices, and goods are available, it seems impossible to model the distribution effects occurring in (14) explicitly. Exact aggregation provides conditions under which these effects do not emerge. Indeed, when the objective f is weakly separable in $\mathscr{J}$, ρ_x does not appear as an argument of the aggregate transformation function. Under weak homogeneous separability of f in $\mathscr{J}$, the term $\partial \bar{\rho}_x / \partial \mathbf{x}_j$ vanishes on both sides of (14) and the first-order condition for optimality becomes:[9]

$$\mathbf{p}_j \equiv \sum_{h \in \mathscr{J}_j} p_{jh} \frac{x_{jh}}{\bar{\mathbf{x}}_j} = -\lambda \frac{\partial \mathbf{f}^{\text{whs}}(\mathbf{x}, z; \boldsymbol{\alpha})}{\partial \bar{\mathbf{x}}_j}$$

where the superscript whs denotes a function being weakly homogeneously separable in $\mathscr{J}$. In this case, the way elementary goods x_{jh} are chosen affects the first-order conditions only through the aggregate quantity $\mathbf{x}_j$ and price $\mathbf{p}_j$, which can then be seen as sufficient statistics for modeling aggregate first-order conditions.

However, nothing ensures that the transformation function satisfies such restrictions and that the first-order condition can be simplified as described above. What do the first-order conditions look like when the weak homogeneous separability restriction does not hold? How should the aggregate prices arising in the definition of aggregate profits (10) be specified? In order to find the aggregate price corresponding to marginal impact of $\mathbf{x}_j$, let us write Equation (14) in terms of elementary goods and prices making use of (6):

$$\sum_{h \in \mathscr{J}_j} \frac{p_{jh} x_{jh}}{\bar{\mathbf{x}}_j} + p' \left(\tilde{\mathbf{x}} * \frac{\partial \bar{\rho}_x}{\partial \bar{\mathbf{x}}_j} \right) = -\lambda \frac{\partial f(x, z; \alpha)}{\partial x'} \left(\frac{\partial \tilde{\mathbf{x}}}{\partial \bar{\mathbf{x}}_j} * \bar{\rho}_x + \tilde{\mathbf{x}} * \frac{\partial \bar{\rho}_x}{\partial \bar{\mathbf{x}}_j} \right) \tag{15}$$

The impact of a variation in aggregate quantity $\bar{\mathbf{x}}_j$ is decomposed into two effects in (15). A first effect conveys the impact of a marginal change in the aggregates, the distribution of the elementary goods remaining unchanged, and a second effect is implied by the shifts in the distribution of elementary goods following a change in

[8] The "over bars" are introduced for denoting fixed levels for aggregate goods $\bar{\mathbf{x}}_j$ and for characterizing the optimal levels of elementary goods $\bar{x}_j$ and optimal shares of elementary goods $\bar{\rho}_x$, for given levels of the aggregate $\bar{\mathbf{x}}_j$.

[9] See, for example, Koebel (1998) for a proof of this assertion.

the aggregate $\bar{\mathbf{x}}_j$. The left-hand side of (15) represents the marginal relative price of a change in the aggregate $\bar{\mathbf{x}}_j$. Using the equality

$$p'\left(\tilde{\mathbf{x}} * \frac{\partial \bar{\rho}_x}{\partial \bar{\mathbf{x}}_j}\right) = p'\frac{\partial \bar{x}}{\partial \bar{\mathbf{x}}_j} - \sum_{h \in \mathscr{J}_j} p_{jh}\frac{\bar{x}_{jh}}{\bar{\mathbf{x}}_j}$$

the left-hand side of (15) can be rewritten $p'\partial\bar{x}/\partial\bar{\mathbf{x}}_j$. This result tells us that without weak separability and additive aggregation assumptions, the marginal value of the good $\bar{\mathbf{x}}_j$ corresponds to the weighted sum of elementary prices, with $\partial\bar{x}_j/\partial\bar{\mathbf{x}}_j$ as weights. Thus, the aggregate price conforming with economic theory should be defined as $p'\partial\bar{x}/\partial\bar{\mathbf{x}}_j$, which is not commonly available from statistical offices.[10] Notice that when the weak homogeneous separability is satisfied, $p'\partial\bar{x}/\partial\bar{\mathbf{x}}_j = p_j'\bar{x}_j/\bar{\mathbf{x}}_j$ and unit values $\mathbf{p}_j \equiv p_j'\bar{x}_j/\bar{\mathbf{x}}_j$ can be retained for modeling first-order conditions.

Is there no other possibility for representing aggregate first-order conditions than to make strong restrictions on $f(x, z; \alpha)$ or to retain "monsters" as aggregate prices? By using the optimality conditions for the elementary goods, the impact of the change in the distribution of elementary goods related to a change in $\mathbf{x}_j$ can be netted out from the left and right sides of (15), which can be written as

$$\sum_{h \in \mathscr{J}_j} p_{jh}\frac{x_{jh}}{\bar{\mathbf{x}}_j} = -\lambda \sum_{h \in \mathscr{J}_i} \frac{\partial f(x, z; \alpha)}{\partial x_{jh}}\frac{x_{jh}}{\bar{\mathbf{x}}_j} \tag{16}$$

Thus, defining unit values as $\mathbf{p}_j \equiv p_j'x_j/\bar{\mathbf{x}}_j$, Equation (16) can be used for estimating the parameters of $\mathbf{f}^a(\mathbf{x}, z; \alpha, \rho_x)$. The system (16) can be retained for the estimation, even when weak homogeneous separability (or additive price aggregation) is not satisfied. Briefly, it has been shown that the aggregate marginal conditions (14) can be modeled indirectly by (16), as it is usually done in studies neglecting completely the problem of aggregation. However, the function $\mathbf{f}^a$ will not necessarily fulfill the second-order conditions for a maximum with respect to $\mathbf{x}$. These results are summarized in the following proposition.

Proposition 1

(i) *If* x^* *is a solution to the disaggregate optimization problem* (11), *then* $\mathbf{x}^a = a_x(x^*)$ *is a solution to the aggregate optimization problem* (13).

(ii) *The first-order optimality conditions of problem* (13) *can be represented by* $-\lambda\partial\mathbf{f}^a/\partial\bar{\mathbf{x}}_j = \mathbf{p}_j$ *where* $\mathbf{p}_j \equiv p_j'x_j/\mathbf{x}_j$ *and by* $\mathbf{f}^a(\bar{\mathbf{x}}, z; \alpha, \bar{\rho}_x) = 0$.

(iii) *The function* $\mathbf{f}^a$ *is quasi-concave in* $\mathbf{x}$ *if and only if for all vector* $v \perp \mathrm{d}\mathbf{f}^a/\mathrm{d}\bar{\mathbf{x}}$,

$$v'\left(\frac{\partial^2\mathbf{f}^a}{\partial\bar{\mathbf{x}}\,\partial\bar{\mathbf{x}}'} + \frac{\partial\bar{\rho}_x'}{\partial\mathbf{x}}\frac{\partial^2\mathbf{f}^a}{\partial\rho_x\,\partial\rho_x'}\frac{\partial\bar{\rho}_x}{\partial\bar{\mathbf{x}}'}\right)v \leq -v'\left(\frac{\partial^2\mathbf{f}^a}{\partial\bar{\mathbf{x}}\,\partial\rho_x'}\frac{\partial\bar{\rho}_x}{\partial\bar{\mathbf{x}}'} + \frac{\partial\bar{\rho}_x'}{\partial\bar{\mathbf{x}}}\frac{\partial^2\mathbf{f}^a}{\partial\rho_x\,\partial\bar{\mathbf{x}}'} + \left[\frac{\partial\mathbf{f}^a}{\partial\rho_x'}\frac{\partial\bar{\rho}_x}{\partial\bar{\mathbf{x}}_j\,\partial\bar{\mathbf{x}}_k}\right]\right)v \tag{17}$$

[10] For the aggregate price $\mathbf{p}_j^v = p'\partial\bar{x}^p/\partial\mathbf{x}_j$, the product $\mathbf{p}_j^v\mathbf{x}_j$ will generally be different from the profit $p_j'x_j$.

PROOF. Point (i) follows from (13). If x^* is a solution of $\max_x\{p'x\colon f(x, z; \alpha) \geq 0\}$ then $\mathbf{x}^a = a_x(x^*)$ is solution of $\max_{\bar{\mathbf{x}}}\{\mathbf{p}'\bar{\mathbf{x}}\colon \mathbf{f}^a(\bar{\mathbf{x}}, z; \alpha, \bar{\rho}_x) \geq 0\}$, where $\bar{\mathbf{x}} = a_x(\bar{x})$. Point (ii) follows from (16) and from the fact that $\partial\mathbf{f}^a/\partial\bar{\mathbf{x}}_j = \partial f/\partial x_j'\big(x_j/\bar{\mathbf{x}}_j\big)$. Point (iii): from point (i), it follows that the Hessian matrix $\mathrm{d}^2\mathscr{L}/\mathrm{d}\bar{\mathbf{x}}\,\mathrm{d}\bar{\mathbf{x}}'$ of the Lagrangian $\mathscr{L} = \mathbf{p}'\bar{\mathbf{x}} + \lambda\mathbf{f}^a(\bar{\mathbf{x}}, z; \alpha, \bar{\rho}_x)$ is negative semidefinite on the hyperplane orthogonal to $\mathrm{d}\mathbf{f}^a/\mathrm{d}\bar{\mathbf{x}}$. The expression of this Hessian matrix is given by

$$\frac{\mathrm{d}^2\mathscr{L}}{\mathrm{d}\bar{\mathbf{x}}\,\mathrm{d}\bar{\mathbf{x}}'} = \frac{\mathrm{d}^2(\mathbf{p}'\bar{x})}{\mathrm{d}\bar{\mathbf{x}}\,\mathrm{d}\bar{\mathbf{x}}'} + \lambda\frac{\mathrm{d}^2\mathbf{f}^a}{\mathrm{d}\bar{\mathbf{x}}\,\mathrm{d}\bar{\mathbf{x}}'} = \frac{\mathrm{d}\mathbf{p}'}{\mathrm{d}\bar{\mathbf{x}}} + \frac{\mathrm{d}\mathbf{p}}{\mathrm{d}\bar{\mathbf{x}}'} + \left[(p * \bar{\mathbf{x}}')\frac{\partial\bar{\rho}_x}{\partial\bar{\mathbf{x}}_k\,\partial\bar{\mathbf{x}}_\ell}\right]$$
$$+ \lambda\left(\frac{\partial^2\mathbf{f}^a}{\partial\bar{\mathbf{x}}\,\partial\bar{\mathbf{x}}'} + \frac{\partial\bar{\rho}_x'}{\partial\bar{\mathbf{x}}}\frac{\partial^2\mathbf{f}^a}{\partial\rho_x\,\partial\rho_x'}\frac{\partial\bar{\rho}_x}{\partial\bar{\mathbf{x}}'} + \frac{\partial^2\mathbf{f}^a}{\partial\bar{\mathbf{x}}\,\partial\rho_x'}\frac{\partial\bar{\rho}_x}{\partial\bar{\mathbf{x}}'} + \frac{\partial\bar{\rho}_x'}{\partial\bar{\mathbf{x}}}\frac{\partial^2\mathbf{f}^a}{\partial\rho_x\,\partial\bar{\mathbf{x}}'} + \left[\frac{\partial\mathbf{f}^a}{\partial\rho_x'}\frac{\partial\bar{\rho}_x}{\partial\bar{\mathbf{x}}_k\,\partial\bar{\mathbf{x}}_\ell}\right]\right)$$

Using definitions (6), (10), and the first-order conditions for optimality, the above expression can be simplified to

$$\frac{\mathrm{d}^2\mathscr{L}}{\mathrm{d}\bar{\mathbf{x}}\,\mathrm{d}\bar{\mathbf{x}}'} = \lambda\left(\frac{\partial^2\mathbf{f}^a}{\partial\bar{\mathbf{x}}\,\partial\bar{\mathbf{x}}'} + \frac{\partial\bar{\rho}_x'}{\partial\bar{\mathbf{x}}}\frac{\partial^2\mathbf{f}^a}{\partial\rho_x\,\partial\rho_x'}\frac{\partial\bar{\rho}_x}{\partial\bar{\mathbf{x}}'}\right)$$

Therefore,

$$\frac{\mathrm{d}^2\mathbf{f}^a}{\mathrm{d}\bar{\mathbf{x}}\,\mathrm{d}\bar{\mathbf{x}}'} = \frac{1}{\lambda}\frac{\mathrm{d}^2\mathscr{L}}{\mathrm{d}\bar{\mathbf{x}}\,\mathrm{d}\bar{\mathbf{x}}'} + \frac{\partial^2\mathbf{f}^a}{\partial\bar{\mathbf{x}}\,\partial\rho_x'}\frac{\partial\bar{\rho}_x}{\partial\bar{\mathbf{x}}'} + \frac{\partial\bar{\rho}_x'}{\partial\bar{\mathbf{x}}}\frac{\partial^2\mathbf{f}^a}{\partial\rho_x\,\partial\bar{\mathbf{x}}'} + \left[\frac{\partial\mathbf{f}^a}{\partial\rho_x'}\frac{\partial\bar{\rho}_x}{\partial\bar{\mathbf{x}}_k\,\partial\bar{\mathbf{x}}_\ell}\right] \tag{18}$$

is negative semidefinite on the hyperplane $(\mathrm{d}\mathbf{f}^a/\mathrm{d}\mathbf{x})^\perp$ if and only if inequality (17) is satisfied. ■

Points (i) and (ii) of Proposition 1 give some hope for achieving aggregation by simply neglecting the problem. Indeed, these points mean that only information is lost through aggregation, but the validity of the microeconomic reasoning and theory is not contradicted by using aggregate goods and prices. The reciprocal of (i) is in general not satisfied: the knowledge of $\mathbf{x}^a$ does not allow to infer x^* (information on disaggregates is lost). Point (ii) also tells us that the aggregate marginal productivity (utility) $\partial\mathbf{f}^a/\partial\mathbf{x}_j$ is a weighted average of the elementary marginal productivities (utilities) of the goods within the subset $\mathscr{J}_j$. Contrary to the former results, point (iii) of Proposition 1 raises doubts about the microeconomic property of quasi-concavity to hold in the aggregate. Condition (17) may not appear very appealing. In fact, this result means that the distributive impact on the Hessian matrix of $\mathbf{f}^a$ should not destroy the negative semidefiniteness of the Hessian matrix of $\mathscr{L}$ in expression (18). Whether condition (17) is satisfied or not is an empirical question that cannot be answered in general.

Although the content of point (iii) appears somewhat pessimistic, note that the inequality (17) is satisfied when the conditions for an exact aggregate representation are fulfilled. Thus, condition (17) can be seen as weaker than those underlying the theories of exact aggregation.

COROLLARY 1. (i) *Under weak separability of f in the partition $\mathcal{J}$, $\mathbf{f}^a$ is quasi-concave in suitably defined aggregates* $\mathbf{x}$. (ii) *When the shares of elementary goods in the aggregate are independent of the aggregate goods*, $\mathbf{f}^a$ *is quasi-concave in* $\mathbf{x}$.

PROOF. If f is weakly separable in the partition $\mathcal{J}$, $\bar{\rho}_x$ does, indeed, no longer parameterize $\mathbf{f}^a$ and condition (17) boils down to $v'[\partial^2\mathbf{f}^a/\partial\bar{\mathbf{x}}\,\partial\bar{\mathbf{x}}']v \leq 0$, for any $v \perp \partial\mathbf{f}^a/\partial\bar{\mathbf{x}}$, a condition that is fulfilled since f is quasi-concave in x. The choice of the aggregates can, however, not be arbitrary, but is dictated by the exact aggregation approach (see Example 1 for an illustration). Under the condition of point (ii) in Corollary 1, $\partial\bar{\rho}_x/\partial\mathbf{x}'$ and $\partial^2\bar{\rho}_x/\partial\mathbf{x}\,\partial\mathbf{x}'$, and therefore the whole right-hand side of expression (17) is simply zero. ■

This corollary means that when weak separability is imposed on f or when the shares ρ_x of elementary quantities in the aggregate are independent of $\mathbf{x}$, the function $\mathbf{f}^a$ inherits the microeconomic properties with respect to aggregate goods.

COROLLARY 2. *For given ρ_x, the function $\mathbf{f}^a$ satisfies $-\lambda\partial\mathbf{f}^a/\partial\bar{\mathbf{x}}_j = \mathbf{p}_j$ (where $\mathbf{p}_j \equiv p_j'x_j/\mathbf{x}_j$) at the optimum and is quasi-concave in* $\mathbf{x}$.

Corollary 2 means that when the shares ρ_x are known (but not necessarily independent of $\mathbf{x}$ or constant), the usual "*ceteris paribus* device" can be applied for deriving the usual *microeconomic* properties for $\mathbf{f}^a$ with respect to *aggregates*. The main problem is that information on elementary goods and prices is usually not available, and therefore the properties of Proposition 1 rather than those of Corollary 2 apply to $\mathbf{f}(\mathbf{x}, z; \boldsymbol{\alpha})$.

It is, however, not often possible to derive the analytical expressions of the demand and supply functions from the first-order optimality solutions. Nevertheless, Proposition 1 may be of interest for its economic meaning. Moreover, in empirical work, first-order optimality conditions are often used for parameter estimation (in models with Euler equations, market power, efficiency wages, etc.). Instead of first-order conditions, the next section discusses aggregation within the profit function π^*.

5. NEGLECTING AGGREGATION IN OPTIMIZED RELATIONSHIPS

In this section, the conditions under which regularities of elementary optimal transformation and profit functions f^* and π^* are transmitted to $\mathbf{f}^*$ and $\boldsymbol{\pi}^*$ are studied. The aggregate profit function $\boldsymbol{\pi}^{*a}$ can be related to the elementary profit function π^* through the following transformations:

$$\pi^*(p, z; \alpha) = \pi^*\big(\tilde{\mathbf{p}} * \rho_p, z, \alpha\big) = \boldsymbol{\pi}^{*a}\big(\mathbf{p}, z; \alpha, \rho_p\big)$$

Note that the aggregate profit function is not parameterized by the same parameters as $\mathbf{f}^a$: ρ_x no longer appears in the expression of $\boldsymbol{\pi}^{*a}$, which depends instead on the elementary price ratios ρ_p. When ρ_p is not observed, the simplified function $\boldsymbol{\pi}^*$ depends on the variables $\mathbf{p}$, z, and $\boldsymbol{\alpha}$. How are $\boldsymbol{\pi}^*$ and $\boldsymbol{\pi}^{*a}$ related? As discussed above, the equality $\boldsymbol{\pi}^*(\mathbf{p}, z; \boldsymbol{\alpha}) = \boldsymbol{\pi}^{*a}(\mathbf{p}, z; \alpha, \rho_p)$ is always possible when the aggregate parameters $\boldsymbol{\alpha}$ are functions of elementary parameters α and relative prices ρ_p. Then, $\boldsymbol{\alpha}$ will not be constant unless elementary prices are strictly proportional to the aggregate prices. In econometric studies $\boldsymbol{\alpha}$ is often assumed to be constant and thus the equality $\boldsymbol{\pi}^* = \boldsymbol{\pi}^{*a}$ cannot hold exactly. I follow Lewbel (1993, 1996) and consider that $\boldsymbol{\pi}^*$ and $\boldsymbol{\pi}^{*a}$ are related by

$$\boldsymbol{\pi}^*(\mathbf{p}, z; \boldsymbol{\alpha}) = \mathrm{E}[\boldsymbol{\pi}^{*a}(\mathbf{p}, z; \alpha, \rho_p) \mid \mathbf{p}, z] = \boldsymbol{\pi}^{*a}(\mathbf{p}, z; \alpha, \bar{\rho}_p)$$

where $\boldsymbol{\alpha}$ is a vector of constant parameters and E denotes the expectation operator relative to the conditional distribution of ρ_p. This last equality expresses implicitly a stochastic dependence between ρ_p and $\mathbf{p}$ and z. Thereby, for determining the regularity properties of $\boldsymbol{\pi}^*$ one shall consider those of $\boldsymbol{\pi}^{*a}$. The microeconomic profit function π^* is linearly homogeneous and convex in prices. Since $\pi^*(p, z; \alpha) = \boldsymbol{\pi}^{*a}(\mathbf{p}, z; \alpha, \rho_p)$, the last function will also be linearly homogeneous and convex in p. But which properties does $\boldsymbol{\pi}^{*a}$ satisfy with respect to aggregate prices $\mathbf{p}$?

PROPOSITION 2.

(i) *The function* $\boldsymbol{\pi}^{*a}$ *is linearly homogeneous in* $\mathbf{p}$ *if and only if*

$$\frac{\partial \boldsymbol{\pi}^{*a}}{\partial \rho_p'} \frac{\partial \bar{\rho}_p}{\partial \mathbf{p}'} \mathbf{p} = 0 \tag{19}$$

(ii) *The aggregate version of Hotelling's lemma*, $\mathrm{d}\boldsymbol{\pi}^{*a}/\mathrm{d}\mathbf{p}_j = \mathbf{x}_j^{*a}$, *where* $\mathbf{x}_j^{*a} \equiv \sum_h p_{jh} x_{jh}^* / \mathbf{p}_j$, *is obtained if and only if*

$$\frac{\partial \boldsymbol{\pi}^{*a}}{\partial \rho_p'} \frac{\partial \bar{\rho}_p}{\partial \mathbf{p}_j'} = 0 \tag{20}$$

(iii) *The function* $\boldsymbol{\pi}^{*a}$ *is convex in* $\mathbf{p}$ *if and only if for all vector* v,

$$v'\left(\frac{\partial^2 \boldsymbol{\pi}^{*a}}{\partial \mathbf{p} \partial \mathbf{p}'} + \frac{\partial \bar{\rho}_p'}{\partial \mathbf{p}} \frac{\partial^2 \boldsymbol{\pi}^{*a}}{\partial \rho_p' \partial \rho_p} \frac{\partial \bar{\rho}_p}{\partial \mathbf{p}'}\right) v \geq -v'\left(\frac{\partial^2 \boldsymbol{\pi}^{*a}}{\partial \mathbf{p} \partial \rho_p'} \frac{\partial \bar{\rho}_p}{\partial \mathbf{p}'} + \frac{\partial \bar{\rho}_p'}{\partial \mathbf{p}} \frac{\partial^2 \boldsymbol{\pi}^{*a}}{\partial \mathbf{p}' \partial \rho_p} + \left[\frac{\partial \boldsymbol{\pi}^{*a}}{\partial \rho_p'} \frac{\partial^2 \bar{\rho}_p}{\partial \mathbf{p}_j \partial \mathbf{p}_k}\right]\right) v \tag{21}$$

PROOF.

(i) $\boldsymbol{\pi}^{*a}$ is linearly homogeneous in $\mathbf{p}$ if and only if

$$\boldsymbol{\pi}^{*a}(\kappa \mathbf{p}, z; \alpha, \bar{\rho}_p(\kappa \mathbf{p})) = \kappa \boldsymbol{\pi}^{*a}(\mathbf{p}, z; \alpha, \bar{\rho}_p)$$

where $\kappa \in \mathbb{R}$. Differentiation with respect to κ and evaluation at $\kappa = 1$ leads to

$$\frac{\partial \boldsymbol{\pi}^{*a}}{\partial \mathbf{p}'} \mathbf{p} + \frac{\partial \boldsymbol{\pi}^{*a}}{\partial \rho_p'} \frac{\partial \bar{\rho}_p}{\partial \mathbf{p}'} \mathbf{p} = \boldsymbol{\pi}^{*a}(\mathbf{p}, z; \alpha, \bar{\rho}_p)$$

As $\partial \boldsymbol{\pi}^{*a}/\partial \mathbf{p}' = \mathbf{x}^{*a\prime}$ and $\mathbf{p}' \partial \boldsymbol{\pi}^{*a}/\partial \mathbf{p} = \boldsymbol{\pi}^{*a}$, condition (19) follows

(ii)

$$\frac{\mathrm{d}\boldsymbol{\pi}^{*a}}{\mathrm{d}\mathbf{p}_j} = \frac{\partial\boldsymbol{\pi}^{*a}}{\partial\mathbf{p}_j} + \frac{\partial\boldsymbol{\pi}^{*a}}{\partial\rho_p'}\frac{\partial\bar{\rho}_p}{\partial\mathbf{p}_j} = \mathbf{x}_j^{*a} + \frac{\partial\boldsymbol{\pi}^{*a}}{\partial\rho_p'}\frac{\partial\bar{\rho}_p}{\partial\mathbf{p}_j}$$

(iii)

$$\frac{\mathrm{d}^2\boldsymbol{\pi}^{*a}}{\mathrm{d}\mathbf{p}\,\mathrm{d}\mathbf{p}'} = \frac{\mathrm{d}}{\mathrm{d}\mathbf{p}}\left(\frac{\partial\boldsymbol{\pi}^{*a}}{\partial\mathbf{p}'} + \frac{\partial\bar{\rho}_p'}{\partial\mathbf{p}}\frac{\partial\boldsymbol{\pi}^{*a}}{\partial\rho_p}\right)$$
$$= \frac{\partial^2\boldsymbol{\pi}^{*a}}{\partial\mathbf{p}\,\partial\mathbf{p}'} + \frac{\partial^2\boldsymbol{\pi}^{*a}}{\partial\mathbf{p}\,\partial\rho_p'}\frac{\partial\bar{\rho}_p}{\partial\mathbf{p}'} + \frac{\partial\bar{\rho}_p'}{\partial\mathbf{p}}\frac{\partial^2\boldsymbol{\pi}^{*a}}{\partial\mathbf{p}'\,\partial\rho_p} + \frac{\partial\bar{\rho}_p'}{\partial\mathbf{p}}\frac{\partial^2\boldsymbol{\pi}^{*a}}{\partial\rho_p'\,\partial\rho_p}\frac{\partial\bar{\rho}_p}{\partial\mathbf{p}'} + \left[\frac{\partial\boldsymbol{\pi}^{*a}}{\partial\rho_p'}\frac{\partial^2\bar{\rho}_p}{\partial\mathbf{p}_j\,\partial\mathbf{p}_k}\right]$$

Only the positive semidefiniteness of the matrix $\partial^2\boldsymbol{\pi}^{*a}/\partial\mathbf{p}\,\partial\mathbf{p}' + \partial\bar{\rho}_p'/\partial\mathbf{p}$ $[\partial^2\boldsymbol{\pi}^{*a}/\partial\rho_p'\,\partial\rho_p]\partial\bar{\rho}_p/\partial\mathbf{p}'$ can be deduced from the convexity of π^* in p. ■

Proposition 2 means that in general, the aggregate profit function does not have to satisfy any microeconomic regularity properties. From (19) and (20), it can be directly seen that when aggregate Hotelling's lemma holds for all goods, the aggregate profit function is linearly homogeneous in $\mathbf{p}$. Proposition 2 may appear pessimistic at first sight. However, under the usual aggregative assumptions, the following corollary is obtained.

COROLLARY 3. (i) *Under weak separability of* $\pi^*(p, z; \alpha)$ *in the partition* $\mathcal{J}$, $\boldsymbol{\pi}^{*a}$ *satisfies Hotelling's lemma and is linearly homogeneous and convex in suitably defined aggregate prices* $\mathbf{p}_j$. (ii) *When the ratios* $\bar{\rho}_{pj}$ *of elementary to aggregate prices are independent of the aggregate prices*, $\boldsymbol{\pi}^{*a}$ *satisfies Hotelling's lemma and is linearly homogeneous and convex in* $\mathbf{p}$.

PROOF. (i) Under weak separability of $\pi^*(p, z; \alpha)$ in the partition $\mathcal{J}$, the term ρ_p does not parameterize anymore $\boldsymbol{\pi}^{*a}$, and all partial derivatives of $\boldsymbol{\pi}^{*a}$ with respect to ρ_p vanish in the expressions of Proposition 2. Condition (iii) becomes $[\partial^2\boldsymbol{\pi}^{*a}/\partial\mathbf{p}\,\partial\mathbf{p}']$ positive semidefinite. Since π^* is convex in p, this last requirement is fulfilled. Part (ii) of Corollary 3 is obtained by seeing that under independence assumption, the expressions $\partial\bar{\rho}_p/\partial\mathbf{p}'$ and $\partial^2\bar{\rho}_p/\partial\mathbf{p}_j\,\partial\mathbf{p}_k$ vanish in Proposition 2. ■

Proposition 2 is therefore not as negative as it sounds: the usual regularity properties are inherited in the aggregate under weaker conditions than those of the usual aggregation approaches. Indeed, Proposition 2 does neither require that $\partial\boldsymbol{\pi}^{*a}/\partial\rho_p = 0$ nor that $\partial\bar{\rho}_p/\partial\mathbf{p}' = 0$, but only that both impacts be compensated for the regularity properties to hold in the aggregate. Besides, even if the impacts (19) and (20) do not vanish, the aggregate model is not pure nonsense, but the usual regularity properties should not be imposed on the profit function. Under Lewbel's (1996) Assumption 2, $\bar{\rho}_p$ is independent of $\mathbf{p}$, part (ii) of Corollary 3 applies, and the microeconomic properties will hold in the aggregate.

Point (ii) of Proposition 2 also dictates how aggregate quantities must be defined in order to retrieve Hotelling's lemma in the aggregate: either $\mathbf{x}_j \equiv \sum_{h\in\mathcal{J}_j} p_{jh}x_{jh}/\mathbf{p}_j$, for any specification of the aggregate price $\mathbf{p}_j$, or $\mathbf{p}_j \equiv \sum_{h\in\mathcal{J}_j} p_{jh}x_{jh}/\mathbf{x}_j$, for any specification of the aggregate quantity $\mathbf{x}_j$. Then, $\partial\boldsymbol{\pi}^{*a}/\partial\mathbf{p}_j = \mathbf{x}_j^{*a}$ is retrieved at the aggregate level (although $\mathrm{d}\boldsymbol{\pi}^{*a}/\mathrm{d}\mathbf{p}_j^u = \mathbf{x}_j^{*a}$ may not hold).

The distribution of relative prices ρ_p may also depend on noncorresponding variables z. Then, the marginal profit of z can also be decomposed into a scale and a distributional impact:

$$\frac{\mathrm{d}\boldsymbol{\pi}^{*a}}{\mathrm{d}z} = \frac{\partial\boldsymbol{\pi}^{*a}}{\partial z} + \frac{\partial\boldsymbol{\pi}^{*a}}{\partial\rho_p'}\frac{\partial\bar{\rho}_p}{\partial z}$$

where $\partial\boldsymbol{\pi}^{*a}/\partial z$ is equal to the disaggregate marginal profit $\partial\pi^*/\partial z$. Since $\partial\boldsymbol{\pi}^{*a}/\partial\rho_p'[\partial\bar{\rho}_p/\partial z]$ corresponds to the difference between disaggregate and aggregate marginal profits, it may be called aggregation bias. Similarly, I determine the aggregation biases affecting the partial derivatives $\mathrm{d}\boldsymbol{\pi}^{*a}/\mathrm{d}z$, $\mathrm{d}^2\boldsymbol{\pi}^{*a}/\mathrm{d}z\,\mathrm{d}z'$, and $\mathrm{d}^2\boldsymbol{\pi}^{*a}/\mathrm{d}z\,\mathrm{d}\mathbf{p}'$. The following result is also of interest for the latter empirical investigation.

COROLLARY 4. *For given* ρ_p, the function $\boldsymbol{\pi}^{*a}$ *satisfies Hotelling's lemma and is linearly homogeneous and convex in* $\mathbf{p}$.

This corollary is the counterpart of Corollary 2 and means that if information on elementary prices p (and thus on relative prices ρ_p) was available, the scale impact and the distributional effect could be identified separately. Indeed, in this case, the usual "*ceteris paribus* device" can be applied in order to obtain the impact of a variation in $\mathbf{p}$ for given ρ_p. As a consequence, the usual microeconomic properties apply to $\boldsymbol{\pi}^{*a}$. However, if no information on ρ_p is available, both scale and distributional impacts can no longer be disentangled. For this reason, Proposition 2 rather than Corollary 4 applies to the simplified profit function $\boldsymbol{\pi}^*$.

Briefly, it has been shown that important properties of profit functions such as linear homogeneity, Hotelling's lemma, and convexity in prices may be lost when goods and prices are aggregated. To which extent these violations are cumbersome is the theme of the next section.

6. TOWARD EMPIRICAL INVESTIGATION: THE MODEL

Input price changes, wage variations, growth, and technical change have rather dissimilar impacts on the demands for different skills of labor. Therefore, when only information on the aggregate quantity of labor and an aggregate wage is available, some interesting information on the demand for different labor skills is lost. A further consequence of neglecting disaggregate information may be bias in the estimates relying on aggregate data. The following sections provide an empirical application, based on the above framework and results, that illustrates how aggregation is achieved when neglected. The extent and impact of aggregation bias on microeconomic regularity conditions are studied in order to empirically assess the damage caused by the negligence.

Aggregation of heterogeneous labor inputs is studied. More precisely, the consequences of representing the categories of labor h_{nt}, s_{nt}, and u_{nt}, which denote, respectively, "high-skilled," "semiskilled" and "unskilled" labor, by a scalar quantity of labor $\mathbf{x}_{\ell nt}$ are investigated. The subscript n is introduced for characterizing the production unit and t for time. The other arguments of the transformation function are energy e_{nt}, materials m_{nt}, and capital k_{nt}, which are assumed to be flexible inputs, whereas output y_{nt} is exogenous; therefore, $x_{nt} \equiv (k_{nt}, h_{nt}, s_{nt}, u_{nt}, e_{nt}, m_{nt})$ and $z_{nt} \equiv y_{nt}$ in the expression of $f(x_{nt}, z_{nt}; \alpha_n) \geq 0$. The data set used consists of a small panel of 27 German industries over the period 1978–1990. These data are described in detail by Falk and Koebel (1999).

The aggregate labor input is defined as the sum of elementary labor inputs as commonly defined by statistical offices and retained in empirical studies: $\mathbf{x}_{\ell nt} = h_{nt} + s_{nt} + u_{nt}$. Aggregate wages are given by the unit wage $\mathbf{p}_{\ell nt} = (p_{hnt}h_{nt} + p_{snt}s_{nt} + p_{unt}u_{nt})/\mathbf{x}_{\ell nt}$. For the use of such aggregates to be possible, the "exact aggregation" approach would require that an aggregate function $\mathbf{f}^s$ exists such that

$$f(x_{nt}, y_{nt}; \alpha_n) = \mathbf{f}^s(k_{nt}, h_{nt} + s_{nt} + u_{nt}, e_{nt}, m_{nt}, y_{nt}; \alpha_n)$$

In this case, however, a rational production unit would use only one labor input: the cheapest one. This last situation is clearly not met with my data and therefore exact aggregation cannot be achieved. This short reasoning leads to reject the strategy relying on exact aggregation theory for modeling aggregate labor demand. Is therefore any empirical study using aggregates useless? Not necessarily, because the production process can be represented by $\mathbf{f}^a(k_{nt}, \mathbf{x}_{\ell nt}, e_{nt}, m_{nt}, y_{nt}; \alpha_n, \rho_{xnt})$, with $\rho_{xnt} = (h_{nt}/\mathbf{x}_{\ell nt}, s_{nt}/\mathbf{x}_{\ell nt}, u_{nt}/\mathbf{x}_{\ell nt})'$ and approximately represented by $\mathbf{f}(k_{nt}, \mathbf{x}_{\ell nt}, e_{nt}, m_{nt}, y_{nt}; \boldsymbol{\alpha}_n)$.

6.1. *The Specification of the Models.* Since in applied production analysis the determinants of input demands are often of interest, a disaggregate profit function and its associated input demand system $x^*(p_{nt}, y_{nt}; \alpha_n)$ are considered. Since in this empirical application all variable goods are inputs, from now on it is more convenient to consider cost functions c^* instead of profit functions. Both concepts are related by $c^*(p_{nt}, y_{nt}; \alpha_n) = -\pi^*(p_{nt}, y_{nt}; \alpha_n)$. Thus, the properties of c^* are directly derived from those of π^*. The normalized quadratic cost function is chosen for its local flexibility property (see Diewert and Wales, 1987, 1992):

$$c^*(p_{nt}, y_{nt}; \alpha_n) = p'_{nt}A_{pn} + \frac{1}{2}\frac{p'_{nt}A_{pp}p_{nt}}{\theta'_n p_{nt}} + p'_{nt}A_{py}y_{nt} + \frac{1}{2}\left(\theta'_n p_{nt}\right)y_{nt}\alpha_{yy}y_{nt} \tag{22}$$

The vector $A_{pn} = [\alpha_{pn}]$ is of size 6×1, the symmetric matrix $A_{pp} = [\alpha_{pp}]$ is of size 6×6, the matrix A_{py} is 6×1, and α_{yy} is a scalar. The vector θ_n (of size 6×1) is solely introduced for normalization and is arbitrarily fixed to $\theta_n = x_{n1}/c_{n1}$ so that $\theta'_n p_{nt}$ gives the Laspeyres price index for total costs, normalized to one in the basis year for which $t = 1$. Linear homogeneity in prices is thereby directly imposed on c^*. Concavity in prices can also be directly imposed, as shown by Diewert and Wales (1987), by constraining the matrix A_{pp} to be negative semidefinite. The parameters of the matrix A_{pn} and the 21 parameters of A_{pp} cannot all be identified without additional constraints. For this reason, the following six equality constraints are put on the matrix A_{pp}:

$$(1, \ldots, 1)A_{pp} = 0$$

An interesting property of the normalized quadratic cost function that was proved by Diewert and Wales (1987) is that even after all these restrictions being imposed, the normalized quadratic cost function remains flexible; that is, it can still approximate (locally) an arbitrary cost function as well as all its first- and second-order derivatives. The application of Hotelling's lemma leads to the expression of input demands:

$$(23) \qquad -x^*(p_{nt}, y_{nt}; \alpha_n) = A_{pn} + \frac{A_{pp}p_{nt}}{\theta_n' p_{nt}} - \frac{1}{2}\theta_n \frac{p_{nt}' A_{pp} p_{nt}}{(\theta_n' p_{nt})^2} + A_{py} y_{nt} + \frac{1}{2}\theta_n y_{nt} \alpha_{yy} y_{nt}$$

For a given production unit n, there are 28 parameters involved in (23). Since production units may be heterogeneous, system (23) also entails fixed effects in each equation. For overall production units, there are 184 parameters to be estimated.

Using the transformation $p_{nt} = \rho_{pnt} * \tilde{\mathbf{p}}_{nt}$, where $\rho_{pnt} \equiv (1, p_{hnt}/\mathbf{p}_{\ell nt}, p_{snt}/\mathbf{p}_{\ell nt}, p_{unt}/\mathbf{p}_{\ell nt}', 1, 1)$ and $\tilde{\mathbf{p}}_{nt} \equiv (p_{knt}, \mathbf{p}_{\ell nt}, \mathbf{p}_{\ell nt}, \mathbf{p}_{\ell nt}, p_{ent}, p_{mnt})'$, the former disaggregate cost function can be reparameterized into

$$\mathbf{c}^{*a}(\mathbf{p}_{nt}, y_{nt}; \alpha_n, \rho_{pnt}) = (\rho_{pnt} * \tilde{\mathbf{p}}_{nt})' A_{pn} + \frac{1}{2} \frac{(\rho_{pnt} * \tilde{\mathbf{p}}_{nt})' A_{pp} (\rho_{pnt} * \tilde{\mathbf{p}}_{nt})}{\theta_n' (\rho_{pnt} * \tilde{\mathbf{p}}_{nt})}$$
$$+ (\rho_{pnt} * \tilde{\mathbf{p}}_{nt})' A_{py} y_{nt} + \frac{1}{2} (\theta_n' (\rho_{pnt} * \tilde{\mathbf{p}}_{nt})) y_{nt} \alpha_{yy} y_{nt}$$

The vector $\tilde{\mathbf{p}}_{nt}$ can be represented by the matrix multiplication $D\mathbf{p}_{nt}$, where $\mathbf{p}_{nt} \equiv (p_{knt}, p_{\ell nt}, p_{ent}, p_{mnt})'$ and D is a matrix of zeros and ones, of size 6×4; each column of D contains a unique element equal to one. That is,

$$\tilde{\mathbf{p}}_{nt}' \equiv (p_{knt}, \mathbf{p}_{\ell nt}, \mathbf{p}_{\ell nt}, \mathbf{p}_{\ell nt}, p_{ent}, p_{mnt}) = \mathbf{p}_{nt}' \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} = \mathbf{p}_{nt}' D'$$

Then, the elementary price vector can be expressed as

$$p_{nt} = \tilde{\mathbf{p}}_{nt} * \rho_{pnt} = D\mathbf{p}_{nt} * \rho_{pnt} = D_{\rho_{pnt}} \mathbf{p}_{nt}$$

where $D_{\rho_{pnt}}$ is a matrix of size 6×4 with zero and $\rho_{p_{njt}}$ as components. The above cost function can then be rewritten as

$$(24) \qquad \mathbf{c}^{*a}(\mathbf{p}_{nt}, y_{nt}; \alpha_n, \rho_{pnt}) = \mathbf{p}_{nt}' B_{pn} + \frac{1}{2} \frac{\mathbf{p}_{nt}' B_{pp} \mathbf{p}_{nt}}{(\theta_{nt}' \mathbf{p}_{nt})^{-1}} + \mathbf{p}_{nt}' B_{py} y_{nt} + \frac{1}{2} (\theta_{nt}' \mathbf{p}_{nt}) y_{nt} \alpha_{yy} y_{nt}$$

where $\theta_{nt} \equiv D_{\rho_{pnt}}' \theta_n$, and thus $\theta_{nt}' \mathbf{p}_{nt} = \theta_n' p_{nt}$.[11] The different matrices involved in expression (24) are related to the elementary variables and parameters by

[11] Note that θ_{nt} has now n and t as subscripts.

$$B_{pn} = D'_{\rho_{pnt}} A_{np}$$
$$B_{pp} = D'_{\rho_{pnt}} A_{pp} D_{\rho_{pnt}}$$
$$B_{py} = D'_{\rho_{pnt}} A_{py}$$

Thus, the aggregate cost function can still be written in the form of a normalized quadratic function. However, the different matrices involved depend on a large number of unknown variables and parameters whose stability may be questioned. In the extreme case of price proportionality, Corollary 3 applies and the usual properties of the cost function are satisfied. If the variables ρ_{pnt} are known in the expression of $\mathbf{c}^{*a}$, Corollary 4 applies and the aggregate input demand system associated to $\mathbf{c}^{*a}$ is obtained through the application of Hotelling's lemma:

$$(25)\qquad -\mathbf{x}^{*a}(\mathbf{p}_{nt}, y_{nt}; \alpha_n, \rho_{pnt}) = B_{pn} + \frac{B_{pp}\mathbf{p}_{nt}}{(\boldsymbol{\theta}'_{nt}\mathbf{p}_{nt})^{-1}} - \frac{1}{2}\frac{\mathbf{p}'_{nt}B_{pp}\mathbf{p}_{nt}}{(\boldsymbol{\theta}'_{nt}\mathbf{p}_{nt})^{-2}}\boldsymbol{\theta}_{nt} + B_{py}y_{nt} + \frac{1}{2}\boldsymbol{\theta}_{nt}(y_{nt}\alpha_{yy}y_{nt})$$

Alternatively, this system is also obtained from the elementary demand equation (23) as

$$\mathbf{x}^{*a}(\mathbf{p}_{nt}, y_{nt}; \alpha_n, \rho_{pnt}) = D'_{\rho_{pnt}} x^{*}\left(D_{\rho_{pnt}}\mathbf{p}_{nt}, y_{nt}; \alpha_n\right) = D'_{\rho_{pnt}} x^{*a}(\mathbf{p}_{nt}, y_{nt}; \alpha_n, \rho_{pnt})$$

where $x^{*a} = -\partial\mathbf{c}^{*a}/\partial p_{nt}$.

6.2. *The Aggregation Biases.* Several strategies could be adopted for checking whether the simplified relationship $\mathbf{c}^{*}(\mathbf{p}_{nt}, y_{nt}; \boldsymbol{\alpha}_n)$ satisfies the regularity properties. A possibility is to consider a general specification for the aggregate cost function $\tilde{\mathbf{c}}^{*}$ with no regularity properties imposed *a priori*. The different regularity properties can then be investigated by testing whether the estimated parameters of $\tilde{\mathbf{c}}^{*}$ satisfy the restrictions corresponding to the regularity properties. (See Appelbaum (1978) and Berndt (1991, Chapter 9),) for example. However, this approach does not allow to identify whether regularity violations are solely implied by aggregation across goods (or inputs). Indeed, departure from individual rationality, a bad representation of the technology, neglect of aggregation across individuals, etc. may represent other sources of regularity violations. In order to identify the consequences of neglecting aggregation across goods, the theoretical link between the microeconomic and the simplified models should be exploited. In this section, the aggregation biases that also represent the statistics of interest for some regularity tests are explicitly derived on the basis of the fundamental relationship (26) between the simplified and the true models.

$$(26)\qquad \mathbf{c}^{*}(\mathbf{p}_{nt}, y_{nt}; \boldsymbol{\alpha}_n) = \mathbf{c}^{*a}(\mathbf{p}_{nt}, y_{nt}; \alpha_n, \bar{\rho}_{pnt})$$

When proceeding to comparative static analysis and considering, for example, the impact of price change on input demand, aggregation biases can be defined as the difference between the comparative static results obtained from the simplified and the true models. In order to give a meaningful interpretation, the different aggregation biases should be expressed in percentage of costs (or input demand). Therefore, the aggregation bias on the output elasticity, for instance, is given by

$$\text{(27)} \qquad \mathcal{B}_y \equiv \left(\frac{\partial \mathbf{c}^*}{\partial y_{nt}} - \frac{\partial \mathbf{c}^{*a}}{\partial y_{nt}}\right)\frac{y_{nt}}{\mathbf{c}^{*a}} = \sum_{j=h,s,u} \frac{\partial \mathbf{c}^{*a}}{\partial \rho_{p_j nt}} \frac{\partial \bar{\rho}_{p_j nt}}{\partial y_{nt}} \frac{y_{nt}}{\mathbf{c}^{*a}}$$
$$= -\sum_j \mathbf{p}_{\ell nt} x^*_{jnt} \frac{\bar{\rho}_{p_j nt}}{\bar{\rho}_{p_j nt}} \frac{\partial \bar{\rho}_{p_j nt}}{\partial y_{nt}} \frac{y_{nt}}{\mathbf{c}^{*a}} = \sum_j \frac{-p_{jnt} x^*_j}{\mathbf{c}^{*a}} \epsilon_{\rho_j y}$$

where the first equality follows from (26) and the second from the fact that $\partial \mathbf{c}^{*a} / \partial \rho_{p_j nt} = -\mathbf{p}_{\ell nt} x^*_j$ by adapting Hotelling's lemma. The aggregation bias $\mathcal{B}_y$ is therefore a weighted mean of the elasticities $\epsilon_{\rho_j y}$ of relative wages for skill j with respect to output. The weights are given by $-p_{jnt} x^{*a}_{jnt} / \mathbf{c}^{*a}$, and their sum, which is less than one, gives the share of labor costs in total costs.

Similar expressions can be obtained for the aggregation bias related to $\mathbf{p}_{int}$ $(i = k, \ell, e, m)$:

$$\text{(28)} \qquad \mathcal{B}_{\mathbf{p}_i} \equiv \left(\frac{\partial \mathbf{c}^*}{\partial \mathbf{p}_{int}} - \frac{\partial \mathbf{c}^{*a}}{\partial \mathbf{p}_{int}}\right)\frac{\mathbf{p}_{int}}{\mathbf{c}^{*a}} = \sum_{j=h,s,u} \frac{-p_{jnt} x^{*a}_j}{\mathbf{c}^{*a}} \epsilon_{\rho_j \mathbf{p}_i}, \quad i = k, \ell, e, m$$

$\mathcal{B}_{\mathbf{p}_i}$ reflects the departure from Hotelling's lemma for input $\mathbf{x}_i$ (capital, aggregate labor, energy, and non-energy material inputs). Tests for the hypothesis H_0: $\mathcal{B}_{\mathbf{p}_i} = 0$ can be used for investigating whether Hotelling's lemma is inherited by $\mathbf{c}^*$. From Proposition 2, a test of the hypothesis that $\mathbf{c}^*$ is linearly homogeneous in $\mathbf{p}_{nt}$ can be based on the statistic

$$\text{(29)} \qquad \mathcal{B}_{h1} = \sum_{j=h,s,u} \frac{\partial \mathbf{c}^{*a}}{\partial \rho_{p_j nt}} \frac{\partial \bar{\rho}_{p_j nt}}{\partial \mathbf{p}'_{nt}} \frac{\mathbf{p}_{nt}}{\mathbf{c}^{*a}} = \sum_{j=e,k,\ell,m} \mathcal{B}_{\mathbf{p}_i}$$

Further economically interesting impacts are given by second-order derivatives of the cost function. The aggregation bias associated to $\partial^2 \mathbf{c}^* / \partial p_{knt} \partial y_{nt}$ for instance is given by (for simplicity, the subscripts nt are omitted)

$$\text{(30)} \quad \mathcal{B}_{p_k y} \equiv -\left(\frac{\partial^2 \mathbf{c}^*}{\partial p_k \, \partial y} - \frac{\partial \mathbf{x}^{*a}_k}{\partial y}\right)\frac{y}{\mathbf{x}^{*a}_k}$$
$$= \left(\frac{\partial^2 \mathbf{c}^{*a}}{\partial y \, \partial \rho'_p} \frac{\partial \bar{\rho}_p}{\partial p_k} + \frac{\partial \bar{\rho}'_p}{\partial y} \frac{\partial^2 \mathbf{c}^{*a}}{\partial p_k \, \partial \rho_p} + \frac{\partial \bar{\rho}'_p}{\partial y} \frac{\partial^2 \mathbf{c}^{*a}}{\partial \rho'_p \, \partial \rho_p} \frac{\partial \bar{\rho}_p}{\partial p_k} + \frac{\partial \mathbf{c}^{*a}}{\partial \rho'_p} \frac{\partial^2 \bar{\rho}_p}{\partial p_k \, \partial y}\right)\frac{y}{\mathbf{x}^{*a}_k}$$
$$= -\left(\mathbf{p}_\ell \frac{\partial x^{*\prime}_\ell}{\partial y} \frac{\partial \bar{\rho}_p}{\partial p_k} + \mathbf{p}_\ell \frac{\partial \bar{\rho}'_p}{\partial y} \frac{\partial x^*_\ell}{\partial p_k} + \frac{\partial \bar{\rho}'_p}{\partial y} \mathbf{p}_\ell \frac{\partial x^*_\ell}{\partial \rho'_p} \frac{\partial \bar{\rho}_p}{\partial p_k} + \mathbf{p}_\ell x^{*\prime}_\ell \frac{\partial^2 \bar{\rho}_p}{\partial p_k \, \partial y}\right)\frac{y}{\mathbf{x}^{*a}_k}$$
$$= -\sum_{j=h,s,u} \frac{p_j x^*_j}{p_k \mathbf{x}^{*a}_k}\left(\epsilon_{x_j y}\epsilon_{\rho_j p_k} + \epsilon_{\rho_j y}\epsilon_{x_j p_k} + \sum_{i=h,s,u} \epsilon_{\rho_j y}\epsilon_{x_j p_i}\epsilon_{\rho_i p_k} + \epsilon_{\rho_j p_k}\epsilon_{(\partial \rho_j / \partial p_k), y}\right)$$

The first equality follows from the definition of the cost function (see the proof of Proposition 2). The second equality is achieved by using the fact that $\partial \mathbf{c}^{*a} / \partial \rho_p = -\mathbf{p}_\ell x^*_\ell$ and $\partial \mathbf{c}^{*a} / \partial p_k = -\mathbf{x}^{*a}_k$. The third equality follows after conversion in terms of elasticities.[12] It does not seem straightforward to give an economic interpretation of the biases on second-order derivatives. However, as could be expected from the former results, when the relative wages are stable over time, the elasticities of ρ_j are small and the aggregation bias may become negligible. The remaining aggregation biases, $\mathscr{B}_{\mathbf{p}_\ell y}$, $\mathscr{B}_{\mathbf{p}_\ell \mathbf{p}_\ell}$, $\mathscr{B}_{\mathbf{p}_\ell p_i}$, $\mathscr{B}_{p_i y}$, $\mathscr{B}_{p_i p_j}$, and $\mathscr{B}_{yy}$, can be derived accordingly.

Given the above decompositions, it seems interesting to investigate (i) how important the aggregation biases are; (ii) how they affect microeconomic regularity properties.

7. PROPORTIONALITY OF WAGES

For determining the biases generated by aggregation across inputs, the parameters of $\mathbf{c}^{*a}$ have to be estimated. This allows to us determine the impacts $\partial \mathbf{c}^{*a}/\partial \rho_{pnt}$, $\partial^2 \mathbf{c}^{*a}/\partial \rho'_{pnt}\, \partial \rho_{pnt}$, and $\partial^2 \mathbf{c}^{*a}/\partial z_{nt}\, \partial \rho'_{pnt}$ in the expression of the aggregation biases. For this aim, the parameter vector α_n is estimated in the following *SUR* regressions:

$$x_{nt}/y_{nt} = x^{*a}(\mathbf{p}_{nt}, y_{nt}; \alpha_n, \rho_{pnt})/y_{nt} + v_{xnt} \tag{31}$$

where the added random vector v_{xnt} has zero mean and constant covariance matrix conditionally on (p_{nt}, y_{nt}). The detailed system x^{*a} is considered instead of the theoretically equivalent system $\mathbf{x}^{*a}$, in order to have exactly identical parameter estimates for $\hat{\alpha}_n$ and its covariance matrix in the different models considered below.

The impacts on relative wages that are necessary to determine the aggregation biases cannot be identified from the estimation of α_n. From which kind of regressions should these estimates come? In Section 5, it was shown that since the simplified specification depends only on $\mathbf{p}$ and y, the equality $\mathbf{c}^* = \mathbf{c}^{*a}$ can only be achieved when relative prices ρ_p depend (stochastically) on $\mathbf{p}$ and y. For simplicity, the specification for relative wages is chosen to be linear in the explanatory variables:

$$\tilde{\rho}_{pj}\left(\mathbf{p}_{nt}, y_{nt}; \gamma_{jn}\right) = \gamma_{0jn} + \gamma'_{\mathbf{p}j}\mathbf{p}_{nt} + \gamma_{yj} y_{nt} \tag{32}$$

for $j = h, s, u$. There are five slope parameters and one dummy intercept involved in (32). For parameter determination, the regression

$$\rho_{p_{jnt}} = \tilde{\rho}_{pj}\left(\mathbf{p}_{nt}, y_{nt}; \gamma_{jn}\right) + \omega_{jnt} \tag{33}$$

is considered, where $j = h, s, u$ and ω_{jnt} is a residual term with zero mean conditional on regressors and constant variance $\sigma^2_{\rho j}$. The parameter vectors $\hat{\gamma}_{jn}$, $j = h, s, u$ are estimated by ordinary least squares, equation by equation.

It is important to remark that although $\rho_{p_{jnt}}$ is a random variable, there is no endogeneity problem affecting (31). Indeed, ρ_{pnt} has been introduced in a purely

[12] The somewhat unusual elasticity $\epsilon_{(\rho_j/p_k),\, y}$ is defined as

$$\epsilon_{(\partial\rho_j/\partial p_k),\, y} \equiv \frac{\partial\left(\partial\bar{\rho}_{jt}/\partial p_{kt}\right)}{\partial y_t}\frac{y_t}{\partial\bar{\rho}_{jt}/\partial p_{kt}}, \qquad j{=}h,\, s,\, u$$

artificial manner as argument of $\mathbf{c}^{*a}$ and x^{*a} through the transformation $p_{nt} = \rho_{pnt} * \mathbf{p}_{nt}$. In fact, the transformed demand system $x^{*a}(\mathbf{p}_{nt}, y_{nt}; \alpha_n, \rho_{pnt})$ is identical to the disaggregate one $x^{*}(p_{nt}, y_{nt}; \alpha_n)$. Hence, the variables p_{nt} and y_{nt} are adequate instrumental variables for the determination of $\hat{\alpha}_n$.

From Corollary 3 (stochastic) proportionality of prices is a sufficient condition for $\mathbf{c}^{*}$ to satisfy the microeconomic regularities, in which case all parameters with the exception of the fixed effect γ_{0jn} vanish in (32). Using data on disaggregate consumption commodities, Lewbel (1992) investigates the stationarity and cointegration properties of relative prices and weakly rejects the assumption of stochastic price proportionality. Here, as I am interested in the properties of $\mathbf{x}^{*}(\mathbf{p}_{nt}, y_{nt}; \boldsymbol{\alpha}_n)$ and in the bias related to aggregation, the study of the stationarity of disaggregate wages appears not very helpful. For this reason, regressions (32) are considered for testing stochastic price proportionality.

The results of the stochastic price proportionality tests are provided in Table 1 (model 1). A Fisher test for the joint nullity of all slopes rejects the assumption of stochastic proportionality of wages over time for the qualifications h and s. This rejection may appear surprising in the light of the high correlations between elementary and aggregate wages (see Table A.1 in the Appendix). For the qualification u, however, stochastic price proportionality could not be rejected.

The same exercise is done for testing the approximate price proportionality for energy and nonenergy material inputs (model 2). The aggregate material input price is constructed as $\mathbf{p}_{\mathbf{m}nt} \equiv (p_{ent}e_{nt} + p_{mnt}m_{nt})/\mathbf{x}_{\mathbf{m}nt}$ where $\mathbf{x}_{\mathbf{m}nt} \equiv e_{nt} + m_{nt}$. The Pearson coefficient of correlation $(r^2(p_{ent}, \mathbf{p}_{\mathbf{m}nt}) = 0.668)$ between the prices of energy and aggregate material is now much lower than in model 1: an oil-shock and a counter-shock occurred over the period. All three labor inputs are explicitly considered in model 2. The whole specification in (31) and (33) must be slightly adapted, since now $\mathbf{p}_{nt} \equiv (p_{knt}, p_{hnt}, p_{snt}, p_{unt}, \mathbf{p}_{\mathbf{m}nt})'$. The regressions (33) consist of two relationships,

TABLE 1

TESTING THE CONSTANCY OF RELATIVE PRICES[1]

Var. Explained	Adjusted R^2	F-Test
Model 1		
$p_{ht}/\mathbf{p}_{\ell t}$	0.964	4.98
$p_{st}/\mathbf{p}_{\ell t}$	0.881	79.75
$p_{ut}/\mathbf{p}_{\ell t}$	0.920	1.60
Model 2		
$p_{et}/\mathbf{p}_{mt}$	0.453	19.66
$p_{mt}/\mathbf{p}_{mt}$	0.475	11.85
Model 3		
$p_{ht}/\mathbf{p}_{\ell t}$	0.964	6.04
$p_{st}/\mathbf{p}_{\ell t}$	0.881	99.19
$p_{ut}/\mathbf{p}_{\ell t}$	0.920	1.80
$p_{et}/\mathbf{p}_{mt}$	0.455	29.50
$p_{mt}/\mathbf{p}_{mt}$	0.475	17.17

[1] F-test for the hypothesis that all the slopes are zero. The critical values at the five percent significance level are for model 1: $F_{0.05}$ (5319) = 2.24, for model 2: $F_{0.05}$ (6318) = 2.13, and for model 3: $F_{0.05}$ (4320) = 2.40.

one for $p_{ent}/\mathbf{p}_{\mathbf{m}nt}$ and one for $p_{mnt}/\mathbf{p}_{\mathbf{m}nt}$. Again, the Fisher test leads to rejection of the null hypothesis of stable relative prices $p_{ent}/\mathbf{p}_{\mathbf{m}nt}$ and $p_{mnt}/\mathbf{p}_{\mathbf{m}nt}$ (Table 1).

In a third model, labor inputs are aggregated into $\mathbf{x}_{\ell nt}$ and simultaneously material inputs are aggregated into $\mathbf{x}_{\mathbf{m}nt}$. Then, there are five regressions of the type (32), with p_{knt}, $\mathbf{p}_{\ell nt}$, $\mathbf{p}_{\mathbf{m}nt}$, and y_t as explanatory variables. It is not surprising, given the results for models 1 and 2, that the assumption of constant relative prices is unambiguously rejected for four of the five relative prices. Note that in model 3, each value of the F-test is above its corresponding value of models 1 and 2. Therefore, within this data set, the more inputs are simultaneously aggregated, the more strongly is price proportionality rejected.

Since, by Corollary 3, price proportionality is a sufficient condition for the aggregation bias to vanish and the validity of microeconomic properties for $\mathbf{c}^*(\mathbf{p}_{nt}, y_{nt}; \alpha_n)$, the rejection of this assumption, even in some favorable cases of high correlation between elementary and aggregate prices, casts some doubts on the meaning of estimates relying on $\mathbf{c}^*$. The rejection of price proportionality means that any changes in the explanatory variables of $\mathbf{c}^*$ cannot be interpreted as "*ceteris paribus*" since they also measure changes in the distribution of the omitted elementary wages and input prices. Estimated parameters can therefore not be interpreted as reflecting solely microeconomic behavior. This is the reason that microeconomic regularities do not need to hold for $\mathbf{c}^*$ even if they are verified by c^*. Note, however, that the rejection of the *sufficient* condition for the microeconomic condition to hold in the aggregate does not imply their rejection: the different distributional effects might compensate each other and vanish in the aggregate. In the next section, the sizes of the aggregation biases are reported and the validity of microeconomic properties is tested.

8. HETEROGENEOUS SKILLS AND AGGREGATE LABOR DEMAND

In the preceding section, the stochastic proportionality of wages—a sufficient condition for the microeconomic regularities to hold in the aggregate—was rejected. Therefore, the elasticities estimated on the basis of $\mathbf{c}^*$ will not only lead to a loss of information, but may also entail the aggregation bias identified in Section 6.2. Moreover, the microeconomic regularity properties may also be violated by the aggregate model. Their *a priori* imposition may thus imply further estimation biases. However, as underlined in this article, the different distributional effects might compensate each other and vanish in the aggregate. The extent of the bias on elasticities and regularity conditions is empirically investigated in this section.

8.1. *The Sensitivity of Elementary Prices.*

In the expression of the aggregation biases given in Section 6.2, the sensitivity of the relative elementary prices with respect to $\mathbf{p}_{nt}$ and y_{nt} plays an important role. Elasticities can be directly computed from regressions (33). Their values and significance levels are reported in Tables 2–4, which correspond, respectively, to models 1–3.

For model 1, only three elasticities out of 15 are significant, and their absolute values are rather low. Moreover, the signs of the elasticities with respect to a given variable often alternate from one equation to the other. Briefly, three favorable

TABLE 2

SENSITIVITY OF RELATIVE WAGES, MODEL 1

	Estimate	t-Value		Estimate	t-Value		Estimate	t-Value
$\epsilon_{\rho_h p_k}$	−0.006	−0.34	$\epsilon_{\rho_s p_k}$	−0.003	−0.30	$\epsilon_{\rho_u p_k}$	0.017	1.39
$\epsilon_{\rho_h \mathbf{p}_\ell}$	0.057	2.65	$\epsilon_{\rho_s \mathbf{p}_\ell}$	−0.080	−6.96	$\epsilon_{\rho_u \mathbf{p}_\ell}$	−0.023	−1.58
$\epsilon_{\rho_h p_e}$	0.011	0.84	$\epsilon_{\rho_s p_e}$	−0.007	−0.95	$\epsilon_{\rho_u p_e}$	0.006	0.73
$\epsilon_{\rho_h p_m}$	−0.016	−0.57	$\epsilon_{\rho_s p_m}$	−0.005	−0.36	$\epsilon_{\rho_u p_m}$	0.006	0.34
$\epsilon_{\rho_h y}$	−0.011	−2.20	$\epsilon_{\rho_s y}$	−0.003	−1.17	$\epsilon_{\rho_u y}$	−0.003	−0.80

TABLE 3

SENSITIVITY OF RELATIVE MATERIAL INPUT PRICES, MODEL 2

	Estimate	t-Value		Estimate	t-Value
$\epsilon_{\rho_e p_k}$	0.516	7.07	$\epsilon_{\rho_m p_k}$	−0.027	−2.89
$\epsilon_{\rho_e p_h}$	−0.064	−0.35	$\epsilon_{\rho_m p_h}$	0.009	0.37
$\epsilon_{\rho_e p_s}$	0.169	0.60	$\epsilon_{\rho_m p_s}$	−0.043	−1.19
$\epsilon_{\rho_e p_u}$	−0.434	−1.68	$\epsilon_{\rho_m p_u}$	0.086	2.55
$\epsilon_{\rho_e \mathbf{p}_m}$	0.319	3.15	$\epsilon_{\rho_m \mathbf{p}_m}$	−0.075	−5.77
$\epsilon_{\rho_e y}$	−0.024	−1.21	$\epsilon_{\rho_m y}$	0.002	0.72

TABLE 4

SENSITIVITY OF RELATIVE INPUT PRICES, MODEL 3

	Estimate	t-Value		Estimate	t-Value		Estimate	t-Value
$\epsilon_{\rho_h p_k}$	−0.001	−0.06	$\epsilon_{\rho_s p_k}$	−0.008	−0.83	$\epsilon_{\rho_u p_k}$	0.021	1.83
$\epsilon_{\rho_h \mathbf{p}_\ell}$	0.052	2.55	$\epsilon_{\rho_s \mathbf{p}_\ell}$	−0.082	−7.50	$\epsilon_{\rho_u \mathbf{p}_\ell}$	−0.022	−1.60
$\epsilon_{\rho_h \mathbf{p}_m}$	−0.002	−0.09	$\epsilon_{\rho_s \mathbf{p}_m}$	−0.005	−0.37	$\epsilon_{\rho_u \mathbf{p}_m}$	0.008	0.49
$\epsilon_{\rho_h y}$	−0.011	−2.23	$\epsilon_{\rho_s y}$	−0.002	−0.95	$\epsilon_{\rho_u y}$	−0.003	−0.93
$\epsilon_{\rho_e p_k}$	0.512	7.08	$\epsilon_{\rho_m p_k}$	−0.026	−2.81			
$\epsilon_{\rho_e \mathbf{p}_\ell}$	−0.400	−4.65	$\epsilon_{\rho_m \mathbf{p}_\ell}$	0.066	5.95			
$\epsilon_{\rho_e \mathbf{p}_m}$	0.327	3.23	$\epsilon_{\rho_m \mathbf{p}_m}$	−0.076	−5.90			
$\epsilon_{\rho_e y}$	−0.016	−0.77	$\epsilon_{\rho_m y}$	0.000	0.13			

conditions for an innocuous negligence of aggregation seem to be fulfilled. These results are not very surprising: given the high correlation between elementary and aggregate wages, there are limited possibilities for other variables to explain the evolution of relative wages.

In model 2, the price of energy is not highly correlated with aggregate material input price. From the regression of $p_{ent}/\mathbf{p}_{mnt}$, two elasticities out of five are significant and their absolute values are rather high. But although they are significant, the terms $\epsilon_{\rho_e p_k}$ and $\epsilon_{\rho_e \mathbf{p_m}}$ enter the expression of the aggregation bias $\mathscr{B}_{\mathbf{p}_i}$ weighted by $p_{ent}x^*_{ent}/\mathbf{c}^{*a}$. For the energy input, these weights are relatively low and therefore some hope for negligible aggregation bias still remains.

In model 3, where both the three skills and two material inputs are aggregated to form two composite inputs, there are now five regressions for relative input prices.

The values of the estimates do not greatly differ from those obtained in models 1 and 2. One elasticity, however, $\epsilon_{\rho_e \mathbf{p}_\ell}$, becomes significant in model 3, although $\epsilon_{\rho_e p_h}$, $\epsilon_{\rho_e p_s}$, and $\epsilon_{\rho_e p_u}$ are not significant in model 2.

These elasticities reflect the sensitivity of elementary wages and prices with respect to the aggregate explanatory variables. In the simplified model 1, for instance, only the aggregate wage level $\mathbf{p}_\ell$ is considered for representing unit labor costs, and the evolution of the relative wages for each skill group is masked. When elementary wages evolve proportionally to $\mathbf{p}_\ell$, aggregate wages are a sufficient statistic for elementary wages, in the sense that $\mathbf{p}_{\ell t}$ represents p_{ht}, p_{st}, and p_{ut}. In fact, wage proportionality has been rejected, and from Table 2, it can be seen that behind an aggregate wage increase of one percent *ceteris paribus*, the wage for high-skilled labor tends to increase a little more than one percent ($\epsilon_{\rho_h \mathbf{p}_\ell}$ is significantly positive in Table 2), whereas the wage for skilled labor increases a little less than one percent. Is this shift in the relative elementary wages important for understanding aggregate labor demand? If they can be neglected, aggregation bias should vanish.

8.2. *Estimation of the Aggregation Biases.* Now the size of the different aggregation biases can be derived. However, the formulae of Section 6.2 cannot be used directly for this purpose. The estimation of (33) provides estimates for the relative wage elasticities that are computed on the basis of $\tilde{\rho}_{pnt} = \mathrm{E}[\rho_{pnt} \mid \mathbf{p}_{nt}, y_{nt}]$. The problem is that the function $\tilde{\rho}_{pnt}$ cannot be identified with the function $\bar{\rho}_{pnt}$ in (26). Defining $\mathbf{c}^*$ as the expectations of $\mathbf{c}^{*a}$ conditionally on $\mathbf{p}_{nt}$, y_{nt} yields

$$\mathbf{c}^*(\mathbf{p}_{nt}, y_{nt}; \boldsymbol{\alpha}_n) = \mathrm{E}[\mathbf{c}^{*a}(\mathbf{p}_{nt}, y_{nt}; \alpha_n, \rho_{pnt}) \mid \mathbf{p}_{nt}, y_{nt}] = \mathbf{c}^{*a}(\mathbf{p}_{nt}, y_{nt}; \alpha_n, \bar{\rho}_{pnt})$$

where the function $\bar{\rho}_{pnt}$ is implicitly defined by this last equality. If $\mathbf{c}^{*a}$ were linear in ρ_{pnt}, one would simply have $\bar{\rho}_{pnt} = \mathrm{E}[\rho_{pnt} \mid \mathbf{p}_{nt}, y_{nt}]$. However, the microeconomic cost function c^* is concave in p_{nt} and thus the aggregate function $\mathbf{c}^{*a}$ is concave in ρ_{pnt}. In this case, by Jensen's inequality

$$\mathbf{c}^*(\mathbf{p}_{nt}, y_{nt}; \boldsymbol{\alpha}_n) \leq \mathbf{c}^{*a}(\mathbf{p}_{nt}, y_{nt}; \alpha_n, \mathrm{E}[\rho_{pnt} \mid \mathbf{p}_{nt}, y_{nt}])$$

Therefore, the function $\bar{\rho}_{pnt}$ should not be identified with the conditional expectation $\mathrm{E}[\rho_{pnt} \mid \mathbf{p}_{nt}, y_{nt}]$, whose determinants have been studied above. However, a second-order approximation to $\mathbf{c}^{*a}$ gives

$$\begin{aligned}
(34) \quad & \mathrm{E}[\mathbf{c}^{*a}(\mathbf{p}_{nt}, y_{nt}; \alpha_n, \rho_{pnt}) \mid \mathbf{p}_{nt}, y_{nt}] \\
&\simeq \mathbf{c}^{*a}(\mathbf{p}_{nt}, y_{nt}; \alpha_n, \mathrm{E}[\rho_{pnt} \mid \mathbf{p}_{nt}, y_{nt}]) \\
&\quad + \mathrm{E}[(\rho_{pnt} - \mathrm{E}[\rho_{pnt} \mid \mathbf{p}_{nt}, y_{nt}])' \\
&\quad \times \frac{\partial^2 \mathbf{c}^{*a}}{\partial \rho_{pnt} \partial \rho'_{pnt}} (\rho_{pnt} - \mathrm{E}[\rho_{pnt} \mid \mathbf{p}_{nt}, y_{nt}]) \mid \mathbf{p}_{nt}, y_{nt}] \\
&= \mathbf{c}^{*a}(\mathbf{p}_{nt}, y_{nt}; \alpha_n, \mathrm{E}[\rho_{pnt} \mid \mathbf{p}_{nt}, y_{nt}]) \\
&\quad + \sum_{j=h,s,u} \sigma^2_{\rho j} \frac{\partial^2 \mathbf{c}^{*a}}{\partial \rho^2_{pjnt}} (\mathbf{p}_{nt}, y_{nt}; \alpha_n, \mathrm{E}[\rho_{pnt} \mid \mathbf{p}_{nt}, y_{nt}])
\end{aligned}$$

TABLE 5
BIASES ON FIRST-ORDER DERIVATIVES

	Value	t-Statistic		Value	t-Statistic
Model 1					
$\epsilon_{\mathbf{c}p_k}$	0.101	124.25	$\mathscr{B}_{\mathbf{c}p_k}$	0.001	0.23
$\epsilon_{\mathbf{c}\mathbf{p}_\ell}$	0.370	176.34	$\mathscr{B}_{\mathbf{c}\mathbf{p}_\ell}$	−0.023	−6.88
$\epsilon_{\mathbf{c}p_e}$	0.019	94.88	$\mathscr{B}_{\mathbf{c}p_e}$	−0.001	−0.55
$\epsilon_{\mathbf{c}p_m}$	0.510	353.95	$\mathscr{B}_{\mathbf{c}p_m}$	−0.001	−0.23
$\epsilon_{\mathbf{c}y}$	0.801	37.08	$\mathscr{B}_{\mathbf{c}y}$	−0.001	−1.51
Model 2					
$\epsilon_{\mathbf{c}p_k}$	0.101	124.25	$\mathscr{B}_{\mathbf{c}p_k}$	−0.005	−1.01
$\epsilon_{\mathbf{c}p_h}$	0.009	60.72	$\mathscr{B}_{\mathbf{c}p_h}$	0.003	0.27
$\epsilon_{\mathbf{c}p_s}$	0.272	215.51	$\mathscr{B}_{\mathbf{c}p_s}$	−0.019	−1.00
$\epsilon_{\mathbf{c}p_u}$	0.090	90.80	$\mathscr{B}_{\mathbf{c}p_u}$	0.036	2.05
$\epsilon_{\mathbf{c}\mathbf{p}_m}$	0.528	356.35	$\mathscr{B}_{\mathbf{c}\mathbf{p}_m}$	−0.033	−4.79
$\epsilon_{\mathbf{c}y}$	0.801	37.08	$\mathscr{B}_{\mathbf{c}y}$	0.001	0.39
Model 3					
$\epsilon_{\mathbf{c}p_k}$	0.101	124.25	$\mathscr{B}_{\mathbf{c}p_k}$	−0.005	−0.86
$\epsilon_{\mathbf{c}\mathbf{p}_\ell}$	0.370	176.34	$\mathscr{B}_{\mathbf{c}\mathbf{p}_\ell}$	0.003	0.45
$\epsilon_{\mathbf{c}\mathbf{p}_m}$	0.528	356.35	$\mathscr{B}_{\mathbf{c}\mathbf{p}_m}$	−0.034	−4.36
$\epsilon_{\mathbf{c}y}$	0.801	37.08	$\mathscr{B}_{\mathbf{c}y}$	−0.001	−0.71

the last term being negative.[13] Briefly, although $\mathbf{c}^{*a}(\mathbf{p}_{nt}, y_{nt}; \alpha_n, \bar{\rho}_{pnt})$ and $\mathbf{c}^{*a}(\mathbf{p}_{nt}, y_{nt}; \alpha_n, \tilde{\rho}_{pnt})$ do not coincide, the aggregation biases can be calculated suitably on the basis of (34). All aggregation biases given in Section 6.2 are adjusted accordingly. Hence, for studying the second-order derivatives of $\mathbf{c}^*$, the computation of some third-order derivatives of the input demand functions is required. In my empirical analysis, the difference between the adjusted and nonadjusted aggregation bias is negligible as a consequence of the very small estimates for $\sigma^2_{\rho j}$.

The estimates $\hat{\alpha}_n$ and $\hat{\gamma}_n$ are then used for calculating the (extended) aggregation biases on the basis of (27)–(30). Under the null hypothesis, the disaggregate explanatory variables are known and the covariance between the estimates $\hat{\alpha}_n$ and $\hat{\gamma}_n$ is zero. The aggregate elasticities obtained from (31) and the corresponding aggregation bias are reported in Table 5. All elasticities are computed for values of the explanatory variables equal to those of the industry producing the median level of output. The Student t-value of the estimated statistic is computed by first-order approximation of the true variance around the point estimate. The unknown variances $\sigma^2_{\rho j}$ are estimated by $\hat{\sigma}^2_{\rho j} = \hat{\omega}_j' \hat{\omega}_j / (NT - K_{\rho j})$, and their covariances by

$$\mathrm{Cov}\left(\hat{\sigma}^2_{\rho i}, \hat{\sigma}^2_{\rho j}\right) = \begin{cases} 0 & \text{for } i \neq j \\ 2\hat{\sigma}^4_{\rho i} / (NT - K_{\rho i}) & \text{for } i = j \end{cases}$$

where $K_{\rho j}$ denotes the number of regressors in $\tilde{\rho}_{pj}$.

Table 5 gives the estimated cost elasticities with respect to $\mathbf{p}_{nt}$ and y_{nt}. All cost elasticities (for models 1–3) are calculated on the basis of $\mathbf{c}^{*a}(\mathbf{p}_{nt}, y_{nt}; \alpha_n, \rho_{pnt})$, that is,

[13] Since $\partial^2 \mathbf{c}^{*a} / \partial \rho^2_{pjnt} = -\mathbf{p}^2_{njt} \partial x^{*a}_j / \partial p_{jnt}$, the last term of (34) is indeed negative.

for given relative input prices ρ_{pnt}. The aggregation bias, computed as described above, is found to be significant for aggregate labor in model 1 and for aggregate material input in models 2 and 3.

What do these biases mean? The aggregation biases measure the difference between the true elasticities (obtained from $\mathbf{c}^{*a}$) and those computed on the basis of the simplified model $\mathbf{c}^{*}$. As the aggregate cost function $\mathbf{c}^{*a}$ is identical to the microeconomic cost function c^{*}, the elasticities computed on the basis of $\mathbf{c}^{*a}$ or c^{*} must also be identical for the inputs that are not aggregated. (That is also the reason that $\epsilon_{\mathbf{c}p_k}$ is equal to 0.101 in each model.) Besides, the cost shares for aggregate inputs, computed on the basis of $\mathbf{c}^{*a}$, are related to the elementary cost shares by $\epsilon_{\mathbf{cp}_\ell} = \epsilon_{cp_h} + \epsilon_{cp_s} + \epsilon_{cp_u}$ and $\epsilon_{\mathbf{cp}_m} = \epsilon_{cp_e} + \epsilon_{cp_m}$. Indeed, for given ρ_{pnt}, it follows from $\mathbf{c}^{*a}(\mathbf{p}_{nt}, y_{nt}; \alpha_n, \rho_{pnt}) = c^{*}(\mathbf{p}_{nt} * \rho_{pnt}, y_{nt}; \alpha_n)$ that

$$\epsilon_{\mathbf{cp}_m} = \frac{\partial \mathbf{c}^{*a}}{\partial \mathbf{p}_m}\frac{\mathbf{p}_m}{\mathbf{c}^{*a}} = \left(\frac{\partial c^{*}}{\partial p_e}\rho_{pent} + \frac{\partial c^{*}}{\partial p_m}\rho_{pmnt}\right)\frac{\mathbf{p}_m}{\mathbf{c}^{*a}} = \epsilon_{cp_e} + \epsilon_{cp_m}$$

Such a relationship is satisfied in Table 5. Briefly, elasticities computed from $\mathbf{c}^{*a}$ only lead to a loss of information. In contrast, comparative statics done on the basis of the simplified model $\mathbf{c}^{*}$ will also measure the consequences of a shift in $\bar{\rho}_{pnt}$. For this reason, the cost elasticities with respect to aggregate prices that are based on $\mathbf{c}^{*}$ could differ from the sum of the disaggregate elasticities. In the empirical application, this is the case for aggregate labor in model 1 and aggregate material input in models 2 and 3. Even the elasticities of some inputs that are not aggregated may be significantly affected by the aggregation of other inputs: in model 2, the bias $\mathscr{B}_{\mathbf{c}p_u}$ on $\epsilon_{\mathbf{c}p_u}$ is significant. Note that the bias $\mathscr{B}_{\mathbf{cp}_\ell}$ is significant in model 1, where only labor inputs are aggregated, but is no longer significant after material inputs have also been aggregated (model 3).

Which conclusions can be drawn concerning the inheritance of regularity properties? From Section 6.2, it follows that Hotelling's lemma is rejected for labor in model 1, for unskilled labor and aggregate material inputs in model 2, and for aggregate material input in model 3. That is, $\partial \mathbf{c}^{*}/\partial \mathbf{p}_{jnt}$ does not approximately coincide with $\partial \mathbf{c}^{*a}/\partial \mathbf{p}_{jnt} = -\mathbf{x}_j^{*a}$ for these inputs. This rejection may appear surprising given the very high correlation between elementary and aggregate wages (Table A.1), but it is also noteworthy that the size of aggregation bias is small in comparison to the corresponding elasticity. For instance, in model 1, the bias on the true labor share $\epsilon_{\mathbf{cp}_\ell}$ was found to be significantly negative: the bias $\mathscr{B}_{\mathbf{cp}_\ell} = -0.023$, however, represents only 6.2 percent of the true labor share. This suggests that even though Hotelling's lemma is statistically rejected, assuming it to hold might not lead to dramatically different results for model 1.

The linear homogeneity in prices of the cost function is also statistically rejected: a look at Table 6 shows that in all three models the "homogeneity of degree one bias" $\mathscr{B}_{h1}$ is found to be significant. By definition, $\sum_{j=k,\ell,e,m} \epsilon_{\mathbf{c}p_j} + \mathscr{B}_{h1} = \sum_{j=k,\ell,e,m} \epsilon^{*}_{\mathbf{c}p_j}$, where $\epsilon^{*}_{\mathbf{c}p_j}$ denotes the share for input j computed on the basis of $\mathbf{c}^{*}$. Therefore, a significant statistic $\mathscr{B}_{h1}$ means that the cost shares computed on the basis of the simplified model $\mathbf{c}^{*}$ may not add to one. Although it is significant, the bias $\mathscr{B}_{h1}$ is

TABLE 6

TESTING LINEAR HOMOGENEITY IN AGGREGATE INPUT PRICES

	Estimates	t-Statistic
Model 1		
$\mathscr{B}_{h1}$	−0.025	−12.42
Model 2		
$\mathscr{B}_{h1}$	−0.017	−3.57
Model 3		
$\mathscr{B}_{h1}$	−0.036	−9.07

relatively small since it represents less than five percent of the sum of the cost shares (which is one).

In my empirical illustration, neglecting aggregation across inputs does seem to imply not only a loss of information but also a loss of microeconomic regularities. The estimated parameter vector $\hat{\boldsymbol{\alpha}}$ of the simplified model will not reflect solely microeconomic behavior; it also measures shifts in the underlying distribution of elementary prices. As a consequence, some microeconomic regularities are invalidated in the aggregate.

Similarly, the biases on second-order derivatives of the cost function may be calculated. The own-price and output elasticities (for constant relative elementary prices) are given in Table 7. The corresponding cross-price elasticities are listed in the Appendix (Tables A.4–A.6). The assumption of vanishing aggregation biases is again often rejected.

It is easy to show that for constant ρ_p, the elasticities $\epsilon_{\ell \mathbf{p}_\ell}$ and $\epsilon_{\ell y}$ are related to the disaggregate elasticities by

TABLE 7

OWN-PRICE, OUTPUT ELASTICITIES, AND THE CORRESPONDING BIAS

Own-Price Elasticities			Biases			Output Elasticities			Biases		
	Value	t-Statistic		Value	t-Statistic		Value	t-Statistic		Value	t-Statistic
Model 1											
ϵ_{kp_k}	−0.038	−2.28	$\mathscr{B}_{kp_k}$	−0.000	−0.28	ϵ_{ky}	0.455	13.53	$\mathscr{B}_{ky}$	0.002	0.15
$\epsilon_{\ell \mathbf{p}_\ell}$	−0.100	−5.13	$\mathscr{B}_{\ell \mathbf{p}_\ell}$	−0.119	−6.79	$\epsilon_{\ell y}$	0.392	10.84	$\mathscr{B}_{\ell y}$	−0.024	−6.56
ϵ_{ep_e}	−0.124	−1.91	$\mathscr{B}_{ep_e}$	0.001	0.21	ϵ_{ey}	0.957	5.77	$\mathscr{B}_{ey}$	−0.015	−0.36
ϵ_{mp_m}	−0.094	−7.55	$\mathscr{B}_{mp_m}$	0.001	0.37	ϵ_{my}	1.162	52.89	$\mathscr{B}_{my}$	−0.001	−0.38
Model 2											
ϵ_{kp_k}	−0.038	−2.28	$\mathscr{B}_{kp_k}$	0.003	0.43	ϵ_{ky}	0.455	13.53	$\mathscr{B}_{ky}$	−0.075	−1.30
ϵ_{hp_h}	−0.276	−1.85	$\mathscr{B}_{hp_h}$	0.002	0.07	ϵ_{hy}	1.666	10.81	$\mathscr{B}_{hy}$	0.470	0.29
ϵ_{sp_s}	−0.085	−1.58	$\mathscr{B}_{sp_s}$	−0.005	−0.77	ϵ_{sy}	0.363	13.92	$\mathscr{B}_{sy}$	−0.084	−1.03
ϵ_{up_u}	−0.681	−5.30	$\mathscr{B}_{up_u}$	0.061	1.90	ϵ_{uy}	0.356	3.72	$\mathscr{B}_{uy}$	0.487	2.14
$\epsilon_{\mathbf{m}\mathbf{p}_\mathbf{m}}$	−0.082	−6.23	$\mathscr{B}_{\mathbf{m}\mathbf{p}_\mathbf{m}}$	−0.111	−4.59	$\epsilon_{\mathbf{m}y}$	1.155	51.70	$\mathscr{B}_{\mathbf{m}y}$	−0.073	−4.60
Model 3											
ϵ_{kp_k}	−0.038	−2.28	$\mathscr{B}_{kp_k}$	0.003	0.34	ϵ_{ky}	0.455	13.53	$\mathscr{B}_{ky}$	−0.071	−1.22
$\epsilon_{\ell \mathbf{p}_\ell}$	−0.100	−5.13	$\mathscr{B}_{\ell \mathbf{p}_\ell}$	−0.105	−6.03	$\epsilon_{\ell y}$	0.392	10.84	$\mathscr{B}_{\ell y}$	0.063	3.40
$\epsilon_{\mathbf{m}\mathbf{p}_\mathbf{m}}$	−0.082	−6.23	$\mathscr{B}_{\mathbf{m}\mathbf{p}_\mathbf{m}}$	−0.112	−4.62	$\epsilon_{\mathbf{m}y}$	1.155	51.70	$\mathscr{B}_{\mathbf{m}y}$	−0.076	−4.72

$$\epsilon_{\ell \mathbf{p}_\ell} = \frac{\partial \mathbf{x}_\ell^{*a}}{\partial \mathbf{p}_\ell} \frac{\mathbf{p}_\ell}{\mathbf{x}_\ell^{*a}} = \sum_{i=h,s,u} \sum_{j=h,s,u} \frac{p_j}{\mathbf{p}_\ell} \frac{\partial x_j^*}{\partial p_i} \frac{p_i}{\mathbf{p}_\ell} \frac{\mathbf{p}_\ell}{\mathbf{x}_\ell^{*a}} = \sum_{i=h,s,u} \sum_{j=h,s,u} \frac{p_j x_j^*}{\mathbf{p}_\ell \mathbf{x}_\ell^{*a}} \epsilon_{jp_i}$$

and

$$\epsilon_{\ell y} = \frac{\partial \mathbf{x}_\ell^{*a}}{\partial y} \frac{y}{\mathbf{x}_\ell^{*a}} = \sum_{j=h,s,u} \frac{p_i}{\mathbf{p}_\ell} \frac{\partial x_i^*}{\partial y} \frac{y}{\mathbf{x}_\ell^{*a}} = \sum_{j=h,s,u} \frac{p_j x_j^*}{\mathbf{p}_\ell \mathbf{x}_\ell^{*a}} \epsilon_{jy}$$

The aggregate elasticities are a weighted mean of the respective disaggregate elasticities, the weights being given by the corresponding cost shares. As skilled-labor costs constitute the more important part of total labor costs, this explains why $\epsilon_{\ell y}$ is near to ϵ_{sy}. In the simplified model relying on $\mathbf{c}^*(\mathbf{p}_{nt}, y_{nt}; \boldsymbol{\alpha}_n)$, however, the elasticities are also subject to an impact of a shift in elementary wages: $\epsilon^*_{\ell \mathbf{p}_\ell} \equiv \epsilon_{\ell \mathbf{p}_\ell} + \mathscr{B}_{\ell \mathbf{p}_\ell}$ and $\epsilon^*_{\ell y} \equiv \epsilon_{\ell y} + \mathscr{B}_{\ell y}$. This last impact was found to be statistically significant in models 1 and 3. This means that the small change in relative wages that was observed during the period of consideration (see Tables 2–4) contributed to increase the own-price responsiveness of aggregate labor. The bias on the output elasticity can be interpreted similarly. Note, however, that this bias $\mathscr{B}_{\ell y}$ is found to be significantly negative in model 1, whereas in model 3 it is positive.

The above empirical results may appear rather pessimistic but not as hopeless as some results of exact aggregation theory are. The occurrence of an aggregation bias, indeed, does not preclude an empirical analysis, but the underlying model should depart in some respect from microeconomic theory. It is important to note that microeconomic theory is not useless, since it was relied on for the determination of the explanatory variables of the aggregate model.

9. CONCLUSION

In this article, it has been shown that it is not pure nonsense to consider a relationship having some aggregates as arguments. Moreover, an aggregate representation of the initial problem is possible without restrictions on the admissible microeconomic functional forms. Such a representation exists whether goods are optimally allocated or not. These findings contrast with the pessimistic conclusions of the exact aggregation approach for which an aggregate representation is only possible under rather implausible restrictions on microeconomic relationships.

However, it was also shown that the aggregate parameters do not in general reflect only economic behavior but also shift in the distribution of elementary goods and prices. Such a feature was underlined by, for example, Stoker (1984) in the context of aggregation across individuals. Therefore, several regularity properties of individual behavior may be lost at the aggregate level: linear homogeneity, Hotelling's lemma, and convexity in the aggregate prices are not necessarily satisfied by profit functions. The imposition of microeconomic regularities at the aggregate level (across goods) may lead to further estimation biases.

When the conditions for an exact aggregation are not satisfied, however, the fact that the parameters to be estimated are "complicated combinations of technological characteristics and the distribution of inputs'' (Blackorby et al., 1986) does not only imply a loss of some regularity properties in the aggregate. Another consequence is that the interpretation of the aggregate estimations must be enlarged: the marginal impact of aggregate variables must be thought of as being composed of a direct and an indirect effect on the distribution of elementary components.

Three main recommendations for economic research can be drawn from my analysis. First, any restriction at the aggregate level that is not motivated by empirical purposes (as identification, collinearity, parsimony of parameterization, etc.) should be avoided, or at least tested. Second, as already underlined by Stoker (1984), the interpretation of aggregate estimates in terms of individual behavior should be given up. Third, since in general aggregate parameters are complex functions of elementary components of the model, their stability over the sample should be investigated. These suggestions are not new: they can be found in several textbooks and are seen as both reasonable and valuable by most economists. Surprisingly, these issues are rarely considered in empirical studies. Although presented in textbooks (e.g., Berndt, 1991), tests of the validity of microeconomic restrictions or of the stability of parameters are not often carried out in economic analysis.

APPENDIX

A.1. *Descriptive Statistics*

TABLE A.1

PEARSON CORRELATION COEFFICIENTS

	p_{knt}	p_{hnt}	p_{snt}	p_{unt}	p_{ent}	p_{mnt}	$\mathbf{p}_{\ell nt}$	$\mathbf{p}_{\mathbf{m}nt}$
p_{knt}	1.000							
p_{hnt}	0.694	1.000						
p_{snt}	0.557	0.857	1.000					
p_{unt}	0.559	0.814	0.945	1.000				
p_{ent}	0.684	0.524	0.423	0.389	1.000			
p_{mnt}	0.677	0.690	0.609	0.595	0.610	1.000		
$\mathbf{p}_{\ell nt}$	0.549	0.857	0.990	0.966	0.397	0.594	1.000	
$\mathbf{p}_{\mathbf{m}nt}$	0.687	0.696	0.611	0.597	0.668	0.989	0.594	1.000

TABLE A.2

AVERAGE (OVER n) RELATIVE PRICES

	1978	1980	1982	1984	1986	1988	1990
$p_{ht}/\mathbf{p}_{\ell t}$	1.790	1.776	1.779	1.794	1.798	1.828	1.802
$p_{st}/\mathbf{p}_{\ell t}$	1.079	1.075	1.062	1.055	1.048	1.037	1.033
$p_{ut}/\mathbf{p}_{\ell t}$	0.885	0.887	0.888	0.887	0.888	0.887	0.886
$p_{et}/\mathbf{p}_{\mathbf{m}t}$	1.000	1.176	1.373	1.382	1.247	1.170	1.213
$p_{mt}/\mathbf{p}_{\mathbf{m}t}$	1.000	0.989	0.976	0.977	0.987	0.994	0.992

TABLE A.3
AVERAGE SHARES (OVER n) OF ELEMENTARY INPUT IN THE AGGREGATE

	1978	1980	1982	1984	1986	1988	1990
$h_t/\mathbf{x}_{\ell t}$	0.020	0.022	0.024	0.026	0.028	0.031	0.033
$s_t/\mathbf{x}_{\ell t}$	0.511	0.511	0.532	0.544	0.555	0.572	0.585
$u_t/\mathbf{x}_{\ell t}$	0.469	0.467	0.444	0.430	0.417	0.397	0.381
$e_t/\mathbf{x}_{\mathbf{m}t}$	0.067	0.063	0.061	0.059	0.055	0.052	0.049
$m_t/\mathbf{x}_{\mathbf{m}t}$	0.933	0.937	0.939	0.941	0.945	0.948	0.951

A.2. *Cross-Price Elasticities*

TABLE A.4
CROSS-PRICE ELASTICITIES AND AGGREGATION BIASES, MODEL 1

	Value	t-Statistic		Value	t-Statistic
$\epsilon_{k\mathbf{p}_\ell}$	−0.012	−0.54	$\mathcal{B}_{k\mathbf{p}_\ell}$	0.000	0.47
ϵ_{kp_e}	0.013	1.66	$\mathcal{B}_{kp_e}$	0.000	0.11
ϵ_{kp_m}	0.037	2.48	$\mathcal{B}_{kp_m}$	0.007	1.35
$\epsilon_{\ell p_k}$	−0.032	−0.54	$\mathcal{B}_{\ell p_k}$	0.000	−0.47
$\epsilon_{\ell p_e}$	−0.007	−1.23	$\mathcal{B}_{\ell p_e}$	−0.003	−0.63
$\epsilon_{\ell p_m}$	0.110	6.57	$\mathcal{B}_{\ell p_m}$	−0.007	−0.68
ϵ_{ep_k}	0.068	1.67	$\mathcal{B}_{ep_k}$	0.000	0.11
$\epsilon_{e\mathbf{p}_\ell}$	−0.139	−1.23	$\mathcal{B}_{e\mathbf{p}_\ell}$	−0.064	−0.63
ϵ_{ep_m}	0.194	1.38	$\mathcal{B}_{ep_m}$	0.011	0.59
ϵ_{mp_k}	0.007	2.49	$\mathcal{B}_{mp_k}$	0.001	1.35
$\epsilon_{m\mathbf{p}_\ell}$	0.080	6.57	$\mathcal{B}_{m\mathbf{p}_\ell}$	−0.005	−0.68
ϵ_{mp_e}	0.007	1.39	$\mathcal{B}_{mp_e}$	0.000	0.59

TABLE A.5
CROSS-PRICE ELASTICITIES AND AGGREGATION BIASES, MODEL 2

	Value	t-Statistic		Value	t-Statistic
ϵ_{kp_h}	−0.003	−0.42	$\mathcal{B}_{kp_h}$	−0.002	−0.94
ϵ_{kp_s}	−0.004	−0.16	$\mathcal{B}_{kp_s}$	−0.014	−1.51
ϵ_{kp_u}	−0.005	−0.18	$\mathcal{B}_{kp_u}$	−0.003	−0.36
$\epsilon_{k\mathbf{p}_\mathbf{m}}$	0.050	2.86	$\mathcal{B}_{k\mathbf{p}_\mathbf{m}}$	−0.036	−0.79
ϵ_{hp_k}	−0.031	−0.42	$\mathcal{B}_{hp_k}$	−0.026	−0.94
ϵ_{hp_s}	0.339	1.45	$\mathcal{B}_{hp_s}$	0.027	0.45
ϵ_{hp_u}	0.429	2.52	$\mathcal{B}_{hp_u}$	0.027	0.25
$\epsilon_{h\mathbf{p}_\mathbf{m}}$	−0.461	−5.43	$\mathcal{B}_{h\mathbf{p}_\mathbf{m}}$	0.342	0.26
ϵ_{sp_k}	−0.002	−0.16	$\mathcal{B}_{sp_k}$	−0.005	−1.51
ϵ_{sp_h}	0.011	1.46	$\mathcal{B}_{sp_h}$	0.001	0.45
ϵ_{sp_u}	0.067	1.60	$\mathcal{B}_{sp_u}$	0.000	0.02
$\epsilon_{s\mathbf{p}_\mathbf{m}}$	0.009	0.41	$\mathcal{B}_{s\mathbf{p}_\mathbf{m}}$	−0.068	−1.03
ϵ_{up_k}	−0.005	−0.18	$\mathcal{B}_{up_k}$	−0.004	−0.36
ϵ_{up_h}	0.042	2.54	$\mathcal{B}_{up_h}$	0.003	0.25
ϵ_{up_s}	0.202	1.60	$\mathcal{B}_{up_s}$	0.000	0.02
$\epsilon_{u\mathbf{p}_\mathbf{m}}$	0.443	6.30	$\mathcal{B}_{u\mathbf{p}_\mathbf{m}}$	0.333	1.82
$\epsilon_{\mathbf{m}p_k}$	0.010	2.86	$\mathcal{B}_{\mathbf{m}p_k}$	−0.007	−0.79
$\epsilon_{\mathbf{m}p_h}$	−0.008	−5.48	$\mathcal{B}_{\mathbf{m}p_h}$	0.006	0.26
$\epsilon_{\mathbf{m}p_s}$	0.005	0.41	$\mathcal{B}_{\mathbf{m}p_s}$	−0.035	−1.03
$\epsilon_{\mathbf{m}p_u}$	0.075	6.32	$\mathcal{B}_{\mathbf{m}p_u}$	0.057	1.82

TABLE A.6
CROSS-PRICE ELASTICITIES AND AGGREGATION BIASES, MODEL 3

	Value	t-Statistic		Value	t-Statistic
$\epsilon_{k\mathbf{p}_\ell}$	−0.012	−0.54	$\mathcal{B}_{k\mathbf{p}_\ell}$	−0.027	−0.98
$\epsilon_{k\mathbf{p}_\mathbf{m}}$	0.050	2.86	$\mathcal{B}_{k\mathbf{p}_\mathbf{m}}$	−0.025	−0.55
$\epsilon_{\ell p_k}$	−0.003	−0.54	$\mathcal{B}_{\ell p_k}$	−0.007	−0.98
$\epsilon_{\ell\mathbf{p}_\mathbf{m}}$	0.103	5.55	$\mathcal{B}_{\ell\mathbf{p}_\mathbf{m}}$	0.049	2.79
$\epsilon_{\mathbf{m}p_k}$	0.010	2.86	$\mathcal{B}_{\mathbf{m}p_k}$	−0.005	−0.55
$\epsilon_{\mathbf{m}\mathbf{p}_\ell}$	0.072	5.55	$\mathcal{B}_{\mathbf{m}\mathbf{p}_\ell}$	0.034	2.79

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