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Aggregation with Cournot Competition: An Empirical Investigation

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This paper empirically investigates the existence of Cournot equilibrium and the validity of the Le Chatelier-Samuelson (LCS) principle in the aggregate. Whereas two well known existence conditions are statistically rejected, we cannot reject a third, original, condition. We also find some empirical evidence for the LCS principle, as well as both increasing and constant returns to scale for two-digit US manufacturing industries. The results highlight the importance of imperfect competition for understanding aggregate growth, investment and employment.*

I. Introduction

A large literature on Cournot competition is devoted to two theoretical difficulties: an equilibrium may not exist; even if it exists, comparative statics at the Cournot equilibrium is nonmonotone and heterogeneous over competing firms. Even the direction of the impact of a change in input prices on individual output supply and input demand is ambiguous. With imperfect competition, and focusing on the price of energy, say, some energy intensive firms will suffer from an increase in energy price and reduce their input and output levels, whereas more efficient competitors will increase their market share and possibly even increase their overall use of energy. However, from an economic viewpoint, the way a specific firm reacts to a change in an input price may not be of primary interest. More relevant is how the whole industry copes with the price shock. When a government increases a value added tax rate or lowers tariffs (for instance) it is mainly interested in the change in the industry price and the possible reduction in production and employment in that industry, but not in how each company within the industry adjusts its price, production and employment levels.

In a companion paper (KOEBEL and LAISNEY [2014]) we show that the ambiguity at the firm level is theoretically resolved if we study aggregate output supply and aggregate input demand. The required conditions on the structure of the economy, in terms of firm heterogeneity and returns to scale, are rather weak. They ensure that the Le Chatelier-Samuelson (LCS) principle is likely to be satisfied at the aggregate Cournot equilibrium: an increase in input prices entails a reduction in aggregate input demands.

To the best of our knowledge, this paper is the first empirical investigation of two well-known sufficient conditions for the existence of a Cournot equilibrium, derived by NOVSHEK

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[1985] and by AMIR [1996]. Surprisingly, both theoretical conditions are empirically rejected. However, a further existence condition derived and discussed in KOEBEL and LAISNEY [2014], is not rejected. Our paper also tackles the second problem of indeterminacy of firm level comparative statics and investigates the empirical validity of the LCS principle when competition is imperfect.

In order to produce an empirical decomposition of the impact of input price changes on input demands into an aggregate substitution effect and an expansion effect, it is necessary to propose an adequate empirical specification compatible with firm heterogeneity. Instead of relying on a representative firm setup, which may imply estimation biases when firms are heterogeneous, we proceed to aggregation using the stochastic aggregation theory developed by LEWBEL [1996] and KOEBEL [2002]. The aggregate cost function is defined as the conditional expectation of the summed microeconomic cost functions, given the available aggregate information. In this context, the aggregate cost function does not generally inherit the properties of the microeconomic cost functions. However, in this paper we show that it is possible to derive the conditional expectation of the input demand system from the cost function by amending Shephard's lemma, and to identify the conditional expectation of the substitution and expansion effects using aggregate data only. We show that there are biases linked with the representative firm specification, we characterise them, provide a parametric way to deal with them and propose a simple test for the validity of the representative firm model.

There is a further methodological focus in this paper: we present a method for dealing with empirical issues raised by the adjustment of inputs in the long run, in the context of increasing or decreasing returns to scale. When the rate of returns and the type of competition on the output market are a priori unknown, empirical contributions (reviewed by Bresnahan, 1989) have often appended a pricing rule – compatible with both perfect and imperfect competition – to the cost and input demand system. However, most of these studies do not derive the optimal output level, nor do they report how output and its price react to input price changes. This shortcoming is surprising because, when output is optimally chosen by the production unit, output-restricted elasticities are only of limited interest for deriving policy implications. A further objective of this paper is thus to estimate elasticities that are not restricted by an output level artificially held constant. Since a closed form solution for the optimal output level can seldom be obtained from the price-margin equation, we rely on the implicit function theorem for deriving these elasticities empirically. This method has been developed and applied by KULATILAKA [1987] to model the adjustment of capital to its optimal long run level. We extend his method by deriving moment conditions which are consistent with the theoretical model and identify the substitution and expansion effects of input price changes. This method allows us to derive consistent estimates of the substitution and the expansion matrices and to test the validity of the LCS principle in the aggregate.

The empirical application relies on a panel for 18 two-digit US manufacturing industries over the period 1949 to 2001 (but some of the results have been confirmed with the more disaggregated NBER-CES Manufacturing Industry Database for 462 industries). Our model is essentially static, because with yearly data available (as in most empirical studies on imperfect competition), it is hardly possible to identify dynamic interactions in price and quantity setting which occur within few weeks. Games implying conjectural variations are usually estimated

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using experimental data (in which strategic interactions occur within minutes). A static model is appropriate to estimate the long-run relationships we are interested in. We obtain several results relative to the rate of returns to scale, the markup, the short and long run adjustment of input demand to input price change, and the impact of input price changes on output adjustment and inflation. The empirical findings confirm the validity of the LCS principle. Whether returns to scale are increasing or not in U.S. manufacturing industries is an important but controversial empirical issue. Whereas many researchers have argued for the increasing returns to scale hypothesis (HALL [1988], DIEWERT and FOX [2008]), just as many researchers have found evidence for the contrary (BURNSIDE [1996], BARTELSMAN [1995], BASU and FERNALD [1997]). We find evidence for both increasing and constant returns to scale across industries and time.

The next section outlines the microeconomic model and derives the LCS principle at the aggregate level, when the output market is imperfectly competitive. SECTION III presents the aggregate statistical model compatible with available aggregate data. SECTION IV sets up the empirical model specification and highlights how it departs from usual models of producer behavior. The empirical results are described in SECTION V, and SECTION VI concludes.

II. Existence of Cournot Equilibrium and Aggregate Comparative Statics

Let $p:(Y,z)\mapsto p(Y,z)$ denote the inverse product demand function for a good produced in some sector of the economy. The aggregate output level Y is the sum of firm's h output level y_h, and the aggregate output of all its competitors, Y_{-h}. Vector z denotes exogenous explanatory variables shifting output demand (as the country's population, the unemployment rate, the level of value added tax and so on). We assume that each individual firm is profit maximizing:

$$\pi_h(w,Y_{-h},z)=\max_{y_h}\{p(y_h+Y_{-h},z)\,y_h-c_h(w,y_h)\}. \tag{1}$$

The cost of producing output y_h at input prices w, $c_h(w,y_h)$, corresponds to the value of J input demands x_h^*: $c_h(w,y_h)=w^\top x_h^*(w,y_h)$. Let $y_h^o(w,Y_{-h},z)$ denote the optimal solution to (1): this represents the output supply correspondence. For a given level of Y_{-h}, the input demand correspondence $x_h^o(w,Y_{-h},z)$ is nonincreasing in w (see KOEBEL and LAISNEY [2014]). The aggregate production level of competitors represents a negative externality for firm h. At the Nash equilibrium, Y_{-h} is endogenous and is written as $Y_{-h}^N(w,z)$. Substituting Y_{-h}^N into y_h^o and x_h^o yields the equilibrium supply and demand correspondences $y_h^N(w,z)$ and $x_h^N(w,z)$, respectively, and these are not monotone in w. Let us also define the aggregate input demand and supply quantities as follows:

$$X^*\left(w,\{y_h\}_{h=1}^H\right)\equiv\sum_{h=1}^H x_h^*(w,y_h).$$

$$X^N(w,z)\equiv\sum_{h=1}^H x_h^N(w,z),\quad\text{and}\quad Y^N(w,z)\equiv\sum_{h=1}^H y_h^N(w,z).$$

Superscript N denotes the quantities at the Cournot-Nash-equilibrium.

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NOVSHEK [1985] illustrated that a Cournot equilibrium does not always exist (for a given value of z). He also showed that a sufficient condition for its existence is that the marginal revenue of each firm is decreasing in the rivals' production level. This condition can be stated as:

$$\frac{\partial p}{\partial Y}(y_h + Y_{-h}, z) + y_h \frac{\partial^2 p}{\partial Y^2}(y_h + Y_{-h}, z) \leq 0 \tag{2}$$

for any value of y_h, Y_{-h}. Novshek shows that this can equivalently be written as

$$\frac{\partial p}{\partial Y}(Y, z) + Y \frac{\partial^2 p}{\partial Y^2}(Y, z) \leq 0 \tag{3}$$

for any Y. This inequality depends on aggregate data only and implies that condition (2) is fulfilled for *any* firm.

Another sufficient condition for the existence of a Cournot equilibrium was proposed by Amir (1996, Theorem 3.1): if

$$p(Y, z) \frac{\partial^2 p}{\partial Y^2}(Y, z) - \left[\frac{\partial p}{\partial Y}(Y, z)\right]^2 \leq 0 \tag{4}$$

for any Y, then a Cournot equilibrium exists. Inequalities (3) and (4) are not nested, despite the fact that they involve the same partial derivatives: Amir (2005, p.4) gives an example of a function satisfying (2) but not (4), and of another function for which (4) is true but not (2). The strength of these existence results is that they are independent on the type of firms' cost functions. The generality of this requirement, however, is also responsible for the severity of the conditions (4) and (3), and we are tempted to restrict somewhat the cost function specification in order to gain less restrictive conditions on the marginal revenue.

This road has been followed in the companion paper, KOEBEL and LAISNEY [2014] who showed that a Cournot equilibrium exists if the cost functions are such that no firm chooses an optimal level of output greater than some value $\bar{y} < Y$. An aggregate sufficient condition implying that (2) is satisfied for any value of y_h compatible with a value $\mathbb{H}$ of the Hirschman-Herfindahl index of concentration, provided that the inverse demand function is convex, is given by

$$\frac{\partial p}{\partial Y}(Y, z) + \sqrt{\mathbb{H}} Y \frac{\partial^2 p}{\partial Y^2}(Y, z) \leq 0. \tag{5}$$

Condition (5) is weaker than condition (3): when output demand is concave, both inequalities (3) and (5) are satisfied. However, when output demand is convex, for given values of (Y, z), condition (5) puts less weight on the positive term than Novshek's condition, and it turns out that (5) may be valid when (3) is not.

One aim of the paper is to empirically test the validity of the nonnested existence conditions (3), (4) and (5). These conditions can be tested using only aggregate data at the level of the industry. A second reason for focusing on condition (5) is that it also ensures monotone comparative statics in the aggregate. Koebel and Laisney (2014, Corollary 1) show that if (5) is satisfied, then

$$\frac{\partial X^N}{\partial w^\top}(w, z) \leq \frac{\partial X^*}{\partial w^\top}\left(w, \{y_h\}_{h=1}^H\right) \leq 0, \tag{6}$$

$$\frac{\partial Y^N}{\partial w}(w, z) \leq 0, \tag{7}$$

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if firms are not "too" heterogenous in their technologies (the precise meaning of "too" is given in their Corollary 1). These results complete those obtained in the context of perfect competition by HEINER [1982] and BRAULKE [1984].

III. Aggregation when the Distribution of Market Shares is Unobserved

We now include a variable t denoting time as an argument of the cost and demand functions. We explicitly derive the aggregate relationships from the disaggregate ones and show that they depend upon the distribution of market shares (and how it changes with w, Y and t). We also discuss identification of the effect of unobserved shifts in market shares on the aggregate cost and demand functions.

III.1. *Aggregate Cost and Input Demand Functions*

Whereas it is natural to define total cost at the level of an industry by $C_t = \sum_{h=1}^{H} w^\top x_{ht}$, the total cost function is given by $C(w, y_{1t}, \ldots, y_H t, t) = \sum_{h=1}^{H} c_h(w, y_h, t)$ and depends upon the whole distribution of output within the industry, a piece of information which is difficult to obtain and not always possible to consider explicitly. In order to set up an aggregate model, we follow LEWBEL [1996] and KOEBEL [2002] and reparameterize $(y_1, \ldots, y_H) = \beta Y$ in terms of the market shares $\beta = (y_1/Y, \ldots, y_H/Y)$. Then, it is always possible to define the aggregate cost and demand functions as the conditional expectations of the true but unobserved functions:

$$\mathbf{C}(w,Y,t) = \mathrm{E}_\beta \left[C(w, \beta Y, t) \,|\, w, Y, t \right], \tag{8}$$

$$\mathbf{X}^*(w,Y,t) = \mathrm{E}_\beta \left[X^*(w, \beta Y, t) \,|\, w, Y, t \right]. \tag{9}$$

The conditional expectation is taken with respect to the conditional density of market shares $f(\beta|w,Y,t)$. The randomness of β given (w,Y,t) results from the (unobserved) heterogeneity of firms' technologies.

The properties of the aggregate cost and input demand functions have been studied by LEWBEL [1996] and KOEBEL [2002] who show that microeconomic properties are not necessarily inherited in the aggregate. Indeed, comparative statics for the aggregate demands depend on the way the distribution of market shares is shifted by changes in (w,Y):

$$\frac{\partial \mathbf{C}}{\partial w}(w,Y,t) = \mathrm{E}_\beta \left[\frac{\partial C}{\partial w}(w,\beta Y,t) \,|\, w,Y,t \right] + \int C(w,\beta Y,t) \frac{\partial f}{\partial w}(\beta|w,Y,t)\, d\beta, \tag{10}$$

$$\frac{\partial \mathbf{C}}{\partial Y}(w,Y,t) = \mathrm{E}_\beta \left[\frac{\partial C}{\partial Y}(w,\beta Y,t) \,|\, w,Y,t \right] + \int C(w,\beta Y,t) \frac{\partial f}{\partial Y}(\beta|w,Y,t)\, d\beta. \tag{11}$$

Whereas the first terms of the right hand side of the equalities denote the expected input demand for (10) and marginal cost for (11), the last terms are *redistribution effects* corresponding to shifts in the distribution of market shares following changes in w and Y. These terms correspond to aggregation biases and can be denoted by $B_{Cw}(w,Y,t)$ and $B_{CY}(w,Y,t)$, respectively. Note, however, that the adding up property is inherited in the aggregate in the sense that $\mathbf{C}(w,Y,t) = w^\top \mathbf{X}^*(w,Y,t)$, through linearity of the expectation operator.

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Without disaggregate data on the distribution of market shares, it is difficult to identify both the expected value of the derivatives of C w.r.t. w and Y and the redistribution effect separately. For instance, if f is homogeneous of degree zero in w, both terms of decomposition (10) are homogenous of degree zero. How is it possible in this context to disentangle both terms without using data specific to the distribution f and $\partial f/\partial w$? Economic theory provides structure identifying (10). Indeed, by Shephard's lemma, the first term on the right hand side of equality (10) corresponds to aggregate input demands:

$$\mathrm{E}_\beta\left[\frac{\partial C}{\partial w}(w,\beta Y,t)\,|w,Y,t\right] = \mathrm{E}_\beta\left[X^*(w,\beta Y,t)\,|w,Y,t\right] = \mathbf{X}^*(w,Y,t), \tag{12}$$

a term which can be estimated when data on inputs is available. Thus it is possible to identify the aggregation biases residually, as the difference $\partial\mathbf{C}/\partial w - \mathbf{X}^*$. Note that in this way identification is achieved even in the case where f is homogeneous of degree zero in w.

Proposition 1. Under the assumption that each firm is cost minimizing, and that the aggregate conditional mean cost function $\mathbf{C}$ and all the partial derivatives $\partial\mathbf{C}/\partial w$ are identified, the aggregation biases B_{Cw} are identified.

Corollary 1. The aggregate cost function $\mathbf{C}$
(i) is homogeneous of degree one in w iff for any (w,Y,t)

$$w^\top B_{Cw}(w,Y,t) = 0 \tag{13}$$

(ii) satisfies Shephards' lemma in the sense that $\partial\mathbf{C}/\partial w = \mathbf{X}^*$ iff for any (w,Y,t)

$$B_{Cw}(w,Y,t) = 0. \tag{14}$$

This corollary is a direct consequence of relationships (10) and (12). Function $\mathbf{C}$ is homogeneous of degree one in w iff $w^\top\partial\mathbf{C}/\partial w = \mathbf{C}(w,Y,t) = w^\top\mathbf{X}^*(w,Y,t)$ and so the claim of Corollary 1(i) directly follows from (10). Corollary 1(ii) follows from (10) and (12).

III.2. *Identification of the Markup, Marginal Cost and Aggregation Biases*

We first derive the aggregate markup pricing relationship and discuss how it relates to the exact aggregation literature. Then we study conditions under which the aggregation bias in the price margin relationship is identified.

The first order condition for profit maximization for firm h is given by:

$$p + y_h\frac{\partial p}{\partial y_h} = \frac{\partial c_h}{\partial y_h}(w,y_h,t). \tag{15}$$

At the industry level with H firms, aggregate output is given by $Y = \sum_{h=1}^H y_h$ and satisfies $pY = p\sum_{h=1}^H y_h$. Aggregating (15) over firms involves a weighted sum with market shares as weights and yields:

$$p + \sum_{h=1}^H \frac{y_h}{Y} y_h \frac{\partial p}{\partial Y} = \sum_{h=1}^H \frac{y_h}{Y}\frac{\partial c_h(w,y_h,t)}{\partial y_h}. \tag{16}$$

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Two approaches lead to a relationship that can be estimated with aggregate data. The first one assumes that microeconomic relationships exhibit properties leading to simplifications in equation (16). When all firms have the same inverse demand elasticity and the same marginal cost function $\partial c_h(w,y_h)/\partial y_h \equiv d(w)$, then (16) can be written in terms of aggregate variables only. Such an assumption, however, is incompatible with our purpose of estimating returns to scale without unwarranted restrictions.

The second approximate aggregation approach, developed by LEWBEL [1996], explicitly defines aggregate functions as conditional expectations of the disaggregate relationships, thereby avoiding restrictions on individual technologies. Doing this, we obtain from (16):

$$p + Y\frac{\partial p}{\partial Y}\mathrm{E}_\beta\left[\sum_{h=1}^{H}\beta_h^2|w,Y,t\right] = \mathrm{E}_\beta\left[\sum_{h=1}^{H}\frac{y_h}{Y}\frac{\partial c_h(w,y_h,t)}{\partial y_h}|w,Y,t\right].$$

So, the conditional expectation of the marginal revenue depends upon

$$\mathbb{H}^a(w,Y,t) \equiv \mathrm{E}_\beta\left[\sum_{h=1}^{H}\beta_h^2|w,Y,t\right],$$

which corresponds to the conditional expectation of the Hirschman-Herfindahl index of industry concentration. Using (11), we can rewrite this equation as:

$$p + \mathbb{H}^a(w,Y,t)\,Y\frac{\partial p}{\partial Y} = \frac{\partial \mathbf{C}}{\partial Y}(w,Y,t) - \int C(w,\beta Y,t)\frac{\partial f}{\partial Y}(\beta|w,Y,t)\,d\beta. \qquad (17)$$

The main advantage of the aggregate specification (17) is that it is compatible with the microeconomic specification (15). If a solution y_h^N to the system (15) exists, then the existence of a solution to (17) is also ensured, because in this case the expectations are conditional on $Y = \sum_h y_h^N(w,z,t)$, the aggregate Nash outcome. If the solution to (17) in Y is unique then $Y^N(w,z,t) = \sum_h y_h^N(w,z,t)$. If (17) exhibits multiple solutions in Y, however, one of them only corresponds to $\sum_h y_h^N(w,z,t)$.

Note that specification (17) obtained by aggregating (15) looks similar to the usual microeconomic specifications for which $p + \mathbb{H}^a Y\partial p/\partial Y$ is interpreted as the marginal revenue as perceived by the firm: $\mathbb{H}^a = 1$ corresponds to a cartel, $\mathbb{H}^a = 1/H$ to a (symmetric) Cournot oligopoly and $\mathbb{H}^a = 0$ to perfect competition. See BRESNAHAN [1989] and REISS and WOLAK [2007] for a survey of this literature. All terms of (17) are identified when data on the Herfindahl index is available: p and $\partial p/\partial Y$ are identified from estimating an inverse demand function, $\mathbb{H}^a$ is identified as the conditional expectation of the Herfindahl index; and $\partial \mathbf{C}/\partial Y$ is identified from the estimation of the cost function, and so the aggregation bias $B_{CY}(w,Y,t)$ can be computed residually. Without such information, however, it seems difficult to identify $\mathbb{H}^a$ separately from B_{CY}. BRESNAHAN [1982] and LAU [1982] discussed the conditions under which a constant parameter $\mathbb{H}^a$ can be uniquely identified under the assumption that a representative firm exists ($B_{CY} \equiv 0$). Lau's result can be extended to our framework:

Proposition 2. Under the above assumptions,

(i) the functions $\mathbb{H}^a$ and B_{CY} are nonparametrically identifiable iff the inverse demand function p is not such that $p(Y,z) = q(Y)R(z) + s(Y)$ in the case where z is a scalar, or

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$p(Y,z) = \mathbf{P}(Y,R(z))$ in the case where z is a vector, which means that p is weakly separable in z;

(ii) for parameterized shifts in the distribution of market shares, the model is identified under broader circumstances than those mentioned in (i).

Proposition 2(i), proven in Appendix A for the sake of completeness, extends LAU's [1982] identification theorem in a context where $\mathbb{H}^a$ is not constant but a function of (w,Y,t). The identifiability condition given in P2(i) can be relatively easily tested from the estimation of an inverse demand function. When the null of separability of p in z cannot be rejected, then by Proposition 2(i) $\mathbb{H}^a$ and B_{CY} are not identifiable, unless one follows the hint given in P2(ii) and specifies adequate functional forms for $\mathbb{H}^a$ and B_{CY}.

The main difference between Lau's and our result is that in our case, information on costs is available and this allows identifying functions $\mathbf{C}$ and $\partial\mathbf{C}/\partial Y$. Supplementing cost data to the analysis allows us to overcome Lau's identification problem concerning the marginal cost function and to extend his result to the problem of identifying the aggregation bias. Indeed, aggregate cost and marginal cost depend upon the way aggregate output is distributed over firms, and this information, which was missing in the Bresnahan-Lau approach, allows identifying both the average market power of an industry and the aggregation bias.

For instance, when the firms' market shares are constant, $B_{CY} = 0$, and (17) allows to link the unobserved marginal cost function $\partial C/\partial Y$ to the estimable marginal cost function $\partial\mathbf{C}/\partial Y$ since then (see equation (11)):

$$\frac{\partial \mathbf{C}}{\partial Y}(w,Y,t) = \mathrm{E}_\beta\left[\frac{\partial C}{\partial Y}(w,\beta Y,t)\,|w,Y,t\right].$$

Thus the estimation of the conditional mean cost function $\mathbf{C}$ is informative about the otherwise unobserved aggregate marginal cost and this information allows to identify the average market power $\mathbb{H}^a$ as the ratio between $\partial\mathbf{C}/\partial Y - p$ and $Y\partial p/\partial Y$.

In the case where B_{CY} and $\mathbb{H}^a$ are constant, (17) can be rewritten as

$$p + \mathbb{H}^a Y\frac{\partial p}{\partial Y} + B_{CY} = \frac{\partial \mathbf{C}}{\partial Y}(w,Y,t).$$

When $\partial\mathbf{C}/\partial Y$ is observed, it is possible to identify B_{CY} and $\mathbb{H}^a$ iff the "regressors" p and $Y\partial p/\partial Y$ are not proportional, which is the case iff $p(Y,z) \neq r(z)Y^\theta$. This example illustrates that, with more information on costs, identification is achieved under broader circumstances than those given in Proposition 2(i).

III.3. *Toward the Specification of Aggregate Cost and Input Demand Functions*

From equation (8) it is clear that most microeconomic properties of C are lost in the aggregate if the conditional distribution of market shares depends upon (w,Y,t). How should we specify the aggregate model in this case? For answering this question, let us write

$$\begin{aligned}\mathbf{C}(w,Y,t) &= \int C(w,\beta Y,t)\,f(\beta|w,Y,t)\,d\beta \\ &= \int C(w,\beta Y,t)\,g(\beta)\,d\beta + \int C(w,\beta Y,t)\,(f(\beta|w,Y,t) - g(\beta))\,d\beta \\ &\equiv \mathbf{C}_0(w,Y,t) + \mathbf{C}_1(w,Y,t),\end{aligned} \tag{18}$$

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where $g(\beta)$ denotes the marginal (joint) density of market shares. This decomposition shows that the aggregate cost function can always be additively decomposed into a function $\mathbf{C}_0$ satisfying some microeconomic properties (especially linear homogeneity in w) and a perturbation function $\mathbf{C}_1$ which depends on the gap between the conditional and marginal distribution of the market shares. This decomposition is useful for the empirical specification of the cost function. $\mathbf{C}_1$ vanishes under LEWBEL's [1996] independence assumption, $f(\beta|w,Y,t) = g(\beta)$, and only a well behaved aggregate cost function $\mathbf{C}_0$ remains.

The microeconomic properties of $\mathbf{C}_0$ are actually helpful for identifying $\mathbf{C}_1$: any departure of $\mathbf{C}$ from linear homogeneity, Shephard's lemma and concavity in prices can be attributed to distributional shifts captured by $\mathbf{C}_1$. However, in most cases, only parts of this function can be identified separately from $\mathbf{C}_0$.[1] This is not really a problem, since we are not interested in identifying $\mathbf{C}_1$, but in modelling the aggregate cost function and its partial derivatives.

According to Corollary 1(ii), the input demand system $\mathbf{X}^*$ cannot be obtained from $\mathbf{C}$ by applying Shephard's lemma. From equations (9) and (10), however, we know that:

$$\mathbf{X}^*(w,Y,t) = \frac{\partial \mathbf{C}}{\partial w}(w,Y,t) - B_{Cw}(w,Y,t), \tag{19}$$

which can be plugged into (23). Although the aggregate cost function loses most microeconomic properties, the adding-up property is still satisfied by $\mathbf{C}$, and this allows us to relate the aggregation bias B_{Cw} on input demands to function $\mathbf{C}_1$. Indeed,

$$\begin{aligned} &\mathbf{C}(w,Y,t) = w^\top \mathbf{X}^*(w,Y,t) \\ &\Leftrightarrow \mathbf{C}_0(w,Y,t) + \mathbf{C}_1(w,Y,t) = w^\top \frac{\partial \mathbf{C}}{\partial w}(w,Y,t) - w^\top B_{Cw}(w,Y,t) \\ &\Leftrightarrow w^\top B_{Cw}(w,Y,t) = w^\top \frac{\partial \mathbf{C}_1}{\partial w}(w,Y,t) - \mathbf{C}_1(w,Y,t), \end{aligned} \tag{20}$$

where the last line follows from the definition of $\mathbf{C}$ and linear homogeneity of $\mathbf{C}_0$ in w.

IV. Empirical Specification

In the empirical application, we use a panel of N industries indexed by $n = 1,\ldots,N$ observed over T time periods indexed by $t = 1,\ldots,T$. We first present the model specification, and then discuss the empirical content of the aggregate LCS decomposition.

1. Consider for instance $\mathbf{C}_0 = \alpha_w^\top w + w^\top (A_{wy}y + A_{wt}t)$, which is linearly homogeneous in w and $\mathbf{C}_1 = \gamma_0 + \gamma_w^\top w + \gamma_y y + \gamma_t t$. Then the parameters α_w and γ_w cannot be separately identified in the expression of the aggregate cost function $\mathbf{C}_0 + \mathbf{C}_1$.

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IV.1. *The Empirical Model*

The whole model consists of a system with a price setting rule, an inverse output demand function, five input demand equations and a cost function:

$$1+\mathbb{H}_n^a(w_{nt},Y_{nt},t)\frac{\partial \ln p}{\partial \ln Y}(Y_{nt},z_{nt})=\frac{1}{p_{nt}}\left(\frac{\partial \mathbf{C}_n}{\partial Y}(w_{nt},Y_{nt},t)-B_{CY}(w_{nt},Y_{nt},t)\right)+u_{nt}^{\partial C}, \quad (21)$$

$$p_{nt}=p(Y_{nt},z_{nt})+u_{nt}^{p}, \quad (22)$$

$$X_{nt}=\mathbf{X}_n^{*}(w_{nt},Y_{nt},t)+u_{nt}^{X}, \quad (23)$$

$$C_{nt}=\mathbf{C}_n(w_{nt},Y_{nt},t)+u_{nt}^{C}. \quad (24)$$

Let $u_{nt}\equiv\left(u_{nt}^{\partial C},u_{nt}^{p},u_{nt}^{X},u_{nt}^{C}\right)^{\top}$. The value of Y_{nt} solving equation (2) given $u_{nt}^{\partial C},w_{nt},z_{nt}$ and t will depend on $u_{nt}^{\partial C}$, and thus we cannot assume mean independence between u_{nt} and (Y_{nt},w_{nt},z_{nt}), but we assume that $\mathrm{E}[u_{nt}|w_{nt},z_{nt}]=0$. Several assumptions on $\mathrm{E}\left[u_{nt}u_{ms}^{\top}|w,z\right]$ compatible with different forms of heteroskedasticity and correlation over n, m, t and s are considered. In order to be able to identify the different parameters of (2), notably the parameters of $\partial p/\partial Y$ and $\partial \mathbf{C}_n/\partial Y$, we estimate it jointly with the inverse output demand (22), the set of input demand functions (23) and the cost function (24). As $C_{nt}=w_{nt}^{\top}X_{nt}$, and $u_{nt}^{C}=w_{nt}^{\top}u_{nt}^{X}$, the cost function does not add any new information not already entailed in the input demands when the whole system is estimated in levels. However, when estimation is in first differences this no longer holds, because for any given n and t, the error terms $u_{nt}^{C}-u_{n,t-1}^{C}$ and $u_{nt}^{X}-u_{n,t-1}^{X}$ are linearly independent when w_{nt} is not proportional to $w_{n,t-1}$. We now detail the specification of the functions appearing in the system (2)-(24).

IV.1.1. *Inverse Output Demand*

Several variables are candidates for inclusion in the output demand shifting vector z_t in the expression of $p(Y_{nt},z_t)$. Most of them are macroeconomic variables like GDP, total population, unemployed rate, government expenditures, exports and imports, interest rate or GDP inflation rate. Specifying z_t explicitly may yield a wrong specification of the inverse demand function, so we prefer instead including unobserved time specific variables f_t^p as well as a time trend t in (25) for representing omitted z_t variables. We also introduce industry-specific indicators f_n^p, and assume that

$$p(Y_{nt},z_t)=\exp\left(\eta_Y\ln Y_{nt}+\frac{1}{2}\eta_{YY}(\ln Y_{nt})^2+\eta_{Yt}t\ln Y_{nt}+f_n^p+f_t^p\right), \quad (25)$$

where $\eta=(\eta_Y,\eta_{YY},\eta_{Yt})^{\top}$ is a vector of three parameters and $z_t=\left(t,f_t^p\right)$. With this specification, p_{nt} is separable in z_t iff $\eta_{Yt}=0$ (see Proposition 2(i)).[2] There are $N+T+2$ free parameters in (25).

2. Admittedly, allowing all η parameters to vary with t could be attractive, but we cannot allow this level of generality for data reasons. A limited way to test for separability would be to test the constancy of the η over time. It is limited in the sense that nonrejection leads to nonrejection of separability, but not conversely, as the parameters could move proportionately.

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IV.1.2. *Aggregate Cost Function and Input Demands*

We specify the aggregation bias B_{Cwj} as:

$$B_{Cwj}(w,Y,t) \equiv \frac{1}{w_j}\left(\delta_j + \delta_{wj}^{\top}\ln w + \delta_{Yj}\ln Y + \delta_{tj}t\right), \qquad j = 1,\ldots,J, \tag{26}$$

which involves $J+3$ free δ-parameters, with J the number of inputs. We refrain from a richer specification, e.g. including interaction terms, because (i) none of the B_{Cwj} is observed, and (ii) this simple introduction of a loglinear bias term in our translog specification already results in a considerable increase in complexity. The advantage of introducing the factor $1/w_j$ is that $\mathbf{C}_1(w,Y,t)$ takes the simple form (see equation (20)):

$$\mathbf{C}_1(w,Y,t) = \gamma_C + \gamma_{\ln w}^{\top}\ln w + \gamma_Y \ln Y + \gamma_t t + h(w,Y,t),$$

where function h is homogeneous of degree one in w. Remember (footnote 1) that it is not always possible to separately identify h from $\mathbf{C}_0$, so for simplicity, we set $h(w,Y,t) \equiv \gamma_w^{\top} w$. The parameters γ of $\mathbf{C}_1$ are linked to the β-parameters of B_{Cwj} by

$$\gamma_C = -\sum_{j=1}^{J}\delta_j - \sum_{j=1}^{J}\iota_J^{\top}\delta_{wj}, \quad \gamma_{\ln w} = -\sum_{j=1}^{J}\delta_{wj}, \quad \gamma_Y = -\sum_{j=1}^{J}\delta_{Yj}, \quad \gamma_t = -\sum_{j=1}^{J}\delta_{tj}.$$

Thus, only J free parameters γ_w enter $\mathbf{C}_1$ in addition of those appearing in B_{Cwj}. According to Corollary 1, the representative firm assumption will be tested using estimates of the B_{Cwj} and B_{CY} and the restrictions (13) and (14). Note that departure of $\mathbf{C}$ from linear homogeneity in w can be measured by:

$$\mathbf{C} - w^{\top}\frac{\partial \mathbf{C}}{\partial w} = \mathbf{C}_1 - w^{\top}\frac{\partial \mathbf{C}_1}{\partial w} = \gamma_C + \gamma_{\ln w}^{\top}\ln w + \gamma_Y \ln Y + \gamma_t t - \iota_J^{\top}\gamma_{\ln w}. \tag{27}$$

The more traditional part of the cost function, $\mathbf{C}_0$, is specified as a translog functional form, a second order approximation of a general cost function in terms of the logs of its arguments. This specification is flexible in (w,Y,t), so that both increasing and decreasing returns to scale are a priori possible:

$$\begin{aligned}\mathbf{C}_{0n}(w,Y,t) \quad = \quad & \exp\Big[\alpha_{0,n} + \alpha_{w,n}^{\top}\ln w + \alpha_{Y,n}\ln Y + \alpha_{t,n}t \\ & + \frac{1}{2}(\ln w)^{\top}A_{ww}\ln w + \alpha_{wY}^{\top}\ln w \ln Y + \alpha_{wt}^{\top}(\ln w)t \\ & + \frac{1}{2}\alpha_{YY}(\ln Y)^2 + \alpha_{Yt}\ln Yt + \frac{1}{2}\alpha_{tt}t^2\Big],\end{aligned} \tag{28}$$

where $\ln w = (\ln w_1,\ldots,\ln w_J)^{\top}$. The parameters α_{0n}, α_{wn}, α_{Yn}, α_{tn} are industry specific, hence the subscript n. We know from (18) that function $\mathbf{C}_{0n}$ is linearly homogenous in w, so the α-parameters satisfy the usual restrictions:

$$\iota_J^{\top}\alpha_{wn} = 1, \quad \iota_J^{\top}A_{ww} = 0, \quad \iota_J^{\top}\alpha_{wY} = \iota_J^{\top}\alpha_{wt} = 0, \tag{29}$$

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where ι_J denotes a J-vector of ones. Matrix A_{ww} is symmetric. There are $N(J+2)+J(J+3)/2+1$ free α-parameters involved in (28).

Finally, the aggregation bias affecting the aggregate marginal cost is specified as

$$B_{CY}(w,Y,t) \equiv \delta_Y + \delta_{wY}^{\top} \ln w + \delta_{YY} \ln Y + \delta_{tY} t, \tag{30}$$

which includes $J+3$ further parameters, and can be seen as a first order approximation in terms of $\ln w$, $\ln Y$ and t.

IV.1.3. *Hirschman-Herfindahl (HH) Index*

Unfortunately, the HH-index is not available in our data and we have to estimate it together with the other parameters of the model. We restrict it to lie in $[0;0.5]$, a plausible restriction for the two digit level of aggregation (at the four digit level the HH is rarely above 0.5 and it does not increase at a higher level of aggregation). We first experimented with the logistic c.d.f. halved. However, this has the drawback that the lower and upper bounds are reached only at infinity, which created numerical convergence problems. We thus consider the following specification:

$$\mathbb{H}_n^a(w_{nt}, Y_{nt}, t) = \left[\cos\left(\lambda_{0n} + \lambda_w^{\top} \ln w_{nt} + \lambda_Y \ln Y_{nt} + \lambda_t t\right) + 1\right]/4, \tag{31}$$

which leads to similar results for the other parameters of the model, but more stable results for the HH-index.[3] The various λ represent parameters to be estimated, they include industry specific fixed effects λ_{0n} to allow for industry differences in the HH-index of concentration. For estimation, (30) and (31) are substituted into equation (2).

IV.2. *Empirical Expansion and Substitution Matrices*

Our purpose is to identify the terms of the aggregate LCS decomposition:

$$\frac{\partial X^N}{\partial w^{\top}}(w,z,t) = \frac{\partial X^*}{\partial w^{\top}}\left(w, \left\{y_h^N\right\}_{h=1}^H, t\right) + \sum_{h=1}^H \frac{\partial x_h^*}{\partial y_h}\left(w, y_h^N, t\right) \frac{\partial y_h^N}{\partial w^{\top}}(w,z,t), \tag{32}$$

which was obtained from the theoretical relationship

$$X^N(w,z,t) = X^*\left(w, \left\{y_h^N\right\}_{h=1}^H, t\right) = \sum_{h=1}^H x_h^*\left(w, y_h^N, t\right).$$

See KOEBEL and LAISNEY [2014] for details. When output quantities are optimally allocated, they are driven by (w,z,t), i.e. $y_h = y_h^N(w,z,t)$ and $Y = Y^N(w,z,t)$, and the distribution of market shares becomes endogenous: $\beta = \beta^N(w,z,t)$ and we can write:[4]

$$X^N(w,z,t) = X^*\left(w, \beta^N(w,z,t) Y^N(w,z,t), t\right) = \mathbf{X}^*\left(w, Y^N(w,z,t), t\right) + u^N. \tag{33}$$

3. The potential drawback of cyclicity of the cosine function is not too important here, since over the range of values of the argument of the cosine for each industry, cos turns out to be bijective for most industries (15 out of 18).
4. The heterogeneity in market shares is mainly driven by the heterogeneity in the cost functions, and becomes endogenous at the Nash equilibrium.

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The last equality follows from (9) when $Y = Y^N$. The term u^N denotes a random aggregation error satisfying $\mathrm{E}\left[u^N | w, z, t\right] = 0$. Keeping u^N fixed, this equation implies that (omitting the arguments of Y^N):

$$\frac{\partial X^N}{\partial w^\top}(w,z,t) = \frac{\partial \mathbf{X}^*}{\partial w^\top}\left(w, Y^N, t\right) + \frac{\partial \mathbf{X}^*}{\partial Y}\left(w, Y^N, t\right)\frac{\partial Y^N}{\partial w^\top}(w,z,t). \tag{34}$$

All terms involved in this expression can be obtained from the estimation of function $\mathbf{X}^*$ and from the aggregate first order condition (17).[5] Applying the implicit function theorem to (17) yields:

$$\frac{\partial Y^N}{\partial w^\top}(w,z,t) = \frac{\dfrac{\partial^2 \mathbf{C}}{\partial Y \partial w} - \dfrac{\partial B_{CY}}{\partial w} - Y\dfrac{\partial p}{\partial Y}\dfrac{\partial \mathbb{H}^a}{\partial w}}{(1+\mathbb{H}^a)\dfrac{\partial p}{\partial Y} + Y\dfrac{\partial p}{\partial Y}\dfrac{\partial \mathbb{H}^a}{\partial Y} + Y\mathbb{H}^a\dfrac{\partial^2 p}{\partial Y^2} - \dfrac{\partial^2 \mathbf{C}}{\partial Y^2} + \dfrac{\partial B_{CY}}{\partial Y}}. \tag{35}$$

This expression is then replaced into (34) in order to obtain matrix $\partial X^N / \partial w^\top$.

IV.3. *Econometric Issues*

We now explain how we propose to deal with the endogeneity of aggregate output in our input demand system. The observed input quantities are related to the demand functions by:

$$\begin{aligned} X_{nt} &= \mathrm{E}\left[X_{nt} | w_{nt}, z_{nt}, t\right] + v_{nt}^X \\ &= X_n^N\left(w_{nt}, z_{nt}, t\right) + v_{nt}^X \\ &= \mathbf{X}_n^*\left(w_{nt}, Y_n^N\left(w_{nt}, z_{nt}, t\right), t\right) + v_{nt}^N \\ &= \mathbf{X}_n^*\left(w_{nt}, Y_{nt} - v_{nt}^Y, t\right) + v_{nt}^N. \end{aligned} \tag{36}$$

All equalities are equivalent representations of the relation of interest. The first equality corresponds to the definition of the conditional mean. The second equality follows from the hypothesis that the conditional means of input quantities (given w_{nt}, z_{nt}, t) correspond to aggregate Nash equilibrium input demands X^N. The random term v_{nt}^X represents random deviations between observations and Nash equilibrium. The third equality follows from (33), with the random term $v_{nt}^N \equiv u_{nt}^N + v_{nt}^X$. The fourth equality is obtained because observed aggregate output is related to the Nash equilibrium output by $Y_{nt} = Y_{nt}^N + v_{nt}^Y$. Unfortunately, these equations are not useful as such for parameter estimation: the second equation does not allow to disentangle substitution and expansion effects; the third equality is useless, because Y_{nt}^N is not known; the fourth equality is not much more helpful because the error term v_{nt}^Y is not observed. Instead of that, we have to rely upon (23). These relationships clarify that the random term u_{nt}^X in our regression (23) is a nonlinear function of Y_{nt}. Comparing (23) and (36) it turns out that:

$$u_{nt}^X = \mathbf{X}_n^*\left(w_{nt}, Y_{nt}^N, t\right) - \mathbf{X}_n^*\left(w_{nt}, Y_{nt}^N + v_{nt}^Y, t\right) + v_{nt}^N. \tag{37}$$

5. In general, there will be no one to one correspondence between the substitution and expansion matrices of (32) and (34), but this seems to be the best approximation we can think of.

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Since

$$\mathrm{E}\left[u_{nt}^{X}Y_{nt}|w_{nt},z_{nt},t\right]=\mathrm{E}\left[u_{nt}^{X}\left(Y^{N}\left(w_{nt},z_{nt},t\right)+v_{nt}^{Y}\right)|w_{nt},z_{nt},t\right]=\mathrm{E}\left[u_{nt}^{X}v_{nt}^{Y}|w_{nt},z_{nt},t\right],$$

it turns out, by (37), that Y_{nt} is not a valid instrument. Valid instruments can be found using the fact that

$$\mathrm{E}\left[X_{nt}|w_{nt},z_{nt},t\right]=\mathbf{X}_{n}^{*}\left(w_{nt},Y_{n}^{N}\left(w_{nt},z_{nt},t\right),t\right). \tag{38}$$

These conditional moments justify the use of the unconditional moments $\mathrm{E}\left[u_{jnt}^{X}Z_{nt}\right]=0$ with instruments $Z_{nt}=I\left(w_{nt},z_{nt},t\right)$, and I denoting a K-vector of arbitrary square integrable functions of $\left(w_{nt},z_{nt},t\right)$.

In order to avoid spurious regression problems linked with nonstationarity, we estimate the cost and input demand equations in first differences. Only the price markup relationship (22) is considered in level as both the markup and the HH-index should be stationary.

For each industry we have 50 periods, so the incidental parameter problem connected with the industry-specific fixed effects does not arise here.[6] Thus, the NJ^{*} first year observations are lost, where $J^{*}=J+3$ denotes the number of regressions in the system (2)-(24). The vector of error terms is denoted by $\Delta u_{nt}\equiv\left(u_{nt}^{\partial C},\Delta u_{nt}^{p},\Delta u_{nt}^{X},\Delta u_{nt}^{C}\right)$ with $\Delta u_{nt}^{j}=u_{nt}^{j}-u_{n,t-1}^{j}$ for $j=p,X,C$, and the moment conditions are given by $\mathrm{E}\left[\Delta u_{nt}Z_{n,t-1}^{\top}\right]=0$, where Z_{nt} denotes the instruments and comprises w_{nt}, $\ln w_{nt}$, $t\ln w_{nt}$, $\left(\ln w_{j,nt}\right)^{2}$, as well as N industry and $T-2$ period indicators. The discussion leading to Proposition 3 suggests that $\mathrm{E}\left[u_{n,t-1}Z_{n,t-1}^{\top}\right]=0$, and so the moment conditions $\mathrm{E}\left[\Delta u_{nt}Z_{n,t-1}^{\top}\right]=0$ are fulfilled if $\mathrm{E}\left[u_{nt}Z_{n,t-1}^{\top}\right]=0$.

The same instruments are used for all GMM estimators and all equations. The whole system comprises 291 parameters to be estimated on the basis of $N\left(T-1\right)J^{*}=7488$ observations. So there are $99\times8=792$ orthogonality conditions imposed for parameter identification. The residual variance matrix is obtained from the residuals of a first stage regression in which the weighting matrix was set to the identity.

V. Empirical Results

The data we use are described in Appendix B. They are used also by DIEWERT and FOX [2008] in a study on imperfect competition. Firm data can also be informative for this kind of study (see Dobbelaere and Mairesse, 2013), but as they are typically not exhaustive, they are not useful for our purpose of investigating aggregate output and input demand. A further drawback of micro data for our purposes is the usual lack of reliable price information. Data for the same level of aggregation are also used by HALL [1988], ROEGER [1995], OLIVEIRA MARTINS, SCARPETTA, and PILAT [1996], MORRISON-PAUL and SIEGEL [1999] and others. We present the empirical results in five groups;[7] first, those concerning the (inverse) output demand function; second, those relative to the rate of return to scale and markups; third, the test results on the

6. Results obtained with regressions in levels or with an AR(1) specification for the error terms are available in KOEBEL and LAISNEY [2010]. The results discussed here correspond to the statistically preferred specification.
7. The results were obtained with TSP 5.1.

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TABLE I. – GMM Estimates for the Inverse Output Demand

Estimation method	η_Y	η_{YY}	η_{Yt}	$\varepsilon(p;Y)$	$\partial p/\partial Y$	$Y\partial^2 p/\partial Y^2$	OIT
System	**−1.258** (−13.0)	**−0.205** (−4.5)	**0.012** (5.0)	**−0.571** (−9.6)	**−38.65** (−9.6)	**61.41** (8.3)	.99 (495)
Single equation	**−1.091** (−4.8)	−0.210 (−1.8)	0.006 (1.0)	**−0.593** (−3.8)	**−36.60** (−3.8)	**59.98** (3.0)	.29 (27)

Columns 5 to 7 report the median value of the corresponding statistic over all observations as well as the median Student statistic in parentheses. Bold entries highlight significant results at the 5% level. The last column reports the p-value of the overidentification test and the number of degrees of freedom. The number of instruments is 99 in the single regression (for 72 parameters), and 8x99 for the system (with 297 parameters).

representative firm; fourth, all results describing shifts at given aggregate output, and fifth, results corresponding to the total impact of input price changes on input demand and output supply (which includes general equilibrium effects).

V.1. *The Inverse Demand Function*

TABLE I reports estimates of the inverse output demand function (22). The first line of results concerns the estimates when all J^* regressions are simultaneously run. The second one reports estimates obtained for the single regression (22).

The estimated values of the parameters exhibit no contradiction between the two estimation methods: the significant coefficients in the single equation are close to the system estimates. Our preferred specification is the system of equations because it is more efficient and enables identification of the markup, the Hirschman-Herfindahl function $\mathbb{H}^a$ and the aggregation biases (see Proposition 2). However, we also report the single estimation estimates, because in many cases only data on output quantity and output prices are available to researchers.[8]

The sensitivity of output price to output quantity is found to be quite important with both estimation methods: the inverse demand elasticity with respect to output, $\varepsilon(p;Y)$, is significantly different from zero and large in absolute value. We tested for industry specific heterogeneity in the elasticities by including heterogeneous parameters η_Y for each industry in the inverse demand specification (25). The test did not reject the homogeneity hypothesis ($\eta_{Y,n} = \eta_Y$). Separability in z_t, however, is strongly rejected (as η_{Yt} is significantly different from zero in the system). The overidentification test does not reject the instruments' validity, which is conform to our discussion below (38).

Turning to the sufficient conditions for the existence of a Nash equilibrium, in all cases, the inverse output demand is decreasing and $\partial p^2/\partial Y^2$ is found to be statistically significant and positive. Columns 6 and 7 of TABLE I suggest that $\partial p/\partial Y + Y\partial p^2/\partial Y^2$ is slightly positive. Indeed it turns out to be significantly positive at the 5% level for the bulk of the observations for the system, so that Novshek's sufficient condition is rejected (for the single equation, however, it is insignificant for most observations). Inference here and in the sequel concerns functions of both the estimated coefficients (together with their asymptotic normal distribution) and observed regressors and uses the delta method. Whereas Amir's sufficient condition for the existence

8. But note that we use cost information in the instrumentation. The necessity of instrumentation is documented by the fact that nonlinear least squares estimation yields a median value of $\varepsilon(p;Y)$ of −0.202 which is strikingly different from the GMM results of TABLE I.

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of a Cournot equilibrium, (4), is rejected (at the 5% threshold) for 79% of the observations, Novshek's condition is rejected in 94% of the cases. In contrast, the weaker aggregate condition (5), which also ensures existence of the Cournot equilibrium, is numerically satisfied for values of the HH index $\mathbb{H}^a$ below 0.66.[9]

V.2. *Rate of Return to Scale, Markup and Industry Concentration*

The magnitude of returns to scale plays an important role in our context. With increasing returns to scale (IRS) perfect competition is not viable, as profits would be negative, and this justifies our focus on imperfect competition. The quartiles (over years and industries) of the estimated values of the rate of return to scale and markup are reported in TABLE II. The overidentifying restrictions are not rejected at the 1% threshold. We find evidence for IRS, since most estimates of $\varepsilon(\mathbf{C}, Y)$ are smaller than one and the IRS hypothesis cannot be rejected for 40% of the observations. There is also a group of observations (about 46%) compatible with the assumption of constant returns to scale. Our estimated rate of returns to scale are often lower than those reported by DIEWERT and FOX [2008], due possibly to fewer restrictions imposed on our cost function (as we do not assume neutral technological change). However, our results are close for the following eight industries: SIC No 22, 23, 25, 26, 30, 24, 35 and 38.[10] The results also confirm the existence of markup pricing (the median value of $p/(\partial\mathbf{C}/\partial Y)$ is 1.15), and the markup is significantly greater than one for 39% of the observations.

TABLE II also reports the 0.25, 0.5 and 0.75 quantiles of the estimated HH-index of concentration and their *t*-values. The median value of the estimated HH-index is 0.26 and it is significantly different from zero for 41% of the observations. If we integrate this estimated value of $\mathbb{H}^a$ into our formulation (5) of the existence of the Cournot-Nash equilibrium, we cannot reject the existence assumption. This empirical finding provides empirical support for Proposition 2 in KOEBEL and LAISNEY [2014].

TABLE III reports the correlations (over 18 industries) between the average values (over time) of different measures of imperfect competition, $\mathbb{H}^a$, $\varepsilon(\mathbf{C}, Y)$, $p/(\partial\mathbf{C}/\partial Y)$, $\varepsilon(p, Y)$, s_π, s_π^a, based on the system estimates with $\mathbf{C}_1 \neq 0$. Variables s_π and s_π^a correspond to two rates of profit (a ratio between profit and sales) whose precise definition is given in Appendix B. These correlations suggest that the implicit HH-index, $\mathbb{H}^a$, is not strongly correlated with other indicators of imperfect competition as the rate of returns to scale, the markup, the output demand elasticity or the profit rate. In contrast, $\varepsilon(\mathbf{C}, Y)$ (column 3) is strongly negatively correlated with the markup and positively correlated with the profit rate. These signs are due to the fact that

$$\varepsilon(\mathbf{C}, Y) = \frac{s_\pi + 1}{p/(\partial\mathbf{C}/\partial Y)}.$$

The high correlation between the rate of returns to scale and the markup is related to the fact that industries with a low elasticity of cost with respect to output, $\varepsilon(\mathbf{C}, Y)$, need to price over their marginal cost to make nonnegative profit: if $\varepsilon(\mathbf{C}, Y) < 1$ and $\partial\mathbf{C}/\partial Y = p$ then the profit is negative. The negative correlation between profit rate and output demand elasticity is also conform to theory.

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TABLE II. – GMM Estimates of the Rate of Return to Scale, Markup, and $\mathbb{H}^a$

	$\mathbf{C}_1 \neq 0$				$\mathbf{C}_1 \equiv 0$			
Quartile	$\varepsilon(\mathbf{C};Y)$	$p/(\partial\mathbf{C}/\partial Y)$	$\mathbb{H}^a$	OIT	$\varepsilon(\mathbf{C};Y)$	$p/(\partial\mathbf{C}/\partial Y)$	$\mathbb{H}^a$	OIT
0.25	**0.79** (−3.1)	1.02 (0.3)	0.08 (0.6)		0.86 (−1.4)	0.94 (−0.9)	0.07 (0.4)	
0.50	0.91 (−1.4)	1.15 (1.6)	0.26 (1.7)	0.99	0.98 (−0.2)	1.06 (0.4)	0.17 (0.9)	0.11
0.75	1.03 (0.6)	**1.32** (3.1)	**0.45** (3.5)		1.06 (1.0)	1.17 (1.3)	0.29 (1.6)	

The table reports the 0.25, 0.50 and 0.75 quantiles (over the $N(T-1) = 936$ observations) of the estimated values of $\varepsilon(C,Y)$, $p/(\partial C/\partial Y)$ and $\mathbb{H}^a$. The t-statistics for the hypotheses $\varepsilon(C,Y) = 1$, $p/(\partial C/\partial Y) = 1$ and $\mathbb{H}^a = 0$ appear in parentheses. The OIT column reports the p-value of the overidentification test.

TABLE III. – Correlation between Different Estimated Measures of Imperfect Competition

	$\mathbb{H}^a$	$\varepsilon(\mathbf{C},Y)$	$p/(\partial\mathbf{C}/\partial Y)$	$\varepsilon(p,Y)$	s_π
$\varepsilon(\mathbf{C},Y)$	−0.10				
$p/(\partial\mathbf{C}/\partial Y)$	−0.18	−0.66			
$\varepsilon(p,Y)$	−0.16	−0.26	0.27		
s_π	0.11	0.43	0.00	−0.43	
s_π^a	0.06	0.51	−0.03	−0.26	0.86

The entries correspond to the empirical correlation between the estimated values of the variables of the respective column and line. In a first stage, the variables are averaged over time, then the correlation is calculated over the 18 industries. The values used for s_π and s_π^a are those depicted in FIGURE B3.

The null hypothesis of perfect competition is characterised by either $\mathbb{H}^a = 0$ or $\varepsilon(p;Y) = 0$ in (2). While the first hypothesis is rejected for 41% of the observations, the second one is rejected for all. Hence the null of perfect competition is rejected for 41% of the cases. Four industries (SIC No 27, 32, 33 and 35) are found to be in perfect competition over the period. Four industries (SIC No 24, 25, 26 and 30) had strong market power. For the other industries, we found evidence of market power over subperiods but not over the whole 52 years, a result which is consistent with the literature on the cyclicality of the markup. Our estimates of the markups are lower than those reported by ROEGER [1995], who assumes that returns to scale are constant, but they are broadly in line with those reported by OLIVEIRA MARTINS, SCARPETTA, and PILAT [1996] and DIEWERT and FOX [2008].

V.3. *Test of the Representative Firm Assumption*

As discussed in SECTIONS III and IV, the representative firm model is obtained for $\mathbf{C}_1(w,Y,t)$ homogeneous of degree one in w, which is the case iff expression (27) is identically 0, but this hypothesis is statistically rejected at the 1% threshold.

A more restrictive version of the representative firm hypothesis imposes that $C_1 = 0$. This restriction yields the classical translog specification. TABLE II (right panel) shows that this restriction yields estimates that are more compatible with the assumption of perfect competion: the rate of returns and markups are closer to one, and the HH index is closer to zero. This

9. Amir and Novshek's existence conditions are also rejected with the more disaggregated NBER data for 462 manufacturing industries.
10. See TABLE B1 in Appendix B for the industry names.

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TABLE IV. – Estimates of the Aggregation Biases

Bias	w_k	w_ℓ	w_e	w_m	w_s	Y
B_{Cj}	0.01 (1.1)	−0.19 (−5.0)	0.90 (3.9)	−0.13 (−8.2)	1.88 (10.5)	0.03 (0.2)

The table reports the median value of the estimated aggregation biases, expressed in proportion of the corresponding input quantity. The aggregation bias on marginal costs, B_{CY}, is expressed as a proportion of marginal cost. The t-statistic for the null that $B_{Cj} = 0$ appears in parentheses.

is due to the fact that the imposition $C_1 = 0$ and $B_{Cj} = 0$ on the specification corresponds to the exclusion of input and output reallocation effects across firms, which is statistically rejected. The value of the different types of aggregation biases, expressed in proportion of the corresponding input quantity (or marginal cost) are reported in TABLE IV. It shows that the gap $B_{Cw}(w,Y,t)$ between input demand and the derivative of the cost function (see the discussion aroud Corollary 1) is often significantly different from zero and important. The distribution of output shares is especially sensitive with respect to the price of intermediate services and energy. Our interpretation is that firms' outsourcing and merging decisions heavily depend on the price of intermediate services: they choose to outsource their production if this price is too low or to merge if it is too important.

Shephards' lemma ($B_{Cj} = 0$) and the hypothesis that C is homogeneous of degree zero in input prices ($w^\top B_{Cj} = 0$) are separately rejected for about 75% of the observations, and these hypotheses are also rejected globally.

Although there is statistical evidence for the existence and significance of aggregation biases, the orthogonality conditions of the tranlog model do not reject the translog specification at the 5% threshold. This shows that the unability to reject the orthogonality conditions does not necessarily validate the specification (see TABLE II).

The bias affecting marginal cost represents about 3% of marginal cost and is statistically insignificant for 93% of the observations. This means that market share reallocations (changes in the distribution of output shares) following an increase in aggregate output Y only marginally affect aggregate cost. Let us consider a decomposition of returns to scale, similar to the one in Basu and Fernald (1997, Equation (3)), and rewrite the conditional expectation of (2) as

$$\frac{\partial \mathbf{C}}{\partial Y}\frac{Y}{C} = \frac{pY}{C} + \frac{pY}{C}\mathbb{H}^a \varepsilon(p;Y) + \frac{Y}{C}B_{CY}.$$

The last term YB_{CY}/C represents the shift in the distribution of market shares triggered by an increase in output. After replacing the different terms of this last relationship with the median value of their corresponding estimates, we obtain

$$0.91 \simeq 1.06 - 0.15 - 0.00.$$

So, according to this decomposition, the cost reduction through output reallocation effect over firms is negligibly small (as the estimate of YB_{CY}/C is virtually nil).

We conclude this analysis by commenting the well-documented empirical contradiction between results obtained from microeconomic and more aggregated data (see for instance BASU and FERNALD [1997]). The within industry reallocation effect, B_{CY}, comparable to the one

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considered by Basu and Fernald (1997, p.265) is found to be small, and consistent with the empirical results of MORRISON-PAUL and SIEGEL [1999]. Instead, we find that the impact of imperfect competition, $pY\mathbb{H}^a\varepsilon(p;Y)/C$, is important and explains the gap between the rate of profit and returns to scale for many observations.

V.4. *Substitution Effects*

TABLE V reports the median value of the input demand elasticities over the 18 industries and over all years. These elasticities are calculated for a given aggregate output level, and correspond to the aggregate substitution effect. They give the input price sensitivity of the aggregate input demand $\varepsilon\left(\mathbf{X}_j^*;w_i\right)$ and differ somewhat from $\varepsilon\left(X_j^*;w_i\right)$ which excludes shifts in the distribution of market shares (see Equation (33)).

TABLE V. – Elasticities for the Aggregate Substitution Effect

$\varepsilon\left(\mathbf{X}_j^*;w_i\right)$	$\mathbf{X}_k^*$	$\mathbf{X}_\ell^*$	$\mathbf{X}_e^*$	$\mathbf{X}_m^*$	$\mathbf{X}_s^*$
w_k	−0.19 (−1.6)	0.02 (1.5)	**0.05** (2.2)	**0.02** (2.5)	0.02 (1.4)
w_ℓ	0.09 (0.9)	**−0.36** (−9.6)	**0.31** (2.8)	**0.08** (2.9)	−0.14 (−1.0)
w_e	−0.04 (−1.3)	**0.03** (2.7)	**−0.43** (−2.9)	**0.04** (4.1)	0.01 (0.7)
w_m	0.03 (0.5)	**0.15** (4.7)	0.01 (0.1)	**−0.32** (−11.5)	**0.24** (3.8)
w_s	0.04 (0.3)	**0.08** (2.7)	0.05 (0.7)	**0.27** (9.9)	−0.08 (−0.4)
Y	0.10 (0.6)	**0.83** (7.1)	**0.56** (2.2)	**1.16** (13.7)	**0.66** (3.1)
t	**0.018** (2.9)	**−0.011** (−2.4)	−0.009 (−0.9)	**−0.017** (−6.5)	0.003 (0.8)

The table reports the median value of the elasticities over all observations and the corresponding t-statistic in parentheses. Bold entries highlight significant results at the 5% level. The subscripts *k, l, e, m, s* refer to capital, labor, energy, materials, and services.

The upper panel of TABLE V gives the input price elasticities with the own-price elasticities on the main diagonal. It can be seen that the own-price elasticities are nonpositive, which is consistent with $\varepsilon\left(\mathbf{X}_j^*;w_j\right)\leq 0$.

Most of the inputs are found to be substitutes, but substitution is rather limited as all cross price elasticities are below 0.31 in absolute value. No inputs are found to be significantly complements. Note that the signs of the elasticities of TABLE V are not symmetric: energy and capital are substitutes for changes in the capital price but not conversely. These findings, are not contradictory: our aggregate model does not imply that matrix $\partial\mathbf{X}^*/\partial w^\top$ is symmetric, because increases in energy and capital input prices may have different impacts on the distribution of market shares $f(\beta|w,Y,t)$. According to (19), input demands can react asymmetrically to cross price variations due to shift in market shares induced by input price changes. This empirical finding could explain why the literature reports a variety of contradictory results on the sign of this elasticity (see for instance Frondel and Schmidt, 2002). Without aggregation bias, the signs of $\varepsilon\left(\mathbf{X}_k^*;w_e\right)$ and $\varepsilon\left(\mathbf{X}_e^*;w_k\right)$ could not be different.

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The lower panel of TABLE V reports the impact of a marginal change in output $\varepsilon\left(\mathbf{X}_j^*;Y\right)$ and the impact of time $\varepsilon\left(\mathbf{X}_j^*;t\right)$.[11] All conditional input demands are found to be nondecreasing in the output level. Technological change is not neutral, but capital intensive, labour saving and intermediate input saving.

V.5. *Expansion Effect*

Our modified version of Novshek's assumption (5) is not rejected for 93% of the observations. So, we expect that the Cournot equilibrium exists and that the LCS principle is satisfied at the aggregate level of the industry, in accordance with inequalities (6) and (7).

However, one difficulty comes from the fact that the micoeconomic second order condition for optimality of output is not always satisified empirically in the aggregate. It turns out that the denominator of (35) was found to be significantly negative only in 54% of the observations. The resulting lack of precision is inherited by the long run elasticities.

TABLE VI gives the median value of the estimates for the total impact of input prices on equilibrium output quantity Y^N and price p^N and input demands X^N. The first column shows that the median values of $\varepsilon\left(Y^N,w_h\right)$ are mostly negative, which conforms with inequality (7), but they are not significantly different from zero. There is also some weak evidence for the inflationary impact of rising input prices as all $\varepsilon\left(p^N,w_h\right)$ are found to be nonnegative (but not significantly). This lack of precision is rather common in applications using GMM. Here it is possibly due to the fact that the denominator of (35), which was never found to be significantly positive, is not always *significantly* negative (it is significantly negative for only 54% of the observations). A careful inspection of the different terms of the denominator shows that $\partial^2 C/\partial Y^2$ is estimated to be significantly negative (and so the marginal cost is found to be decreasing) in more than 30% of the cases. This lack of precision also contaminates the estimates of $\varepsilon\left(\mathbf{X}_j^N;w_i\right)$.

For the median values reported in TABLE V, the LCS principle is satisfied: we can approximate the unrestricted demand elasticities by

$$\varepsilon\left(\mathbf{X}_\ell^N;w_\ell\right)\simeq\varepsilon\left(\mathbf{X}_\ell^*;w_\ell\right)+\varepsilon\left(\mathbf{X}_\ell^*;Y\right)\varepsilon\left(Y^N,w_\ell\right). \tag{39}$$

When the median values of $\varepsilon\left(\mathbf{X}_\ell^*;Y\right)$ and $\varepsilon\left(Y^N,w_\ell\right)$ are positive and nonpositive, respectively (TABLEs 5, line 7 and TABLE VI, column7), it turns out that $\varepsilon\left(\mathbf{X}_\ell^N;w_\ell\right)\leq\varepsilon\left(\mathbf{X}_\ell^*;w_\ell\right)$. However, the median values of the unrestricted elasticities reported in TABLE VI do not always satisfy the LCS principle due to sampling variation and violations of the second order condition for optimality. Notice that the input demand functions $\mathbf{X}_j^N$ are not homogeneous of degree zero in w (the column sums for the elasticities are not 0). The only two significant elasticities, $\varepsilon\left(\mathbf{X}_\ell^*;w_\ell\right)$ and $\varepsilon\left(\mathbf{X}_m^*;w_m\right)$, which concern labor and intermediate material demands, have the expected negative sign and are much larger than their short-run counterparts, in conformity with the version of the LCS principle obtained by Koebel and Laisney (2014, Corollary 1).[12]

11. $\varepsilon\left(\mathbf{X}_j^*;t\right)$ is a growth rate or a semi-elasticity defined as $\varepsilon\left(\mathbf{X}_j^*;t\right)\equiv\partial\ln\mathbf{X}_j^*/\partial t$.

12. Finding significant results mainly on the diagonal of a matrix of price elasticities is a frequent result in demand analysis (whether it concerns firms or households): own price effects seem to be more easily identified than cross price effects.

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TABLE VI. – Elasticities for Total Aggregate Impact of Input Price Changes

$\varepsilon(.;w)$	Y^N	p^N	$\mathbf{X}_k^N$	$\mathbf{X}_\ell^N$	$\mathbf{X}_e^N$	$\mathbf{X}_m^N$	$\mathbf{X}_s^N$
w_k	−0.10 (−0.4)	0.06 (0.4)	−0.21 (−0.6)	−0.07 (−0.4)	−0.0 (−0.1)	−0.10 (−0.3)	−0.02 (−0.1)
w_ℓ	−0.86 (−1.3)	0.50 (1.3)	0.13 (0.2)	**−1.23** (−1.7)	−0.08 (−0.1)	−0.85 (−1.2)	−0.68 (−1.1)
w_e	−0.01 (−0.0)	0.00 (0.0)	−0.06 (−0.5)	0.03 (0.2)	−0.38 (−0.9)	0.10 (0.3)	0.07 (0.4)
w_m	−0.94 (−1.3)	0.55 (1.3)	0.02 (0.1)	−0.67 (−1.2)	−0.44 (−1.0)	**−1.46** (−1.8)	−0.19 (−0.5)
w_s	0.36 (0.4)	−0.20 (0.4)	0.03 (0.1)	0.44 (0.6)	0.38 (0.5)	0.85 (0.9)	0.24 (0.3)

The table reports the median value of the elasticities over all observations and the corresponding t-statistic in parentheses. Bold entries highlight significant results at the 10% level. The subscripts *k, l, e, m, s* refer to capital, labor, energy, materials, and services.

In summary, expansion, or price induced change in the output level, matters for explaining changes in aggregate input demand. The mostly negative elasticities of TABLE VI can be reconciled with the upward trending quantities reported on FIGURE B1: TABLE V showed that most of the growth in input demands is due to output growth. Output growth in turn is hampered by input price increases, and rises with shifts in output demand.

VI. Conclusion

Output adjustments have important consequences on input demands. This impact, however, is rarely quantified, because with imperfectly competitive output markets, IRS and externalities disturb the usual representative firm's comparative statics. This paper makes two contributions to the literature: (i) it provides a framework amending the representative firm model for the specification of aggregate input demand and output supply functions; (ii) it shows that empirically the LCS principle is inherited at the level of two-digit US manufacturing industries. The results suggest that, in US manufacturing in the second half of the 20^{th} century, input substitution has been rather limited for a given output level. Output adjustments imply further important changes in the input mix. We find empirical support for IRS and markup pricing of moderate size. This is important for understanding aggregate growth, investment and employment. There is, however, much heterogeneity over industries and time, and about 50% of the observations are compatible with a rate of returns to scale near to one, and a markup close to zero.

Aggregation effects arise through shifts of market shares over firms within an industry. These shifts seem to play a significant role for explaining the evolution of aggregate input demands, but seem negligible for explaining changes in the marginal cost of production and profit rate. These figures are well explained by complementary aspects of imperfect competition: markup pricing, returns to scale and industry concentration. With only aggregate data at hand, however, these effects are difficult to identify with precision.

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Appendix

A. Proof of Proposition 2

Let us rewrite (17) as:

$$p(Y,z) - Y\frac{\partial p}{\partial Y}(Y,z)\,\mathbb{H}^a(w,Y,t) = \frac{\partial \mathbf{C}}{\partial Y}(w,Y,t) - B_{CY}(w,Y,t), \tag{A.1}$$

which defines $Y^N(w,z,t)$.

Functions $\partial \mathbf{C}/\partial Y(w,Y,t)$, $p(Y,z)$ and $\partial p/\partial Y(Y,z)$ are identified outside (A.1), but $B_{CY}(w,Y,t)$ and $\mathbb{H}^a(w,Y,t)$ are unknown. The proof studies the conditions under which functions B_{CY} and $\mathbb{H}^a$ are unique (that is, separately identified). We extend LAU's [1982] proof, which focused on the case where $\mathbb{H}^a$ was a constant parameter.

Nonidentification means that there exist $B^*_{CY} \neq B_{CY}$ and $\mathbb{H}^* \neq \mathbb{H}^a$ such that for any (w,z,t)

$$p(Y,z) - Y\frac{\partial p}{\partial Y}(Y,z)\,\mathbb{H}^*(w,Y,t) = \frac{\partial \mathbf{C}}{\partial Y}(w,Y,t) - B^*_{CY}(w,Y,t). \tag{A.2}$$

We first note that there then exists a continuum of functions B_{CY} and $\mathbb{H}$ compatible with (A.1): any $\mathbb{H}^{**} = \kappa\mathbb{H}^a + (1-\kappa)\mathbb{H}^*$ and $B^{**}_{CY} = \kappa B_{CY} + (1-\kappa)B^*_{CY}$ satisfy (A.1).[13] Note also that B_{CY} is identified iff $\mathbb{H}^a$ is identified.

We first prove the following lemma:

13. For $\kappa = -\mathbb{H}^*/(\mathbb{H}^a - \mathbb{H}^*)$ we even obtain that $\mathbb{H}^{**} = 0$ which corresponds to perfect competition. This shows that nonindentification would have far reaching consequences for empirical work.

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Lemma 1. Let G and G^* be two smooth functions defined on a set A and depending on (w,Y,t) and strictly monotonic in Y. Then, there exists a function F such that

$$G^*(w,Y,t) = F(w,G(w,Y,t),t). \tag{A.3}$$

Proof of Lemma A1.

Since both G and G^* are monotonic in Y we can write

$$\begin{aligned} Y &= G^{-1}(w,G(w,Y,t),t) \\ Y &= G^{*-1}(w,G^*(w,Y,t),t), \end{aligned}$$

with the inverse function G^{*-1} monotonic in its second argument. Hence

$$\begin{aligned} G^*(w,Y,t) &= G^*\left(w,G^{-1}(w,G(w,Y,t),t),t\right) \\ &\equiv F(w,G(w,Y,t),t). \end{aligned} \tag{A.4}$$

This argument can be extended to nonmonotonic functions by splitting A into subsets on which G and G^* are monotonic in Y. ■

Proof of Proposition 2.

Using the notations of Lemma A1, G is identified if there exists a function F satisfying

$$F(w,G(w,Y,t),t) = G(w,Y,t).$$

For

$$G(w,Y,t) \equiv \frac{\partial \mathbf{C}}{\partial Y}(w,Y,t) - B_{CY}(w,Y,t),$$

(A.3) becomes:

$$p(Y,z) - Y\frac{\partial p}{\partial Y}(Y,z)\,\mathbb{H}^*(w,Y,t) = F\left[w, p(Y,z) - Y\frac{\partial p}{\partial Y}(Y,z)\,\mathbb{H}^a(w,Y,t), t\right].$$

Differentiating this equation w.r.t. z yields

$$\begin{aligned} &\frac{\partial p}{\partial z}(Y,z) + \mathbb{H}^*(w,Y,t)\frac{\partial^2 p}{\partial Y \partial z}(Y,z)\,Y = \left(\frac{\partial p}{\partial z} + \mathbb{H}^a(w,Y,t)\frac{\partial^2 p}{\partial Y \partial z}(Y,z)\,Y\right)F_G \\ \Leftrightarrow\quad &(1-F_G)\frac{\partial p}{\partial z}(Y,z) = \frac{\partial^2 p}{\partial Y \partial z}(Y,z)\,Y\left[\mathbb{H}^a(w,Y,t)F_G - \mathbb{H}^*(w,Y,t)\right], \end{aligned} \tag{A.5}$$

where F_G is the derivative of F w.r.t. its second argument. Under the assumptions of Lemma A1, $F_G \neq 0$. We consider the case where $\mathbb{H}^a$ and G are not identified. Then, by (A.3), there exist triples (w,Y,t) such that $F(w,G(w,Y,t),t) \neq G(w,Y,t)$ which implies $F_G \neq 1$ and by (A.5), $\mathbb{H}^a F_G \neq \mathbb{H}^*$.

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If z is a scalar, (A.5) is satisfied for any (w,Y,t,z) iff $\phi \equiv (1-F_G)/(\mathbb{H}^a F_G - \mathbb{H}^*)$ depends only upon Y, in which case:

$$\begin{aligned} & \frac{\partial^2 p/\partial Y \partial z}{\partial p/\partial z}(Y,z) = \phi(Y) \\ \Leftrightarrow \quad & \frac{\partial}{\partial Y} \ln\left(\frac{\partial p}{\partial z}(Y,z)\right) = \phi(Y) \\ \Leftrightarrow \quad & \frac{\partial p}{\partial z}(Y,z) = q(Y)\, r(z) \\ \Leftrightarrow \quad & p(Y,z) = q(Y)\, R(z) + s(Y), \end{aligned}$$

where R denotes a primitive of r. If z is not a scalar, then (A.5) implies that

$$\frac{\partial p/\partial z_i}{\partial p/\partial z_j}(Y,z) = \frac{\partial^2 p/\partial z_i \partial Y}{\partial^2 p/\partial z_j \partial Y}(Y,z).$$

By LEONTIEF's [1947] Proposition 1 on functional separability, there then exists a real valued function R such that $p(Y,z) = \mathbf{P}(Y, R(z))$. Sufficiency is shown by LAU [1982], p. 97. ■

B. Data Description

This study relies on data provided by the Bureau of Labor Statistics (BLS) for 18 two-digit U.S. manufacturing industries over the period 1949-2001.[14] Unfortunately, this dataset based on the Standard Industrial Classification (SIC) has been superseded after the introduction of the North American Industry Classification in 2006. These SIC data series are not longer updated by the BLS.

Although it would certainly be better to use data at a more disaggregated level, there exist only few datasets comprising quantities and price indices at such level. This data set comprises information on the price and quantity of output (p,Y) and of five inputs (hence $J=5$): capital (w_k, X_k), labour (w_ℓ, X_ℓ), energy (w_e, X_e), intermediate material input (w_m, X_m), and services (w_s, X_s). The evolution of these quantities over time is depicted in FIGURE B1 at the aggregate level over all 18 industries. From FIGURE B1, it seems clear that these variables (except perhaps labour input) are nonstationary. In addition to the endogenous variables, several exogenous variables are also nonstationary.

The profit rate s_π defined as the ratio of (gross) profit to sales, is an important variable for assessing the relevance of imperfectly competitive behavior. Computing this variable is not an easy task because profits depend on the user costs of capital which are not observed and whose definition is not consensual. DIEWERT, HARRISON, and SCHREYER [2004] is a useful reference on this point. In this paper, we follow DIEWERT [2003] and retain the user cost of capital formula,

$$w_{k,nt} \equiv w_{i,nt}(1+r_t) - \mathrm{E}_t\left[(1-\delta_{nt})\, w_{i,n,t+1}\right]. \tag{B.1}$$

This equation shows that the user cost of capital w_k is increasing in the discounting rate r_t and decreasing in the expectation errors on investment goods inflation (given by $w_{i,n,t+1} -$

14. Data are available upon request.

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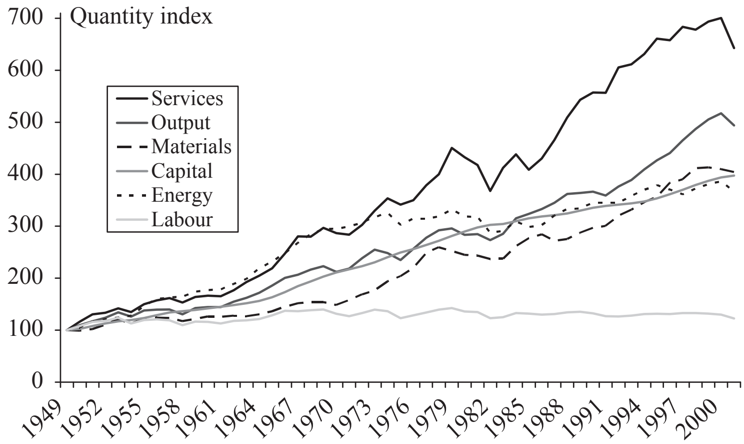

FIGURE B.1. – The Evolution of Input Quantities. Basis 100 in 1949

$\mathrm{E}_t\left[w_{i,n,t+1}\right]$). According to DIEWERT [2003], we set r_t to 5% plus the consumer price inflation rate and assume no expectation errors on $w_{i,n,t+1}$. The few values of the user costs found to be slightly negative (5 out of 954 cases) were replaced with 0.0001.

The time series of the profit rate s_π averaged over all 18 industries is depicted in FIGURE B2. In U.S. manufacturing industries, the profit rate is about 5.3 percent on average over time and industries. This figure is somewhat different from that of BASU and FERNALD [1997] who report an average profit rate of 3 percent. This discrepency is related to differences in the time period and the industries covered, but certainly also to the high sensitivity of the user costs of capital with respect to a priori assumptions upon (i) the expected price change of investment goods and (ii) the discounting rate of future income streams.

Given the sensitivity of the economic definition of the profit rate with respect to a priori choices, it appears helpful to complete the picture with the more robust accounting definition of the gross profit rate, which does not rely on the user cost of capital and the capital stock, but uses instead investment and investment price:

$$s_\pi^a = \frac{p_{nt}Y_{nt} - \left(w_{i,nt}X_{i,nt} + w_{\ell,nt}X_{\ell,nt} + w_{e,nt}X_{e,nt} + w_{m,nt}X_{m,nt} + w_{s,nt}X_{s,nt}\right)}{p_{nt}Y_{nt}}.$$

Notation $X_{i,nt}$ denotes gross investment and $w_{i,nt}$ its price. On average over time and industries s_π^a is equal to 8.7 percent, which seems to confirm the rather high level of profits in US manufacturing.

Average profitability over time and over industries is depicted in FIGURES B2 and B3, respectively. The evolution of average profitability (FIGURE B2) broadly reflects the business cycle, but there is also a downward trend or a break in this picture: whereas the average (economic) profit rate was about 7.8 percent over the 1949-1975 period, it declined to 2.8 percent for the post 1975 period. Such a pattern is actually observed for many industries (not reported).

FIGURE B3 reports the average level of profitability over the period 1949-2001 for each industry and shows that profitability varies a lot across industries. Two industries exhibit a

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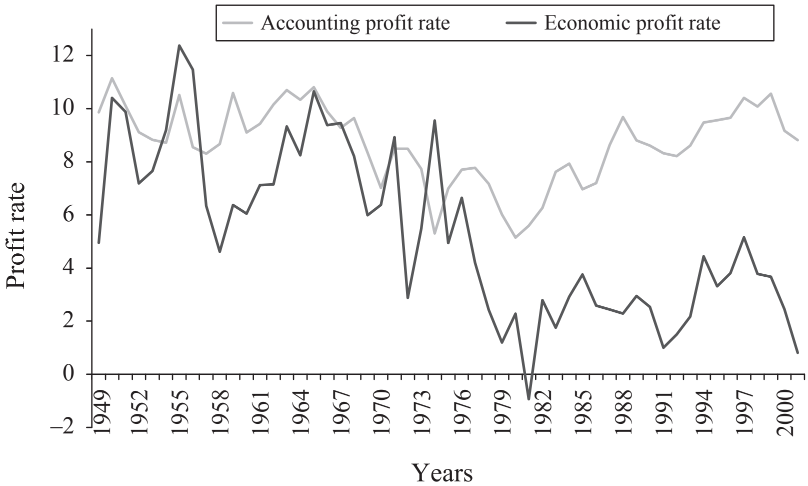

FIGURE B.2. – Profit Rates over Time (Average over Industries)

negative average economic rate of profit: Textile Mills Products and Primary Metal Industry (SIC numbers 22 and 33). For two industries, average profitability exceeds 10 percent: Chemistry & Allied Products and Petroleum Refining (SIC 28 and 29). See TABLE B1 for the list of industry names with their corresponding SIC number.

Let us now compare the accounting and economic profit rates. In general both concepts diverge mainly because the accounting definition neglects the opportunity cost of the investment in capital goods and reduces an investment decision with uncertain returns to a static accounting exercise. From FIGURE B2, it can be seen that both concepts are quantitatively very close for the 1949-75 period, but the accounting profit rate becomes much larger than the economic profit rate after 1976. Similarly, FIGURE B3 shows that on average over the industries, the accounting profit rate is greater than the economic profit rate. There is a simple relationship between s_π^a and s_π explaining these facts. Gross investment is given by $X_{i,nt} = X_{k,nt} - (1-\delta_{nt})X_{k,n,t-1}$. In the steady state, $r_t = \delta_t$ and $w_{i,nt} = w_{i,n,t+1}$ (the user cost becomes $w_{k,nt} = 2\delta_t w_{i,nt}$) and the sole purpose of investment is replacement: $X_{i,nt} = \delta_t X_{k,nt}$. It follows that in the steady state $w_{k,nt}X_{k,nt} = 2w_{i,nt}X_{i,nt}$ and that $s_\pi^a - s_\pi = w_{i,nt}X_{i,nt}/(p_{nt}Y_{nt})$. In the steady state, the difference between the accounting and economic profit is given by the investment share in turn-over. So s_π^a provides an easily calculable upper bound for (steady-state) economic profits, which helps to understand why in most cases $s_\pi^a > s_\pi$ in FIGURES B2 and B3. The fact that for the 1949-1975 period, we observed that $s_\pi^a \simeq s_\pi$ is mainly due to a moderate nominal interest rate combined with high inflation for the investment goods (so that $w_{i,nt} < w_{i,n,t+1}$ in (B.1)).[15]

15. The commodity prices index increased by 3.3% in average over the 1949-1975 period, and by 4.6% over 1976 to 2001. For the price index of investment goods, the figures are respectively 4.3% and 3.5%.

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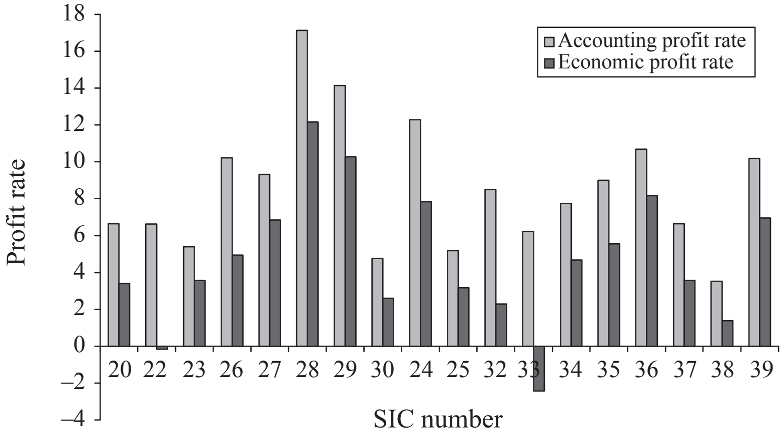

FIGURE B.3. – Profit Rates over Industries (Average over Time)

TABLE B.1. – The Standard Industrial Classification for U.S. Manufacturing Industries

SIC No	Industry name	SIC No	Industry name
20	Food & kindred products	30	Rubber and misc. plastics products
21	Tobacco products industry	31	Leather and leather products
22	Textile mill products industry	32	Stone, clay, glass & concrete products
23	Apparel and other textile products	33	Primary metal industries
24	Lumber and wood products	34	Fabricated metal products
25	Furniture and fixtures	35	Industrial machinery and equipment
26	Paper and allied products	36	Electronic & other electric equipment
27	Printing and publishing	37	Transportation equipment
28	Chemical and allied products industry	38	Instruments, clocks, optical goods
29	Petroleum and coal products	39	Other manufacturing industries

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