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Tests of Representative Firm Models: Results for German Manufacturing Industries

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Abstract

Many studies of producer behavior consider cost and input demand functions derived from microeconomic theory and estimate them on the basis of aggregate data. If the characteristics of the firms differ, the negligence of heterogeneity can lead to estimation bias. An alternative is to restrict individual behavioral functions to being linear in the firm specific parameters. The aim of this paper is to describe aggregate producer behavior without placing too strong restrictions on functional form and to explicitly account for firm heterogeneity. Estimation for German manufacturing sectors confirms that neglected heterogeneity is an important source of bias in representative firm models.

Keywords: exact aggregation, representative firm, heterogeneity, demand system

1. Introduction

Many studies involved in the modeling of producer behavior consider cost and input demand functions derived from *microeconomic* theory and estimate them on the basis of *aggregate* data. Following the work of Diewert (1973), the selected functional forms are often required to provide a second order local approximation to an arbitrary function. However, when the aggregation of individual cost functions is not carried out explicitly and aggregate data appear instead in microeconomic relations in a purely ad hoc fashion, an aggregation bias can emerge as soon as firms differ in any explanatory variable.

In order to alleviate aggregation bias, some authors suggest to restrict individual behavior to be linear in the variables subject to heterogeneity. In this case the aggregate form will only depend on aggregate variables. Doing so, Appelbaum (1982) and Borooah and Van der Ploeg (1986), for example, model producer behavior on the basis of cost functions linear in the production level. The resulting independence of marginal costs from the production level seems restrictive however, and could now lead to an approximation bias.

An alternative solution is to adopt the flexible functional forms from the first approach and also to proceed to linear aggregation as seen in the second approach. This route is followed by Lewbel (1988) or by Heineke (1993) among others in the context of consumer analysis. In production economics, Dickson (1994) offers one of the few contributions along those lines. In particular, he considers both aggregation and approximation issues simultaneously, and shows that when firms differ in the level of production, a Herfindahl index emerges in the aggregated translog cost function. Another originality of this approach is to give economic foundations to the relationship between concentration, costs, and input demands. In his study however, Dickson only considers differences in the level of production. Yet, heterogeneity also characterizes firms' production processes. Moreover, it seems plausible on economic grounds, that both kinds of heterogeneity are interrelated.

The original framework developed by Fortin (1988, 1991) allows heterogeneity in both production processes and explanatory variables. We choose to adapt this latter approach to optimized relations (firms are minimizing costs) and to flexible functional forms for each firm. After exact aggregation across all firms belonging to the same industry, some distributional statistics appear in the aggregate relations. When not available, these statistics are assumed constant over the period and estimated along with technology parameters. Then aggregation bias can be identified and the representative firm model, nested within ours, can be tested. This is done for 29 German manufacturing industries considered over the period 1960 to 1992.

Section 2 is devoted to the discussion of some aggregation results, Section 3 describes the model proposed. The database and available concentration statistics are discussed in Section 4. The estimation procedure and the results are discussed in Section 5.

2. Linear Aggregation of Cost Functions

We consider that every basis unit i has a production process belonging to the same parametric functional family, parameterized by a vector $\alpha_i \in \mathbb{R}^k$:

$$c\left(p_t, t, y_{it}, \alpha_i\right) = \min_{x_{it}} \left\{ p_t' x_{it} : f\left(x_{it}, y_{it}, t, \alpha_i\right) \leq 0 \right\}, \tag{1}$$

where $p_t \in \mathbb{R}_{++}^{S_p}$ and $x_{it} \in \mathbb{R}_{+}^{S_{x_i}}$, where S_v denotes the dimension of a vector v; f is a transformation function, $y_{it} \in \mathbb{R}_+$ is the output level and t a time index also reflecting the effect of technological change on production technology, since it appears explicitly in functions f and c. The subscript i characterizes variables specific to each decision unit: input prices are assumed identical within an industry. Thus, the present analysis relies on the restrictive assumptions of price and product homogeneity (within a given industry as described below).[1] Further work should try to also study aggregation across goods (in addition to aggregation across decision entities), in order to test the composite commodity assumption underlying many analyses.

Total costs of an industry are given by $C_t \equiv \sum_{i=1}^{n_t} p_t' x_{it}$ and coincide with the linear aggregated cost function (across the n_t production units), defined as:

$$C\left(p_t, t, y_{1t}, \ldots, y_{nt}, \alpha_1, \ldots, \alpha_{n_t}\right) \equiv \sum_{i=1}^{n_t} c\left(p_t, t, y_{it}, \alpha_i\right). \tag{2}$$

The amount of information required in order to exactly describe industry costs and associated input demands is large; for modeling, additional assumptions are clearly required. An appealing approach is to consider that the joint distribution of production levels and individual characteristics (y_{it}, α_i) can be parameterized by a finite number of statistics grouped in the vector Υ_t. Then we can rewrite

$$C_t = C^a (p_t, t, \Upsilon_t) = \sum_{i=1}^{n_t} c (p_t, t, y_{it}, \alpha_i) . \tag{3}$$

The variables entering Υ_t will depend on the form of the microeconomic functions c and on the distribution form of the heterogeneity characteristics (y_{it}, α_i).

Gorman (1990), Heineke and Shefrin (1988), Lau (1982) and others show that the microeconomic functional forms c satisfying such a reparameterization (from (2) to (3)) belong to the class of *Gorman polar forms*:

$$c (p_t, t, y_{it}, \alpha_i) = \sum_{j=1}^{J} c_{1j} \left(p_t, t, \alpha_{1j}\right) a \left(y_{it}, \alpha_{i3j}\right) + c_2 (p_t, t, \alpha_{i2}) . \tag{4}$$

The parameter vector α_i' is now decomposed into $\left(\alpha_1', \alpha_{i2}', \alpha_{i3}'\right) \in \mathbb{R}^{S_{\alpha 1}} \times \mathbb{R}^{S_{\alpha 2}} \times \mathbb{R}^{S_{\alpha 3}}$, with α_1 being identical for all production units and α_{2i} and α_{3i} free to vary with for each unit i. Thus, the output aggregators $a \left(y_{it}, \alpha_{i3j}\right)$ can also differ across production units. Here, the validity of such a reparameterization is assumed; the underlying restrictions are discussed by Heineke and Shefrin (1988). Moreover, Gorman (1981) and Heineke and Shefrin (1988) show that under individual rationality, the summation index J is finite (see also Heineke, (1993), for a discussion).

2.1. The Representative Firm and Aggregation Bias

The assumption of a representative firm usually means that a whole industry or economy can be considered as a single optimizing agent. Macroeconomic work relying on this assumption models n_t firms, each producing $y_{it} \in \mathbb{R}_+$ and each having a production process characterized by $\alpha_i \in \mathbb{R}^{S_{\alpha i}}$, as a single unit characterized by a macroeconomic cost function $C^m (p_t, t, Y_t, \alpha^m)$, where $Y_t = Y (y_{1t}, \ldots, y_{nt})$ is a scalar and Y a continuously differentiable, strictly increasing output aggregator function. The dimension of the vector α^m of macroeconomic parameters depends on the functional form retained for C^m; the relationships between the α^m and the underlying microeconomic parameters α_i, are not often explicitly presented in macroeconomic models. Furthermore, many macroeconomic studies consider aggregate output to be:

$$Y_t = \sum_{i=1}^{n_t} y_{it}. \tag{5}$$

Given these specifications, the restrictions underlying macroeconomic studies are more stringent than those required by (3) and (4). C^m is a particular case of C^a and $C^m = C^a$ when the vector Υ_t in equation (3) has Y_t and α^m as its only components. In general, it is not ensured that $C^m (p_t, t, Y_t, \alpha^m)$ is the right object for modeling total costs C_t, but several approaches relying on a specific definition of a representative firm can justify such a practice.

Definitions

The representative agent framework is said to hold when the macroeconomic cost function C^m verifies condition (a), (b) or (c):
(a) $\sum_{i=1}^{n_t} c\,(p_t, t, y_{it}, \alpha_i) = C^m\,(p_t, t, Y_t, \alpha^m)$ for all $p_t,\ y_{1t}, \ldots, y_{nt}$ and some set of α.
(b) $\mathfrak{B} \equiv C^m\,(p_t, t, Y_t, \alpha^m) - C^a\,(p_t, t, \Upsilon_t) = 0$ for some set of $y_{1t}, \ldots, y_{nt}$, and α.
(c) Condition (b) holds and the aggregate cost function C^a verifies all microeconomic properties.

Definition (a) requires that total costs coincide with the macroeconomic cost function C^m defined above. Of course, this occurs when all firms have the same characteristics and production levels, but this extreme case does not fulfill definition (a), which must hold for any production level. A direct adaptation of Gorman (1968) permits us to state necessary and sufficient conditions for (a) to hold, namely, that all microeconomic cost functions take the form:

$$\overline{c} = \widetilde{c}_1\,(p_t, t, \alpha_1)\, y_{it} + c_2\,(p_t, t, \alpha_{2i})\,. \tag{6}$$

This functional form is clearly nested within the class of Gorman Polar forms given by (4) ($J = 1$ and $a\,(y_{it}, \alpha_{3i}) = y_{it}$). Thus, for definition (a) to be valid, restrictions on the form of the microeconomic cost function, the distribution of the firms' characteristics and the output aggregator function $a\,(y_{it}, \alpha_{3i})$ are required. In the following, the function (6) is called the output linear Gorman polar form. If a representative firm framework as defined in (a) exists, then C^a is identical to C^m and the vector Υ_t parameterizing the distribution of individual characteristics is $\Upsilon_t^m \equiv (Y_t, \alpha^m) = \left(Y_t, \alpha_1, \alpha_{21}, \ldots, \alpha_{2n_t}\right)'$. The amount of information α_{2i} required may still appear to be important (α_2 has $S_{\alpha_2} \times n_t$ components), but is largely reduced when the model is linear in the parameters α_{2i} to be estimated, as is usually the case. Then, microeconomic costs can be written as

$$\widetilde{c} = \widetilde{c}_1\,(p_t, t, \alpha_1)\, y_{it} + \widetilde{c}_2\,(p_t, t)'\, \alpha_{2i}\,,$$

where $\widetilde{c}_2\,(p_t, t)$ is a column vector of functions of p_t and t. In this case

$$\Upsilon_t^m = \left(Y_t, \alpha_1', \sum_{i=1}^{n_t} \alpha_{2i}'\right)' \in \mathbb{R}_+ \times \mathbb{R}^{S_{\alpha_1}} \times \mathbb{R}^{S_{\alpha_2}}.$$

This form is adopted for example by Appelbaum (1982) and Borooah and van der Ploeg (1986). The linearity in the output level appears particularly restrictive; firstly, a redistribution of production from one firm to another has no effect on industry costs and secondly, (6) implies identical marginal costs for all firms within the industry. Besides, the manner in which heterogeneity in the production process enters the Gorman polar form is limited to the additive part $c_2\,(p_t, t, \alpha_{2i})$.

When (6) does not hold, definition (a) of a representative firm does not apply. With a more general microeconomic form, however, the amount of information required in order for (3) to hold is no longer limited to Υ_t^m, but entails additional components noted h_t. Thus $\Upsilon_t = \left(\Upsilon_t^{m\prime}, h_t'\right)'$. This leads us to definition (b), presented and discussed by Fortin (1988, 1991). Definition (b) is implied by definition (a). If one requires (b) to hold for every p_t, $y_{1t}, \ldots, y_{nt}$ then (b) coincides with (a). For some y_{it}, however, the gap $\mathfrak{B}$ between the aggregate and the macroeconomic cost function, may vanish. This term $\mathfrak{B}$ is thereafter called *aggregation bias*. Clearly, when the actual cost function is described by C^a rather than C^m, the first order derivatives based on C^m may also be biased and differ from the actual marginal effects: $\partial C^a/\partial p_{ht} = \partial C^m/\partial p_{ht} - \partial\mathfrak{B}/\partial p_{ht}$, $\partial C^a/\partial t = \partial C^m/\partial t - \partial\mathfrak{B}/\partial t$ and $\partial C^a/\partial Y_t = \partial C^m/\partial Y_t - \partial\mathfrak{B}/\partial Y_t$.

Lewbel (1993) requires that in addition to (b), the aggregate cost functions must have the same economic properties as the microeconomic ones (definition (c)).[2] From (2), it can immediately be seen that the aggregate cost function retains some of the properties of its microeconomic counterparts (continuity, positivity, linear homogeneity in prices). Chavas (1993) shows that the assumption of identity of input prices across production units is crucial for the aggregate cost function to be linear homogeneous and concave in the aggregate prices. For C^a to satisfy all properties of c, additional assumptions are necessary and are discussed below.

2.2. Aggregate Marginal Effects

In what follows, we briefly discuss the impact of a marginal variation in the explanatory variables on aggregate costs and how it is related to the corresponding microeconomic impacts. In order for the definition to be verified, Shephard's lemma should hold at the aggregate level. However, the differentiation of C^a with respect to the price p_{ht} yields:

$$\frac{dC^a}{dp_{ht}} = \frac{\partial C^a}{\partial p_{ht}} + \frac{\partial C^a}{\partial \Upsilon_t'}\frac{d\Upsilon_t}{dp_{ht}} = \sum_{i=1}^{n_t}\frac{dc}{dp_{ht}}, \tag{7}$$

which does not exactly correspond to Shepard's lemma for the function C^a: a shift in input prices could also shift the joint distribution of (y_{it}, α_i). By adapting the arguments of Lewbel (1988, 1993), we see that restrictions on the functional form or on the distribution of individual characteristics (to yield, for example, $\partial C^a/\partial\Upsilon_t' = 0$ or $d\Upsilon_t/dp_{ht} = 0$) is sufficient to imply that $dC^a/dp_t = \sum_{i=1}^{n_t}\partial c/\partial p_t$. Since y_{it} is considered as quasi-fixed and the α_i as independent from p_t, $d\Upsilon_t/dp_{ht} = 0$ is verified in the present situation. By assuming both that all firms face the same input prices and that the heterogeneity distribution does not depend on input prices, definitions (b) and (c) become equivalent. Thus, only the definitions (a) and (b) will be considered in the tests of the representative firm.

For other marginal effects however, such an equality is less likely to occur. For instance, total differentiation of (3) with respect to t yields:

$$\begin{aligned}\frac{dC^a}{dt} &= \frac{\partial C^a}{\partial p_t'}\frac{dp_t}{dt} + \frac{\partial C^a}{\partial\Upsilon_t'}\frac{d\Upsilon_t}{dt} = \sum_{i=1}^{n_t}\frac{dc}{dt}\\ &= \frac{\partial C^a}{\partial p_t'}\frac{dp_t}{dt} + \frac{\partial C^a}{\partial t} + \frac{\partial C^a}{\partial Y_t}\frac{dY_t}{dt} + \frac{\partial C^a}{\partial h_t'}\frac{dh_t}{dt}.\end{aligned}$$

The marginal effect of time can be decomposed into four distinct effects: a direct impact of technological change $\partial C^a/\partial t$ and three trend effects: in prices, in total product demand

and in the distributional statistics h_t. Thus, it seems necessary to unravel each influence in order to identify the impact of interest, as also underlined by Gouriéroux (1990). As before, restrictions on the functional form or on the distribution of heterogeneity (on $\partial C^a/\partial \Upsilon_t$ or on $d\Upsilon_t/dt$) can guarantee that $dC^a/dt = \sum_{i=1}^{n_t} \partial c/\partial t$.

Similarly, the effect of a marginal variation in total production on aggregate cost is given by

$$\frac{dC^a}{dY_t} = \frac{\partial C^a}{\partial Y_t} + \frac{\partial C^a}{\partial h_t'}\frac{dh_t}{dY_t} = \sum_{i=1}^{n_t} \frac{\partial c}{\partial y_{it}}\frac{dy_{it}}{dY_t}. \tag{8}$$

Thus, the aggregate marginal costs dC^a/dY_t appear as a weighted average of individual marginal costs. The terms dy_{it}/dY_t reflect how an additional increase in total output is distributed among firms. When the cost functions c take the output linear Gorman polar form (6), ensuring the identity between Υ_t^m and Υ_t (or equivalently, the identity between C^m and C^a), the implicit restrictions on the cost function and on the distribution of characteristics ensure that $dC^a/dY_t = dC^m/dY_t = \sum_{i=1}^{n_t} \partial c/\partial Y_t = c_1(p_t, t, \alpha_1)$. Then, the way in which total production is distributed across firms is irrelevant.

3. From Microeconomic to Aggregate Functions

The objective of this section is to derive, from a microeconomic system of cost and demand functions, the corresponding aggregate system. In order to avoid too many a priori restrictions, we consider flexible functional forms providing a local approximation to an arbitrary cost function (Diewert (1973)). Then we establish the relationship between microeconomic parameters and their aggregate counterparts, and study the potential bias emerging in costs when heterogeneity is neglected.

We assume that microeconomic cost functions belong to the class of normalized quadratic forms described by Diewert and Wales (1987, 1992):

$$\begin{aligned} c(p_t, t, y_{it}, \alpha_i) \;=\; & p_t' A_{ip} + \frac{1}{2}\left(\theta_1' p_t\right)^{-1} p_t' A_{ipp} p_t + p_t' A_{ipt} t \\ & + p_t' A_{ipy} y_{it} + \theta_2' p_t \left(\alpha_{itt} t^2 + \alpha_{ity} t y_{it} + \alpha_{iyy} y_{it}^2\right). \end{aligned} \tag{9}$$

$A_{ip} = [\alpha_{ip}]$, $A_{ipp} = A_{ipp}' = [\alpha_{ipp}]$, $A_{ipy} = [\alpha_{ipy}]$, $A_{ipt} = [\alpha_{ipt}]$, are respectively $S_x \times 1$, $S_x \times S_x$, $S_x \times 1$ and $S_x \times 1$ matrices containing some subset of the parameters α_i to be estimated. The vectors θ_1 and θ_2 (of size $S_x \times 1$) are introduced for normalization and can be estimated or arbitrarily fixed without destroying flexibility as discussed by Diewert and Wales. Among the usual properties of cost functions, the linear homogeneity and the symmetry in prices are directly imposed in (9). Besides, only $(S_x - 1) S_x/2$ parameters among the $(S_x + 1) S_x/2$ parameters of the matrix A_{ipp} can be identified separately from the S_x parameters of A_{ip}. Therefore, following Diewert and Wales, we also impose the following S_x equality constraints for identification purposes:

$$(1, \ldots, 1)\, A_{ipp} = 0. \tag{10}$$

Diewert and Wales (1987) also show that the price concavity of c is equivalent to the negative semi-definiteness of the matrix A_{ipp}. Moreover, they present a method to impose this last condition directly and show that the normalized quadratic form still remains flexible with concavity imposed, contrary to other usual specifications. All these adaptations are adopted in the present study.

When the functional form (9) is aggregated linearly across the n_t firms forming an industry, the resulting aggregate form is

$$\begin{aligned} C_t &= p_t' A_p + \frac{1}{2}\left(\theta_1' p_t\right)^{-1} p_t' A_{pp} p_t + p_t' A_{pt} t \\ &\quad + p_t' \sum_{i=1}^{n_t} A_{ipy} y_{it} + \theta_2' p_t \left(\alpha_{tt} t^2 + \sum_{i=1}^{n_t} \alpha_{ity} t y_{it} + \sum_{i=1}^{n_t} \alpha_{iyy} y_{it}^2\right), \end{aligned} \tag{11}$$

where $A_p = \left[\sum_{i=1}^{n_t} \alpha_{ip}\right]$, $A_{pj} = \left[\sum_{i=1}^{n_t} \alpha_{ipj}\right]$, for $j \in \{p, t\}$, and $\alpha_{tt} = \sum_{i=1}^{n_t} \alpha_{itt}$. The aggregate demand functions $X^* = \sum_{i=1}^{n_t} x_i^*$ are given by:

$$\begin{aligned} X^* &= A_p + \left(\theta_1' p_t\right)^{-1} A_{pp} p_t - \frac{1}{2}\left(\theta_1' p_t\right)^{-2} p_t' A_{pp} p_t \theta_1 + A_{pt} t \\ &\quad + \sum A_{ipy} y_{it} + \theta_2 \left(\alpha_{tt} t^2 + \sum \alpha_{ity} t y_{it} + \sum \alpha_{iyy} y_{it}^2\right). \end{aligned} \tag{12}$$

These aggregate relations (11) and (12) depend on non-observable variables $\sum A_{ipy} y_{it}$, $\sum \alpha_{iyy} y_{it}^2$ and $\sum \alpha_{iy} t y_{it}$. However, following Fortin (1991) and using the relationship between centered and non-centered second order moments $E[ab] = cov(a, b) + E[a]\,E[b]$, we can rewrite

$$\begin{aligned} \sum A_{ipy} y_i &= n\Omega_{py} + E\left[A_{ipy}\right] \sum y_i, \\ \sum \alpha_{ity} y_i &= n\omega_{ty} + E\left[\alpha_{ity}\right] \sum y_i, \\ \sum \alpha_{iyy} y_i^2 &= n\omega_{yy} + E\left[\alpha_{iyy}\right] \sum y_i^2, \end{aligned} \tag{13}$$

where Ω_{py} is a vector with S_x components $\omega_{p_h y}$, $h = 1, \ldots, S_x$. All ω_{jy} for $j = p_h, t, y$, symbolize the covariance between two microeconomic characteristics: the parameter α_{ijy} and the level of production (or its square in the last case).

Using these relations, total costs C_t can be represented by the aggregate cost function

$$\begin{aligned} & C^a\left(p_t, t, Y_t, \alpha, n_t, \sum y_{it}^2, \Omega_{py}, \omega_{ty}, \omega_{yy}\right) \\ &\quad = p_t' A_p + \frac{1}{2}\left(\theta_1' p_t\right)^{-1} p_t' A_{pp} p_t + p_t' A_{pt} t + p_t' A_{py} Y_t + p_t' \Omega_{py} n_t \\ &\quad\quad + \theta_2' p_t \left(\alpha_{tt} t^2 + \omega_{ty} n_t t + \alpha_{ty} t Y_t + \omega_{yy} n_t + \alpha_{yy} \sum y_{it}^2\right) \end{aligned} \tag{14}$$

which depends only on a limited number of distributional statistics. The value of $\sum y_{it}^2$ is deduced from the Herfindahl index H_t as $\sum y_{it}^2 = H_t \left(\sum y_{it}\right)^2$; the other distributional statistics are, however, not easily observable and are thus treated as parameters of our model and estimated along with α. In the present case, like Fortin (1988, 1991) or Heineke and Shefrin (1988), we can parameterize the distribution of (y_{it}, α_i) by a finite vector of moments

$$\Upsilon_t = \left(Y_t, \alpha, n_t, \sum y_{it}^2, \Omega_{py}, \omega_{ty}, \omega_{yy}\right), \tag{15}$$

where n_t is the moment of order zero, $Y_t = \sum_{i=1}^{n_t} y_{it}$ and $\alpha = \sum_{i=1}^{n_t} \alpha_i$ are the first order moments and the remaining ones are second order moments.[3] As discussed in the previous section the assumption of $\left[\partial C^a/\partial \Upsilon_t'\right][d\Upsilon_t/dp_{ht}] = 0$ implies that $\partial C^a/\partial p_t = \sum \partial c/\partial p_t = X^*$.

4. Data Description

Most of the data were provided by the Statistisches Bundesamt, the German federal statistical office. They are available for 29 West-German industrial sectors at the two-digit level of classification and cover the years 1960 to 1992. Three inputs are considered: labor, material and capital so that in our case $S_x = 3$, $x_{it} = (l_{it}, m_{it}, k_{it})'$, $X_t = (L_t, M_t, K_t)'$, and $p_t' = (p_{Lt}, p_{Mt}, p_{Kt})$; the definitions of these variables follow. Conrad and Unger (1987) split the material data into energy and other materials so that they can estimate one additional demand function. However, the energy data come from their own computations, and official energy data are available from the Statistisches Bundesamt only from 1978 onward (at this level of aggregation).

Further computations appear necessary to define some variables. The labor input is evaluated in man-hours. The yearly average hours of work are collected by the Institut für Arbeitsmarkt- und Berufsforschung for the same aggregation level. The price of materials is not published by the statistical office. Since the price index for production is published, we calculate the material input in constant prices as the difference between production and value added (in constant prices) and deduce p_{Mt} by dividing the nominal value of materials by its value in constant prices. All prices are normalized to 1 in 1991.

The capital input has the particularity of not disappearing instantaneously in the production process, but progressively. Thus, to take account of capital consumption, the real net stock of capital is retained. Since these real net values are only available since 1970 for every two-digit industrial sector, we choose to approximate the missing data by using the investment definition based on the permanent inventory rule:

$$\Delta K_t = K_{t+1} - (1 - \delta_t)\, K_t \tag{16}$$

where δ_t is the depreciation rate and $K_t = \sum_{i=1}^{n_t} k_{it}$. Since the aggregate gross real investment values ΔK_t are known for the whole period, the net capital stock can now be computed for 1969, given the value in 1970 (using 16), if the depreciation rate for this year is known. These depreciation rates are available over the whole period only in the aggregate, but by supposing that they vary by the same rate in each branch, we recover the missing depreciation rates (years 1960 to 1969) and, then, the data on the net real capital stock. The user costs of capital are derived according to $p_{Kt} = (1 + \rho_t)\, p_{\Delta Kt} - (1 - \delta_t)\, p_{\Delta Kt+1}$, where $p_{\Delta Kt}$ is the acquisition price of new capital and ρ_t is the nominal long run rate of interest. The data described so far correspond to an updated version of a data set also used by Flaig and Steiner (1993a, 1993b).

In addition, the framework developed above requires data on the number of firms n_t, and on the sum of the squared individual production levels $\sum y_{it}^2$. The first question which emerges concerns the decision unit considered. Since all published data are collected on the basis of firms, this unit is retained rather than establishments. From 1977 onwards, data for firms with more than 20 employees are available for every year; for 1960 to 1976, data pertain to firms with 10 employees or more. For the period 1960-1976, data relating to firms with 20 employees and more are calculated on the basis of the data for 1977 and the variation rates available for the previous years for firms with 10 or more employees.

The number of small firms (with less than 20 employees) is only available for the years 1961, 1970 and 1987, when the census of German firms took place.[4] The number of small firms for the years in between are recovered by linear interpolation. For the years 1987 to 1992, the trend before 1987 is used.

The needed indicator $\sum_i y_i^2$ reflecting the variance of production across firms can be computed through an Herfindahl index H_t. This index unfortunately presents three drawbacks: it refers to nominal production, is only available for the years after 1977 and is only computed for firms with more than 20 employees. The first drawback is neglected here; in fact it is negligible only if production price indices are identical among firms belonging to the same industry. How we recover Herfindahl indexes over the whole period and for all the firms is described in Appendix A.

5. Testing the Representative Firm Assumption

The model consists of the aggregate input demand functions (12), where the unobservable sums are replaced using (13). The covariances ω are assumed to be constants over the estimation period and estimated along with other parameters. Since the cost and input demand functions are linearly dependent, only the three demand functions are retained for estimation. Further, the input demand functions are divided by the production level to make the homoscedasticity of the added disturbance vector ν_t more plausible. Thus the system becomes

$$\begin{aligned}\frac{X^*}{Y_t} = {} & \left(A_p + \left(\theta_1' p_t\right)^{-1} A_{pp} p_t - \frac{1}{2}\left(\theta_1' p_t\right)^{-2} p_t' A_{pp} p_t \theta_1 + A_{pt} t\right) / Y_t \\ & + n_t \Omega_{py} / Y_t + A_{py} \\ & + \theta\left(\alpha_{tt} t^2 + \left(n_t \omega_{ty} + \alpha_{ty} Y_t\right) t + n_t \omega_{yy} + \alpha_{yy} \left(Y_t\right)^2 H_t\right) / Y_t + \nu_t. \qquad (17)\end{aligned}$$

The variable t is defined as a time trend equal to 1 in 1960 and increasing yearly by 1. The vectors θ_1 and θ_2 are both defined as X_{1991}/C_{1991} so that $\theta_1' p_t = \theta_2' p_t$ can be interpreted as Laspeyres price indices for total costs. With this specification, however, Ω_{py} and ω_{yy} are no longer identified separately. Thus, ω_{yy} is deleted and the model contains 19 parameters which will be estimated on the basis of 33 observations of the variables occurring in 3 equations.[5] Note that the approximations described at the end of the previous section suggest the need for an explicit econometric treatment of measurement errors. However, this is a difficult task within the nonlinear framework of equation (17) and, besides, it is not at all clear that any of the other explanatory variables is error free, an aspect which is seldom treated satisfactory in the literature.

The parameters are estimated in two stages. First, the concavity unrestricted model (A_{pp} is not restricted to being negative semi-definite) is estimated according to the *SUR* procedure, iterating on the residual covariance matrix; this provides the estimations $\widehat{\phi} \equiv \left(\hat{\alpha}', \widehat{\omega}'\right)'$ and the associated covariance matrix. In the second stage, the restricted parameters of the matrix A^0_{pp} satisfying concavity, stacked in a vector $\alpha^0_{pp}(u)$, are calculated using minimum distance:

$$\widehat{\alpha}^0_{pp} = \arg\min_u \left(\widehat{\alpha}_{pp} - \alpha^0_{pp}(u)\right)' \left[n_{obs} V(\widehat{\alpha}_{pp})\right]^{-1} \left(\widehat{\alpha}_{pp} - \alpha^0_{pp}(u)\right), \tag{18}$$

where n_{obs} is the number of observations in the model (33 in our case). The details of the procedure are presented in Appendix B. The resulting estimator is asymptotically equivalent to the constrained *SUR* estimator (see e.g. Gouriéroux and Monfort, (1989)).[6] This procedure only affects the estimates of α_{pp} needed for the computation of own price and substitution elasticities. The other estimates remain unchanged and thus, since the following discussion focuses mainly on $\mathfrak{B}$ and output elasticities, the related conclusions are not conditional to the concavity restriction.

Imposing concavity has, however, the consequence of increasing the sum of squared residuals in an important manner for some industries, casting some doubt on the specification of the model. When observed choices are in fact driven by virtual prices instead of market prices p_t or when firms actually face different prices, the concavity of the cost function is no longer ensured.[7] This result suggests that further studies on the specification of the model and of the functional form retained appear necessary as well as explicit aggregation over heterogeneous prices.

We now turn to the test of the representative firm assumption relying on definitions (a) and (b). The aggregate form (17) nests both models described above. Definition (a) requires that the output linear Gorman polar form (6) holds for each individual. In this case the parameters A_{ipy}, α_{ity} are identical for all firms, that is, all covariances ω vanish in (17), and $\alpha_{iyy} = 0$. The second test refers to the weaker definition (b) of the representative firm (or equivalently (c), given our assumptions). The aggregation bias $\mathfrak{B}$ corresponds to the difference between the aggregate cost function (14) and the corresponding aggregate output linear Gorman polar form:[8]

$$\begin{aligned} C^m(p_t, t, Y_t, \alpha^m) \;=\; & p_t' A_p + \frac{1}{2}\left(\theta_1' p_t\right)^{-1} p_t' A_{pp} p_t + p_t' A_{pt} t \\ & + p_t' A_{py} Y_t + p_t' \theta \left(\alpha_{tt} t^2 + \alpha_{ty} t Y_t\right). \end{aligned} \tag{19}$$

Thus

$$\mathfrak{B} = -p_t' \Omega_{py} n_t - p_t' \theta \left(n_t \omega_{ty} t + \alpha_{yy} \sum_i y^2_{it} \right). \tag{20}$$

Clearly, rejection of definition (a) does not imply the rejection of (b) since even if some parameters Ω_{py}, ω_{ty}, or α_{yy} are significantly different from zero, they may be compensated in

(20) by opposite effects and $\mathfrak{B}$ may vanish. The results of the tests of the two assumptions are summarized in the first three columns of Table 1 (see Appendix C). The test on the significance of $\mathfrak{B}$ is reported for 1976, the middle year of our sample.

Definition (a), a widely accepted definition of the representative agent, is always rejected except for two industries (oil refining and steel, No. 15 and 21).

Tests of definition (b) could appear to yield more optimistic results, although the aggregation bias is still significative in 15 out of 29 industries. The bias $\mathfrak{B}$ is shown to be important in absolute value ($\mathfrak{B}$ represents more than a fourth of total costs in 16 industries), results which seem comparable to those of Fortin (1991, Table 1), although she estimated aggregation bias in a different framework. It is worth noting that the test results do not contradict the nesting of assumptions (a) and (b): this was not mechanically guaranteed since the test of assumption (a) relies on estimated parameters only, whereas the test of assumption (b) also involves data pertaining to the year 1976 (see equation (20)).

Apart from the value of $\mathfrak{B}$ itself, it is also interesting to evaluate the impact of $\mathfrak{B}$ on the first derivatives of the cost function as elasticities and rates of return to scale. The fourth column of Table 1 reports the estimates of the elasticity of aggregate costs with respect to output, $\epsilon_{C^aY} \equiv \partial C^a/\partial Y_t \times Y_t/C^a$ (the inverse of the rate of return to scale). These estimates can be compared with those based on the output linear Gorman polar form (column five) to obtain an idea of the impact of neglecting aggregation. Although an eyeball test suggests that differences between both models may not be too important, for nine industries the tests for constant return to scale lead to diverging conclusions (industries No. 15, 18, 27, 31, 32, 35, 37, 40 and 41), yet in three of these cases $\mathfrak{B}$ turned out to be insignificant (No. 15, 31 and 32).

The estimation of the elasticity of costs with respect to the Herfindahl index is reported in the sixth column of Table 1. This elasticity is small (in absolute value) for all industries and significative in only nine industries. Four industries having a significant ϵ_{C^aH}, also have an insignificant aggregation bias. Again, this remark underlines that even when $\mathfrak{B}$ is not significant in absolute value, its first derivatives may be significant. In comparison to Dickson (1994), the impact of the Herfindahl index on costs is found to be less important and more often positive. This may result from the fact that Dickson does not consider heterogeneity in production processes. However his study presents the advantage of retaining four digit industries producing relative homogeneous products. In this context, neglecting aggregation across different inputs, outputs and prices appears less problematic than here (Dickson, (1994 p.450), however, also reports cases of concavity violations).

Technological change, measured by the time trend t, reduces costs in most of the cases (see Table 1): ϵ_{C^mt} is significant positive for only one industry (No. 15), whereas ϵ_{C^at} is positive for three cases (industries No. 15, 35 and 40). It seems worthwile to remind at this place that the significations of ϵ_{C^mt} and ϵ_{C^at} differ; ϵ_{C^at} measures the impact of technological change on cost for a given number of firms, whereas ϵ_{C^mt} also includes the shift in the population of firms forming an industry (see also Section 2). This argument may explain why the rate of technological change differ in magnitude and significativity. Note that the significance of the impact is, in general, statistically higher in the macroeconomic model: in 25 out of 29 industries the absolute t-values are greater for ϵ_{C^mt} than for ϵ_{C^at}. Briefly, when computed on the basis of an aggregated model, the different elasticities must be interpreted differently as when referring to a macroeconomic model: in the latter case, they also implicitly measure the shift in the joint distribution of heterogeneity (the vector Υ_t). With aggregate models it is possible to compute both effects: a direct impact calculated for a given distribution of the heterogeneity characteristics and an indirect impact constituted by the shift in the heterogeneity distribution consecutively to an increase in Y_t and t.

The output elasticities of input demands and the impact of technological change on input demands are reported in Table 2 of Appendix C. It is interesting to notice that no input is inferior (significantly), i.e. $\epsilon_{jY} \geq 0$, for $j = L, M, K$. The main characteristic of technological change is that it is often labor saving: in 17 cases ϵ_{Lt} is significantly negative and significantly positive in two cases only. For the other inputs, technological change is significantly material saving in 4 cases and significantly capital saving in 13 cases.

Looking at the price elasticities (Table 3 of Appendix C), we also find some similarities across the industries considered. First, labor is the input exhibiting the largest own-price responsiveness in all industries excepted Ceramics (No. 19). Second, the impact of a variation of the user price of capital on the demand of capital is quite small. These estimates could convey the difficulties related to the measurement of the stock of capital as well as those related to the specification of the user cost of capital (see the data description). A third regularity found for these industrial sectors is the similarity of the substitution pattern: materials and labor; labor and capital are found to be substitutable, whereas materials and capital are found to be complementary in almost all industries. It is important to underline that such a result was not a priori constrained to be so, since the input demand functions retained for the estimation satisfy the flexibility criteria. By comparing, the German manufacturing sector to non-produced good sectors, on the basis of the same functional form and estimation procedure, Falk and Koebel (1997) found that a different pattern holds for these sectors. A somewhat unpalatable result is that in six sectors, the matrix A_{pp} is estimated to be a matrix of zero: the corresponding price elasticities are therefore zero for the industries number 16, 21, 22, 27, 32 and 40 (Table 3). For six further industries (No. 15, 18, 34, 36, 37, 43) all price elasticities are found to be near zero. This reflects the fact that the concavity constraint imposed in the estimation is often binding and could be rejected when tested. Violation of price concavity could arise when prices are subject to measurement errors or when their heterogeneity across firms is neglected. Again, this underlines the importance of two other neglected issues in production analysis: aggregation across different goods and prices.

6. Conclusion

Restrictions on microeconomic functional forms are often part of a strategy to avoid the occurrence of distributional statistics (not usually available from statistical offices) in the aggregate relations. Thus, output linear Gorman polar forms satisfying such restrictions are often assumed for individuals. In this paper, on the basis of a flexible quadratic cost function at the firm level, we show that the required conditions are rejected for 27 out of 29 German industries considered.

Some less restrictive assumptions could also avoid the apparition of distributional statistics in the aggregate. Several distributional effects may in fact net out, and the aggregation bias may then vanish without necessarily postulating Gorman polar forms for individual production units. The importance of aggregation bias has been estimated; this approach appears a little more optimistic, but the hypothesis of vanishing aggregation bias is still rejected for 15 industries.

In the light of these results, the best way to avoid aggregation bias seems to be to consider aggregation explicitly in economic modeling in order to account for the heterogeneity of the firms (here in their production process and in the level produced). Aggregation bias expressed as a percentage of total cost was shown to be important. Moreover this may also have a significant impact on the estimation of first order derivatives. In the present study, aggregation across different products and prices is neglected; the importance of these issues —underlined throughout this study— demands further consideration.

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Appendix A

Conditionally on the 6-firm concentration ratios C_{R6} available over the whole period, the Herfindahl index is within the interval $\left[H_{\geq 20}^{\min}, H_{\geq 20}^{\max}\right]$, where $H_{\geq 20}^{\min}$ is the conditional minimal Herfindahl index equal to:

$$H_{\geq 20}^{\min} = (C_{R6})^2/6 + (1 - C_{R6})^2 / \left(n_{\geq 20} - 6\right).$$

The upper limit results from the application of the transfer principle, and a direct adaptation from the results of Sleuwaegen and Dehandschutter (1986) leads to:

$$H_{\geq 20}^{\max} = \begin{cases} \left(C_{R6} - 5\frac{1-C_{R6}}{n_{\geq 20}-6}\right)^2 + \left(n_{\geq 20} - 1\right)\left(\frac{1-C_{R6}}{n_{\geq 20}-6}\right)^2 & \text{if} \quad C_{R6} \geq s \\ C_{R6}/6 & \text{if} \quad C_{R6} < s \end{cases}$$

where the step s is given by:

$$s = \frac{1/6 + 5/\left(n_{\geq 20} - 6\right)}{1 + 5/\left(n_{\geq 20} - 6\right)}.$$

When the number of firms $n_{\geq 20}$ tends to infinity, the case considered by Sleuwaegen and Dehandschutter is retrieved.

Herfindahl indices are only available for firms with 20 employees and more for the period 1977 to 1992. To recover the missing values for the period 1960 to 1976, I regressed the Herfindahl index on variables C_{R6}, $Y_{\geq 20}$ and $n_{\geq 20}$ available for the whole period.[9] Then I used this estimated relation to “forecast” the values for 1960 to 1976. The estimated relation for $H_{\geq 20}$ is chosen as a convex combination of its extreme values as:

$$H_{\geq 20} = \frac{1}{1+\exp(-m)} H_{\geq 20}^{\min} + (1 - \frac{1}{1+\exp(-m)}) H_{\geq 20}^{\max},$$

with $m = a_0 + a_1 C_{R6} + a_2 C_{R6}^2 + a_3 Y_{\geq 20} + a_4 n_{\geq 20}$. The coefficient of correlation between actual and predicted value was below 0.9 for only five industries. Now Herfindahl indices for firms with more than twenty employees are recovered for the whole period and $\sum_i y_{\geq 20i}^2$ is deduced by multiplying $H_{\geq 20}$ by $\left(Y_{\geq 20}\right)^2$.

Data on the production level of firms with less than twenty employees, given by $Y_{<20} = Y_t - Y_{\geq 20}$ are available for the census years for several employment classes (see the data description). We compute $\sum_i y_{<20i}^2$ by distributing $Y_{<20}$ in the four employment classes assuming that firms with less than twenty employees produce proportionally to their number of employees. Further we suppose that inside each employment class the firms are uniformly distributed. Such an assumption is often made in industrial economics to compute surrogates of concentration indices, even for the distribution of the whole production among firms (see Schmalensee (1977)). Here, the way this correction is made does not appear to matter much; in fact, $\sum_i y_{<20i}^2$ always represents less than 1% of $\sum y_i^2$, which results from the very small production per firm ratio for small firms.

Appendix B

Diewert and Wales (1987, 1992) prove that imposing the concavity on C^a amounts to impose negative semi-definiteness of A_{pp}. Further, the authors propose to use the Cholesky decomposition $A_{pp} = U^{'}U$ where U is an upper triangular matrix of parameters of format $S_x \times S_x$, to impose concavity directly on the cost function. In the present case, $S_x = 3$ and the restricted matrix A_{pp}^0 (which is symmetric and verifies $(1, 1, 1)\, A_{pp}^0 = 0$) is given by

$$A_{pp}^0 = \begin{bmatrix} -\left(u_{12}+u_{13}\right)^2 & u_{12}^2 + u_{13}u_{12} & u_{13}^2 + u_{12}u_{13} \\ u_{12}^2 + u_{12}u_{13} & -u_{12}^2 - u_{23}^2 & u_{23}^2 - u_{12}u_{13} \\ u_{13}^2 + u_{12}u_{13} & u_{23}^2 - u_{12}u_{13} & -u_{13}^2 - u_{23}^2 \end{bmatrix}.$$

This reparameterization ensures that the first principal minors of A_{pp}^0 are negative and the second principal minors all positive. The third principal minor, $\det\left(A_{pp}^0\right)$, is constraind to be zero. These nonlinear inequality constraints which corresponds to the negative semi-definite restriction of A_{pp}^0, are imposed on the restricted model simply by using the above reparameterization.

In the first stage, the estimation of the (concavity) unrestricted model provides the estimates of all components of the vector ϕ and, thus, also of $\alpha_{p_L p_M}$, $\alpha_{p_L p_K}$ and $\alpha_{p_M p_K}$, the three free parameters of A_{pp}, stacked in the column vector $\widehat{\alpha}_{pp}$. The second stage incorporates the above concavity restrictions in the construction of the vector $\alpha_{pp}^0(u) \equiv \left(u_{12}^2 + u_{12}u_{13},\right.$ $\left.u_{13}^2 + u_{12}u_{13}, u_{23}^2 - u_{12}u_{13}\right)'$, subvector of the whole restricted parameter vector ϕ^0. Then, the minimization of the distance between $\widehat{\phi}$ and $\phi^0(u)$ formalizes the problem of imposing concavity.

Since the parameters u are exactly identified in the relation $\alpha_{pp} - \alpha^0_{pp}(u)$, and the remaining parameters of ϕ^0 are also exactly identified in $\phi - \phi^0$, we can apply the results of Kodde, Palm and Pfann (1990, Theorem 5) and minimize only over the arguments u of α^0_{pp} using (18), rather than over all parameters in $\phi^0(u)$. Under the null of concavity, the statistic Ψ of the value of the objective $\Psi = n_{obs}\left(\widehat{\alpha}_{pp} - \widehat{\alpha}^0_{pp}\right)'\left[n_{obs}V(\widehat{\alpha}_{pp})\right]^{-1}\left(\widehat{\alpha}_{pp} - \widehat{\alpha}^0_{pp}\right)$ follows a mixture of chi-square and with weights that have to be estimated (see Gouriéroux and Monfort (1989) on this point). This makes formal tests of concavity quite complex.

Appendix C: Tables of Empirical Results

Table 1. Tests of representative firm models

No.	Industry[1]	Definition (a) χ^2 test[2]	Definition (b) $\mathfrak{B}/C^a$ in %	Definition (b) t-test[3]	Elasticities[4] ϵ_{C^aY}	Elasticities[4] ϵ_{C^mY}	Elasticities[4] ϵ_{C^aH}	Rate of technical change[3] ϵ_{C^at}	Rate of technical change[3] ϵ_{C^mt}
14	Chemical products	453.6	55.3	2.5	0.70 (12.1)	0.73 (11.0)	−0.088 (−3.3)	−0.003 (−1.8)	−0.003 (−3.2)
15	Mineral oil refining	**7.2**	**−10.1**	−0.2	0.61 (2.1)	0.81 (1.6)	−0.051 (−0.5)	0.019 (3.2)	0.011 (3.3)
16	Plastic products	51.7	**−46.0**	−1.9	0.78 (4.1)	0.90 (4.2)	0.023 (0.8)	−0.009 (−1.9)	−0.005 (−2.8)
17	Rubber products	48.4	**3.7**	0.3	0.82 (6.0)	0.85 (4.8)	0.069 (2.4)	−0.011 (−3.5)	−0.008 (−12.0)
18	Stones and clay	167.3	−84.5	−7.7	0.86 (4.7)	0.99 (0.4)	0.010 (0.3)	0.003 (1.4)	−0.015 (−23.4)
19	Ceramics	390.9	24.8	2.5	0.71 (2.6)	0.61 (5.0)	−0.012 (−0.9)	0.002 (0.3)	−0.008 (−7.8)
20	Glass	83.3	**11.1**	0.6	0.94 (2.2)	0.90 (2.8)	−0.213 (−7.3)	−0.014 (−14.8)	−0.013 (−11.1)
21	Iron and steel	**13.0**	**−7.0**	−0.8	0.72 (8.2)	0.68 (12.9)	0.036 (1.8)	−0.012 (−5.9)	−0.010 (−16.9)
22	Non-ferrous metal	74.1	**1.5**	0.1	0.75 (6.1)	0.74 (7.0)	−0.006 (−0.1)	−0.000 (−0.0)	−0.003 (−2.7)
23	Founderies	127.7	−96.5	−5.0	0.71 (5.5)	0.77 (4.5)	−0.033 (−2.0)	−0.007 (−6.8)	−0.011 (−20.3)
24	Drawing plants, etc.	26.4	26.5	2.3	0.94 (3.0)	0.91 (5.1)	−0.030 (−2.4)	−0.009 (−9.1)	−0.007 (−16.2)
25	Structural metal products	53.8	**−1.9**	−0.1	0.79 (4.0)	0.74 (5.7)	−0.002 (−1.0)	−0.008 (−1.1)	−0.005 (−3.9)
26	Mechanical engineering	19.2	**−14.3**	−1.6	0.87 (3.7)	0.91 (3.6)	−0.011 (−1.4)	−0.005 (−3.7)	−0.006 (−10.1)
27	Office machinery[5]	123.9	91.5	2.1	0.85 (1.5)	0.83 (3.7)	0.034 (3.3)	0.010 (0.5)	−0.040 (−9.0)
28	Road vehicles	141.5	26.4	4.3	0.92 (3.1)	0.82 (3.7)	0.077 (3.5)	0.000 (0.1)	0.001 (0.3)
29	Shipbuilding	28.3	−46.6	−2.1	0.73 (7.1)	0.70 (9.4)	−0.013 (−0.5)	−0.004 (−1.8)	−0.004 (−3.5)
31	Electrical engineering	60.4	**36.6**	1.8	0.97 (0.7)	0.86 (3.2)	0.006 (0.7)	−0.009 (−1.8)	−0.012 (−5.6)
32	Precision, optical instruments	184.4	**−2.3**	−0.3	0.99 (0.3)	0.85 (3.9)	0.024 (2.1)	−0.011 (−3.7)	−0.006 (−4.0)
33	Finished metal goods	50.5	−58.2	−2.1	0.85 (3.7)	0.91 (3.7)	−0.016 (−0.7)	−0.000 (−0.1)	−0.009 (−14.2)
34	Musical instruments, toys, etc.	93.4	**31.5**	2.1	0.80 (4.0)	0.54 (7.1)	−0.001 (−0.2)	0.003 (1.5)	−0.000 (−0.2)
35	Wood working	150.0	−309.0	−8.0	0.94 (1.6)	0.80 (5.4)	0.012 (0.4)	0.051 (6.8)	−0.008 (−9.6)
36	Wood products	143.8	**−5.4**	−0.4	0.92 (3.9)	0.90 (4.6)	0.025 (0.8)	−0.003 (−0.6)	−0.008 (−9.6)
37	Paper manufacturing	104.9	−35.0	−2.8	0.79 (4.0)	0.89 (1.7)	−0.042 (−1.6)	−0.004 (−1.2)	−0.010 (−4.3)
38	Paper processing	74.5	−85.1	−2.0	0.89 (2.4)	0.87 (3.7)	0.035 (2.0)	0.008 (1.5)	−0.003 (−3.6)
39	Printing and duplicating	32.5	**11.3**	0.3	0.99 (0.2)	0.93 (1.5)	0.021 (0.9)	−0.005 (−0.7)	−0.008 (−5.8)
40	Leather	446.7	−33.6	−6.6	0.88 (2.4)	0.97 (0.3)	0.034 (1.8)	0.024 (4.6)	−0.005 (−2.7)
41	Textile	450.3	−31.9	−2.5	0.96 (1.4)	0.69 (6.2)	0.119 (4.8)	−0.009 (−2.7)	−0.013 (−21.0)
42	Clothing	518.8	**−6.2**	−1.9	0.88 (2.1)	0.81 (6.6)	0.038 (5.2)	−0.008 (−7.2)	−0.008 (−26.9)
43	Food and beverages	153.9	**8.1**	0.9	1.11 (−2.4)	1.29 (−6.2)	−0.013 (−1.3)	−0.006 (−2.9)	−0.009 (−6.8)

(1) The names retained here are shortened versions of the full denominations given in Statistiches Bundesamt (1987). For the aicraft industry and the tobacco industry (number 30 and 45), no producer prices are available, thus we leave these industries out. For food and beverages, product prices are not available separately, thus we aggregate these industries together (number 43 and 44).

(2) χ^2 values for the hypothesis that the five coefficients reflecting aggregation bias are simultaneously zero, the critical value at the 5% level is 11.07 and at the 1% level 15.09.

(3) Student tests for the hypothesis that the corresponding statistic is zero.

(4) t-values are in brackets: the null hypothesis is, respectively $\epsilon_{C^aY} = 1$, $\epsilon_{C^mY} = 1$ (constant returns to scale), and $\epsilon_{C^aH} = 0$ (no impact of the Herfindahl index).

(5) For the office machinery industry, the data are only available from 1970 ownward.

Table 2. Output elasticities and impact of technological change[3]

No.	Industry[1]	ϵ_{C^aY}	ϵ_{M^aY}	ϵ_{L^aY}	ϵ_{K^aY}	ϵ_{C^at}	ϵ_{M^at}	ϵ_{L^at}	ϵ_{K^at}
14	Chemical products	0.70 (28.9)	1.01 (39.9)	0.07 (0.8)	0.17 (2.0)	−0.003 (−1.8)	0.001 (0.6)	−0.009 (−3.2)	−0.019 (−7.2)
15	Mineral oil refining	0.61 (3.2)	0.60 (2.9)	0.26 (1.7)	0.94 (4.2)	0.019 (3.2)	0.023 (3.5)	−0.025 (−5.8)	−0.022 (−3.9)
16	Plastic products	0.78 (14.5)	1.17 (18.8)	0.15 (0.9)	0.09 (1.1)	−0.009 (−1.9)	−0.017 (−3.3)	0.012 (0.7)	−0.033 (−4.2)
17	Rubber products	0.82 (26.7)	1.00 (25.2)	0.64 (8.1)	0.15 (1.1)	−0.011 (−3.5)	−0.004 (−1.0)	−0.022 (−7.7)	−0.019 (−4.1)
18	Stones and clay	0.86 (28.7)	1.22 (24.6)	0.29 (3.4)	0.48 (5.7)	0.003 (1.3)	0.005 (1.7)	0.014 (3.1)	−0.036 (−5.7)
19	Ceramics	0.71 (6.2)	0.99 (5.7)	0.56 (5.0)	0.14 (2.2)	0.002 (0.3)	0.020 (2.0)	−0.020 (−3.7)	0.043 (6.1)
20	Glass	0.94 (32.5)	1.12 (28.7)	0.87 (14.5)	0.10 (1.1)	−0.014 (−14.8)	0.002 (−1.6)	−0.039(−17.3)	0.010 (3.2)
21	Iron and steel	0.72 (21.0)	1.18 (45.8)	−0.22 (−1.9)	0.63 (3.9)	−0.012 (−5.9)	0.003 (1.5)	−0.039 (−8.7)	−0.027 (−4.9)
22	Non-ferrous metal	0.75 (18.0)	1.00 (19.5)	−0.08 (−0.6)	0.32 (2.8)	−0.000 (−0.0)	0.001 (0.1)	−0.008 (−1.6)	0.016 (3.2)
23	Founderies	0.71 (13.7)	1.00 (18.7)	0.46 (5.1)	0.17 (2.1)	−0.007 (−6.8)	0.000 (0.0)	−0.014 (−7.1)	−0.011 (−5.0)
24	Drawing plants, etc.	0.94 (46.2)	1.18 (41.1)	0.66 (10.4)	0.05 (1.0)	−0.009 (−9.1)	−0.006 (−3.5)	−0.015 (−5.8)	−0.012 (−4.9)
25	Structural metal products	0.79 (14.8)	1.07 (15.7)	0.31 (3.2)	0.57 (6.6)	−0.008 (−1.1)	−0.006 (−0.7)	−0.020 (−1.5)	0.052 (4.2)
26	Mechanical engineering	0.87 (24.5)	1.05 (24.2)	0.68 (10.4)	0.15 (1.9)	−0.005 (−3.7)	−0.002 (−0.8)	−0.010 (−3.5)	0.002 (0.5)
27	Office machinery[5]	0.85 (8.4)	3.02 (7.6)	0.31 (2.9)	0.05 (1.2)	0.010 (0.5)	0.218 (2.7)	−0.088 (−4.0)	0.097 (11.4)
28	Road vehicles	0.92 (33.9)	1.03 (49.0)	0.86 (7.9)	0.02 (0.2)	0.000 (0.1)	0.001 (0.3)	−0.011 (−1.8)	0.039 (6.6)
29	Shipbuilding	0.73 (18.8)	0.96 (29.8)	0.34 (4.5)	0.33 (5.0)	−0.004 (−1.8)	0.008 (4.0)	−0.028 (−10.6)	0.011 (3.5)
31	Electrical engineering	0.97 (25.6)	1.09 (29.3)	0.88 (8.3)	0.32 (3.9)	−0.009 (−1.8)	−0.015 (−2.3)	−0.003 (−0.5)	0.012 (1.4)
32	Precision, optical instruments	0.99 (24.0)	1.36 (24.6)	0.67 (10.7)	0.11 (3.0)	−0.011 (−3.7)	0.006 (1.4)	−0.036 (−9.1)	0.031 (8.8)
33	Finished metal goods	0.85 (21.8)	1.01 (17.7)	0.80 (14.6)	−0.18 (−1.8)	−0.000 (−0.1)	0.014 (2.6)	−0.028 (−5.7)	0.023 (3.1)
34	Musical instruments, toys, etc.	0.80 (15.7)	1.01 (19.5)	0.53 (4.9)	0.08 (0.8)	0.003 (1.5)	0.014 (5.3)	−0.023 (−9.5)	0.043 (11.8)
35	Wood working	0.94 (24.2)	1.14 (19.0)	0.63 (6.9)	0.17 (1.6)	0.051 (6.8)	0.059 (6.5)	0.043 (6.5)	0.008 (1.0)
36	Wood products	0.92 (44.2)	1.41 (28.0)	−0.06 (−0.6)	1.52 (11.0)	−0.003 (−0.6)	−0.005 (−0.7)	0.017 (1.5)	−0.122 (−6.2)
37	Paper manufacturing	0.79 (15.2)	0.99 (20.0)	0.33 (2.8)	0.55 (2.5)	−0.004 (−1.1)	0.003 (−0.9)	−0.003 (−0.4)	−0.012 (−0.8)
38	Paper processing	0.89 (18.7)	1.12 (18.2)	0.53 (10.1)	0.32 (3.8)	0.008 (1.5)	0.026 (3.8)	−0.026 (−6.6)	−0.013 (−2.0)
39	Printing and duplicating	0.99 (18.3)	1.20 (14.6)	0.89 (9.5)	0.15 (1.9)	−0.005 (−0.7)	−0.001 (−0.1)	−0.019 (−3.0)	0.036 (4.6)
40	Leather	0.88 (17.3)	1.17 (19.0)	0.37 (3.6)	0.42 (5.2)	0.024 (4.6)	0.042 (6.5)	−0.007 (−1.5)	−0.010 (−1.8)
41	Textile	0.96 (34.1)	1.07 (46.5)	0.87 (8.6)	0.42 (4.3)	−0.009 (−2.7)	−0.011 (−3.0)	0.006 (1.4)	−0.043 (−8.4)
42	Clothing	0.88 (16.2)	1.03 (45.8)	0.63 (3.7)	0.11 (1.0)	−0.008 (−7.2)	−0.001 (−1.2)	−0.020 (−6.2)	−0.033 (−12.4)
43	Food and beverages	1.11 (24.1)	1.31 (19.2)	0.29 (3.0)	0.51 (3.2)	−0.006 (−2.9)	0.001 (0.3)	−0.026 (−8.1)	−0.061 (−14.0)

See notes for Table 1.

Table 3. Price elasticities(3)

No.	Industry(1)	$\epsilon_{M^a p_M}$	$\epsilon_{L^a p_L}$	$\epsilon_{K^a p_K}$	$\epsilon_{M^a p_L}$	$\epsilon_{M^a p_K}$	$\epsilon_{L^a p_M}$	$\epsilon_{L^a p_K}$	$\epsilon_{K^a p_M}$	$\epsilon_{K^a p_L}$
14	Chemical products	-0.03 (-1.9)	-0.16 (-3.3)	-0.04 (-1.4)	0.04 (2.7)	-0.01 (-2.3)	0.10 (2.7)	0.05 (2.7)	-0.09 (-2.3)	0.13 (2.7)
15	Mineral oil refining	-0.00 (-0.2)	-0.10 (-1.8)	-0.03 (-0.5)	0.00 (1.0)	-0.00 (-0.4)	0.04 (1.0)	0.06 (1.2)	-0.02 (-0.4)	0.05 (1.2)
16	Plastic products	-0.00 (-0.0)	-0.00 (-0.0)	-0.00 (-0.0)	0.00 (0.0)	-0.00 (-0.0)	0.00 (0.0)	0.00 (0.0)	-0.00 (-0.0)	0.00 (0.0)
17	Rubber products	-0.06 (-1.4)	-0.19 (-2.4)	-0.10 (-1.5)	0.08 (1.8)	-0.02 (-1.4)	0.14 (1.8)	0.06 (2.0)	-0.13 (-1.4)	0.23 (2.0)
18	Stones and clay	-0.04 (-0.6)	-0.13 (-0.9)	-0.02 (-0.0)	0.05 (0.8)	-0.01 (-0.9)	0.10 (0.8)	0.03 (1.1)	-0.06 (-0.9)	0.08 (1.1)
19	Ceramics	-0.29 (-6.7)	-0.25 (-7.2)	-0.00 (-0.0)	0.29 (7.0)	-0.00 (-0.2)	0.25 (7.1)	0.00 (0.3)	-0.01 (-0.2)	0.01 (0.3)
20	Glass	-0.07 (-1.8)	-0.23 (-3.8)	-0.06 (-1.3)	0.10 (2.7)	-0.03 (-2.7)	0.17 (2.7)	0.06 (4.0)	-0.16 (-2.7)	0.22 (4.0)
21	Iron and steel	-0.00 (-0.0)	-0.00 (-0.0)	-0.00 (-0.0)	0.00 (0.0)	0.00 (0.0)	-0.00 (-0.0)	0.00 (0.0)	0.00 (0.0)	0.00 (0.0)
22	Non-ferrous metal	-0.00 (-0.0)	-0.00 (-0.0)	-0.00 (-0.0)	-0.00 (-0.0)	-0.00 (-0.0)	0.00 (0.0)	0.00 (0.0)	-0.00 (-0.0)	0.00 (0.0)
23	Founderies	-0.12 (-3.6)	-0.19 (-4.3)	-0.02 (-0.5)	0.14 (4.1)	-0.02 (-1.9)	0.17 (4.0)	0.02 (2.0)	-0.11 (-1.9)	0.13 (2.0)
24	Drawing plants, etc.	-0.12 (-3.5)	-0.35 (-4.8)	-0.03 (-1.0)	0.14 (4.2)	-0.02 (-2.5)	0.30 (4.2)	0.05 (2.9)	-0.17 (-2.5)	0.19 (2.9)
25	Structural metal products	-0.26 (-3.3)	-0.47 (-3.2)	-0.00 (-0.0)	0.26 (3.3)	0.00 (0.0)	0.47 (3.3)	-0.00 (-0.0)	0.00 (0.0)	-0.00 (-0.0)
26	Mechanical engineering	-0.18 (-3.9)	-0.32 (-5.0)	-0.01 (-0.2)	0.19 (4.5)	-0.01 (-1.1)	0.31 (4.5)	0.02 (1.3)	-0.10 (-1.1)	0.11 (1.3)
27	Office machinery(5)	-0.00 (-0.0)	-0.00 (-0.0)	-0.00 (-0.0)	0.00 (0.0)	0.00 (0.0)	0.00 (0.0)	0.00 (0.0)	0.00 (0.0)	0.00 (0.0)
28	Road vehicles	-0.09 (-2.9)	-0.26 (-3.0)	-0.00 (-0.1)	0.10 (3.1)	-0.01 (-0.7)	0.25 (3.1)	0.01 (0.7)	-0.06 (-0.7)	0.06 (0.7)
29	Shipbuilding	-0.05 (-2.4)	-0.13 (-3.2)	-0.03 (-1.4)	0.06 (2.8)	-0.01 (-1.9)	0.11 (2.8)	0.02 (2.4)	-0.11 (-1.9)	0.13 (2.4)
31	Electrical engineering	-0.21 (-4.0)	-0.32 (-4.1)	-0.00 (-0.0)	0.21 (4.2)	0.00 (0.2)	0.32 (4.1)	-0.00 (-0.2)	0.02 (0.2)	-0.02 (-0.2)
32	Precision, optical instruments	-0.00 (-0.0)	-0.00 (-0.0)	-0.00 (-0.0)	0.00 (0.0)	-0.00 (-0.0)	0.00 (0.0)	0.00 (0.0)	-0.00 (-0.0)	0.00 (0.0)
33	Finished metal goods	-0.14 (-3.9)	-0.42 (-5.5)	-0.15 (-3.0)	0.19 (4.5)	-0.05 (-4.4)	0.31 (4.4)	0.11 (5.3)	-0.41 (-4.5)	0.55 (5.3)
34	Musical instruments, toys, etc.	-0.04 (-1.0)	-0.09 (-1.2)	-0.01 (-0.3)	0.04 (1.1)	-0.00 (-0.7)	0.08 (1.1)	0.01 (0.8)	-0.05 (-0.7)	0.06 (0.8)
35	Wood working	-0.11 (-3.6)	-0.67 (-5.3)	-0.16 (-2.6)	0.16 (4.7)	-0.05 (-5.5)	0.48 (4.5)	0.20 (4.9)	-0.38 (-5.6)	0.54 (4.9)
36	Wood products	-0.03 (-0.4)	-0.05 (-0.3)	-0.00 (-0.0)	0.03 (0.3)	-0.00 (-0.0)	0.05 (0.3)	0.00 (0.0)	-0.00 (-0.0)	0.00 (0.0)
37	Paper manufacturing	-0.02 (-0.7)	-0.05 (-0.6)	-0.00 (-0.0)	0.02 (0.8)	-0.00 (-0.0)	0.05 (0.8)	0.00 (0.0)	-0.00 (-0.0)	0.00 (0.0)
38	Paper processing	-0.03 (-0.9)	-0.14 (-2.3)	-0.06 (-1.8)	0.04 (1.5)	-0.01 (-1.8)	0.09 (1.5)	0.05 (3.0)	-0.12 (-1.8)	0.18 (3.0)
39	Printing and duplicating	-0.17 (-3.1)	-0.26 (-4.1)	-0.01 (-0.6)	0.19 (3.5)	-0.02 (-2.1)	0.24 (3.6)	0.03 (2.3)	-0.12 (-2.1)	0.14 (2.3)
40	Leather	-0.00 (-0.0)	-0.00 (-0.0)	-0.00 (-0.0)	0.00 (0.0)	-0.00 (-0.0)	0.00 (0.0)	0.00 (0.0)	-0.00 (-0.0)	0.00 (0.0)
41	Textile	-0.07 (-3.6)	-0.17 (-3.4)	-0.00 (-0.1)	0.07 (3.7)	-0.00 (-0.6)	0.16 (3.7)	0.01 (0.6)	-0.00 (-0.6)	0.04 (0.6)
42	Clothing	-0.15 (-7.0)	-0.36 (-6.7)	-0.01 (-0.3)	0.16 (7.0)	-0.01 (-2.1)	0.35 (6.8)	0.01 (2.2)	-0.13 (-2.1)	0.13 (2.2)
43	Food and beverages	-0.00 (-0.0)	-0.02 (-0.4)	-0.04 (-1.0)	0.00 (0.1)	-0.00 (-0.1)	0.00 (0.1)	0.02 (1.0)	-0.01 (-0.1)	0.04 (1.0)

See notes for Table 1.

Notes

1. Micro units are also heterogeneous in their decision rules and the assumption that all units are cost minimizing is restrictive. When some inputs are fixed (or quasi–fixed), we must consider restricted cost functions, and quasi–fixed inputs must also be aggregated (across micro units) in the same way as the production. Furthermore, in the presence of quasi–fixed inputs, one can interpret the observed choices as if these resulted from an optimization program parameterized by *virtual* prices p_{it}^v, instead of market prices p_t. If these virtual prices are identical across micro units (and can thus be noted p_t^v), the minimization program for which the quasi–fixed inputs $x_i^v(p_t^v, t, y_{it}, \alpha_i)$ can be seen as solutions is $c^v(p_t^v, t, y_{it}, \alpha_i) = \min_{x_{it}}\{p_t^{v\prime} x_{it} : f(x_{it}, y_{it}, t, \alpha_i) \leq 0\}$. The aggregation problem then boils down to the one presently considered. Of course, assuming the identity of virtual prices is very restrictive; this assumption was only made to underline the fact that it permits setting the aggregation of x_{it} across production units aside. Thus the assumption of *identity* of (virtual) prices facilitates aggregation in a more decisive manner than the assumption of static equilibrium cost minimizing behavior postulated in (1) (see also section 5).
2. Lewbel presents the definition in the consumer context, here we adapt it to the producer.
3. Strictly speaking, α should bear the index t as well, but we will assume constancy over time for simplicity.
4. For these years, the numbers of small firms are listed for several classes of employment (with 1, 2 to 4, 5 to 9, 10 to 19 employees).
5. If the number of firms were constant over the period or distributed around a constant mean, no heterogeneity parameters ω would be identifiable. The same conclusion would be reached if, instead of estimating parameters α corresponding to sums of micro-parameters, one were to estimate averages of the corresponding micro-parameters (noted $\overline{\alpha}$). In this latter case, we would simply have to reparameterize (17) using $A_j = n_t \overline{A}_j$, $j = p,\ pp,\ pt$, and $\alpha_{tt} = n_t \overline{\alpha}_{tt}$, and then estimate $\overline{A}_j$ and $\overline{\alpha}_{tt}$ instead of A_j and α_{tt}. However, the macroeconomic models presented in the literature usually do not use any data on the number of firms n_t, so that to measure the aggregation bias emerging in these models, the specification (17) is appropriate.
6. Convergence turns out to be much more difficult to obtain when iterations on both the restricted parameters and the residual covariance matrix (the constrained *SUR* model) are done simultaneously. This drawback is avoided with the procedure adopted here.
7. It seems to me that the differences in prices (or in virtual prices) between firms are more susceptible to be at the origin of concavity violations than a misspecification of the model for the reason of quasi-fixity of some inputs. Indeed, assume that the observed input allocation is driven by the virtual prices p_t^v, identical for each firm. Then, instead of X^* specified as in (12), the actual demand functions X^v would be obtained after reparameterizing X^* by multiplying each price by p_{jt}^v / p_{jt} to obtain $X^v(p_t^v, t, \Upsilon_t) = X^*(p_{1t}\frac{p_{1t}^v}{p_{1t}}, \ldots, p_{\ell t}\frac{p_{\ell t}^v}{p_{\ell t}}, t, \Upsilon_t)$. The associated aggregate virtual cost function $C^v \equiv p_t^{v\prime} X^v$ must also be concave in p_t^v in this case. With a normalized quadratic cost function as in (9), this amounts to imposing the negative semi-definiteness of the matrix $DA_{pp}D$, where the matrix D is diagonal, with p_{jt}^v / p_{jt} as components ($j = M, L, K$). Since A_{pp} negative semi-definite is equivalent to $DA_{pp}D$ negative semi–definite, the imposition of concavity in the potentially misspecified model is not misleading. When the virtual prices are specified as $p_{jt}^v = \beta_j p_{jt}$, as in many models studying allocative inefficiency (Lovell and Sickles (1983) for example), the system (17) is well specified, although the inefficiency parameters β_j are not explicitly estimated here. In this case, the aggregation bias $\mathfrak{B}$ corresponds to the difference between the macroeconomic virtual cost function and the aggregate virtual cost function: $C^{vm} - C^{va}$.
8. This is the aggregate of $\tilde{c}$ with

$$\widetilde{c}_2(p_t, t)'\alpha_{2i} = p_t' A_{ip} + \frac{1}{2}(\eta' p_t)^{-1} p_t' A_{ipp} p_t + p_t' A_{ipt} t + \theta' p_t \alpha_{itt} t^2,$$

and

$$\widetilde{c}_1(p_t, t, \alpha_1) = (p_t' A_{py} + \theta' p_t \alpha_{ty} t).$$

9. In fact these values are only partially available from 1960 to 1976; for some years C_{R6} and $n_{\geq 20}$ have been linearly interpolated and $Y_{\geq 20}$ deduced from available information on Y_t.

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