Exports and Labor Demand: Searching for Functional Structure in Multi-Output Multi-Skill Technologies

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To simplify the representation of a technological relationship between inputs and outputs, a production unit's technology must typically satisfy some restrictive conditions, some of them being well known in the literature. This article presents new results for aggregating labor inputs and outputs, in terms of restrictions on elasticities of scale and substitution. These conditions are then empirically investigated, in a framework that is flexible and does not lose its flexibility after separability being imposed. The empirical findings of the exact approach to aggregation are found to be rather pessimistic on the possibility of providing a simplified representation.

KEY WORDS: Aggregation; Box–Cox; Exports; Flexibility; Labor demand; Separability; System serial correlation.

1. INTRODUCTION

The theory of input demand is based on aggregate quantities and prices whose existence is often debated. From the work of Leontief (1947) and Gorman (1995), it is clear that the conditions enabling to represent *exactly* a bunch of goods by a composite commodity are stringent. Whether or not these requirements are fulfilled by production technologies and consumer preferences can be investigated empirically once the corresponding theoretical conditions have been given an observable content. Since the work of Berndt and Christensen (1973), Russell (1975), and Blackorby and Russell (1976), it is well known that inputs and prices can be aggregated within a technology—a structure known as *homothetic separability*—if and only if (iff) the elasticities of scale and substitution satisfy some restrictions. As the elasticities can be estimated, the road to empirical investigations is paved, and following Berndt and Christensen (1974), a substantial number of empirical studies have been published.

Two problems related to homothetic separability are addressed in this article. First, aggregates that are implicitly defined by homothetic separability are nonlinear functions of elementary goods or prices and do not correspond to the aggregate variables provided by statistical offices. Available aggregate quantities and prices are usually additive and/or linearly homogeneous in elementary quantities (or prices), as Laspeyres and Paasche indices, for example. They can be consistently used in empirical research only if the production technology is additively and/or homothetically separable. The second problem with homothetic separability goes in the reverse direction. Requiring aggregators to be homothetic implies restrictions on the production technology that are not *necessary* for an aggregate representation of the technology. Tests can therefore reject homothetic aggregation although, in fact, an aggregate quantity exists, as highlighted by Blackorby, Schworm, and Fisher (1986).

This article addresses these problems and provides a formal characterization of eight different structures that allow one to exactly represent a bundle of inputs (or outputs) by a scalar. These characterizations are then used for empirically investigating the possibility to aggregate three different types of labor inputs and two kinds of outputs. A further contribution of the article is that it considers aggregation of both inputs that are optimally allocated and quasi-fixed outputs. It is shown that different types of variable labor inputs can be aggregated iff these demands react "similarly" to a change in the explanatory variables. The quasi-fixed outputs can be aggregated iff they exert a "similar" impact on all input demand functions. These conditions restrict the way the educational structure of labor and the output structure are allowed to be related. If the observed shift in the educational structure of labor is, in fact, related to the shift in the output structure, then results obtained from aggregate models can be suspected of aggregation biases.

The empirical part of the article investigates the validity of different aggregate representations for German manufacturing industries. Aggregation is considered over three flexible labor inputs (workers with a university degree, those with a vocational degree, and those without a formal diploma) and over two quasi-fixed outputs (production sold in Germany and exports). Not only is the share of exports in total production in German manufacturing high (representing 23% of total output in 1978), but also its importance grew over time, reaching 31% in 1994. During this period, the educational structure of labor inputs also drastically changed. Whereas the number of workers without any formal degree declined by about 46%, the number of workers with a vocational degree increased by 12% and the number of those with a university degree almost doubled. This article shows that the question about the validity of aggregates for labor and output is linked to the degree of interrelation between the *structure* of outputs and labor inputs.

Empirical investigations searching for separable structures, however, face a further important problem first highlighted by Blackorby, Primont, and Russell (1977): The imposition of separable structures leads locally flexible functional forms to lose their flexibility property. In that context, it may occur that separability is rejected not because it is de facto invalid, but just because the restricted function is no longer able to approximate

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an arbitrary separable function. This problem was tackled by Diewert and Wales (1995), who presented a specification that does not lose its local flexibility once homothetic separability is imposed. This approach is adopted here to handle separable, homothetic, and additive functional structures.

Starting with a multi-output and multi-labor inputs cost function, we first give necessary and sufficient conditions enabling the outputs and labor inputs to be exactly represented by a scalar aggregate quantity (Sec. 2). Five different types of output aggregation and three types of labor input aggregation are characterized. As within the cost minimization framework, outputs are fixed and labor inputs flexible, the conditions under which outputs and labor inputs can be aggregated are distinct. The necessary and sufficient conditions for eight types of input and output aggregation are all stated nonparametrically, mainly in terms of elasticities of scale and substitution. These results can then be used for empirical investigation (Sec. 4). For that purpose, we rely on a Box–Cox specification for the cost function (Sec. 3), and discuss the outcome of the tests for functional structure in Section 5.

2. THE STRUCTURE OF MULTI–OUTPUT TECHNOLOGIES

After introducing the notations and defining several structures that allow us to aggregate goods within a technological relationship, we discuss their implications for input demand adjustments.

2.1 The Technology

Let $y = (y_d, y_x)^\top$ be a vector of outputs, where y_d and y_x denote production sold on the domestic market and exports. Similarly, the vector $z = (\ell^\top, v^\top)^\top$ groups all variable inputs: labor inputs, denoted by the subvector $\ell = (\ell_h, \ell_s, \ell_u)^\top$, and nonlabor inputs $v = (v_k, v_m)^\top$, composed of capital and intermediate material. The subscript h represents high-skilled labor, s represents skilled labor, u represents unskilled workers, v_k denotes capital, and v_m denotes intermediate materials. The corresponding price vectors are $w = (w_h, w_s, w_u)^\top$ for wages and $q = (q_k, q_m)^\top$ for input prices. A time trend, t, is also included as explanatory variable in the technology for indicating that it may change over time in an a priori unrestricted way.

The technological relationship between inputs and outputs is implicitly represented by a well-behaved transformation function f,

$$f(z, y, t) = 0. \tag{1}$$

Note that outputs y_x and y_d both appear as arguments of the technology. In some cases (as discussed in Hall 1973; and Kohli 1983), the technology (1) exhibits some additional structure that has interesting implications for the relationship between labor demands and exports. The technology f is said to be *nonjoint in inputs* when there exist functions F_d and F_x such that (1) can be equivalently represented by

$$y_d = F_d(\ell^d, v^d, t) \qquad \text{and} \qquad y_x = F_x(\ell^x, v^x, t), \tag{2}$$

with $\ell^d + \ell^x = \ell$ and $v^d + v^x = v$. In this case the domestic and exported outputs are produced independently, using their own inputs, (ℓ^d, v^d) and (ℓ^x, v^x), and their own production functions, F_d and F_x. There are neither economies nor diseconomies of joint production.

The technology is *nonjoint in* (v, y) when the labor input requirement of a given qualification is independent from any other labor qualifications or, more precisely, when there exist functions F_h, F_s, and F_u, such that (1) can be equivalently represented by

$$\ell_h = F_h(v^h, y^h, t), \tag{3}$$

$$\ell_s = F_s(v^s, y^s, t), \tag{4}$$

and

$$\ell_u = F_u(v^u, y^u, t), \tag{5}$$

with $v^h + v^s + v^u = v$ and $y^h + y^s + y^u = y$. This structure arises when, for example, production is executed in different plants. In the first plant (3), only labor type h is required, and combined with inputs v^h, it produces the quantities y^h of the products with technology F_h. The second and third plants, (4) and (5), use labor types s and u.

Structures that are more interesting in this article are those allowing one to represent outputs and labor inputs by a scalar aggregate. Those structures have been thoroughly discussed by Blackorby et al. (1978) and Gorman (1995). A technology is *separable in outputs* when there exist some continuously differentiable functions G_y and g_y such that for all values of the arguments,

$$f(\ell, v, y_d, y_x, t) = G_y\big(\ell, v, g_y(y_d, y_x), t\big). \tag{6}$$

The real-valued aggregator function $g_y : \mathbb{R}^2_+ \to \mathbb{R}_+$, which is increasing in both arguments, aggregates both types of outputs into a single composite output, so that G_y has one argument less than f. When the technology is separable in y, the marginal rate of substitution between exports and domestic outputs is independent of any other input.

Similarly, the technology is *separable in labor inputs* when

$$f(\ell, v, y, t) = G_\ell\big(g_\ell(\ell_h, \ell_s, \ell_u), v, y, t\big), \tag{7}$$

where the labor input aggregator $g_\ell : \mathbb{R}^3_+ \to \mathbb{R}_+$ is increasing and continuously differentiable.

Most aggregate measures provided by statistical offices are homogeneous and additive in the elementary quantities. For that reason, it is useful to specialize the foregoing definitions and allow the function g_y or g_ℓ to be linearly homogeneous, in which case the technology is said to be *homothetically separable* in the respective partition. When g_y is additive, the technology is *additively separable in outputs*, and

$$f(\ell, v, y_d, y_x, t) = G_y\big(\ell, v, g_x(y_x) + g_d(y_d), t\big). \tag{8}$$

Similarly, when g_ℓ is additive in (7),

$$f(\ell, v, y_d, y_x, t) = G_\ell\big(g_h(\ell_h) + g_s(\ell_s) + g_u(\ell_u), v, y, t\big), \tag{9}$$

and f is said to be *additively separable in labor inputs*. In (8), exports and domestic outputs measured in "efficient units" $g_x(y_x)$ and $g_d(y_d)$ are perfectly substitutable. Both the requirements of homogeneity and additivity can be combined to obtain a technology that is *homothetically additively separable* in y (resp. ℓ). When the functions g_x and g_d are the identity, exports and domestic outputs are perfectly substitutable; from the technological standpoint, both goods are identical.

2.2 Implications for Cost and Labor Demand Functions

The minimal variable costs for producing a bundle y are given by the cost function

$$c(w,q,y,t) = \min_{\ell,v}\{w^\top \ell + q^\top v : f(\ell,v,y,t)=0\}$$
$$= w^\top \ell^*(w,q,y,t) + q^\top v^*(w,q,y,t). \quad (10)$$

Because input demand functions are easily obtained from c, the cost function is commonly relied on in empirical analysis. From the definition (10), it is clear that any structure imposed on f will be inherited in some way by the cost function. When production is nonjoint in inputs, for example, we can write

$$\begin{aligned} &c(w,q,y_d,y_x,t)\\ &= \min_{\ell^d,\ell^x,v^d,v^x}\big\{w^\top(\ell^d+\ell^x)+q^\top(v^d+v^x):\\ &\qquad y_d = F_d(\ell^d,v^d,t)\wedge y_x = F_x(\ell^x,v^x,t)\big\}\\ &= \min_{\ell^d,v^d}\{w^\top \ell^d + q^\top v^d : y_d = F_d(\ell^d,v^d,t)\}\\ &\quad + \min_{\ell^x,v^x}\{w^\top \ell^x + q^\top v^x : y_x = F_x(\ell^x,v^x,t)\}\\ &= C_d(w,q,y_d,t) + C_x(w,q,y_x,t). \end{aligned} \quad (11)$$

A simple test for the technology to be nonjoint in inputs involves checking whether the marginal cost of domestic production is independent of exports,

$$\frac{\partial^2 c}{\partial y_d\,\partial y_x} = 0. \quad (12)$$

By Shephard's lemma, the optimal labor demands obtained under input nonjointness are given by

$$\ell_j^*(w,q,y_d,y_x,t) = \ell_j^d(w,q,y_d,t) + \ell_j^x(w,q,y_x,t), \qquad j\in\{h,s,u\}.$$

The demand for each input can be seen as the sum of two components, the input demand required for producing domestic goods and the one used for producing exports.

Similarly, it can be shown that the technology is nonjoint in outputs and nonlabor inputs iff

$$c(w,q,y,t) = C_h(w_h,q,y,t) + C_s(w_s,q,y,t) + C_u(w_u,q,y,t), \quad (13)$$

or equivalently, in terms of labor demands,

$$\ell_j^*(w,q,y,t) = L_j(w_j,q,y,t), \qquad j\in\{h,s,u\}.$$

In that case there is no substitution between the different labor inputs; that is, the cross-price labor elasticities $\varepsilon(\ell_j^*, w_i)$ are identically 0,

$$\varepsilon(\ell_j^*, w_i) \equiv \frac{\partial \ell_j^*}{\partial w_i}\frac{w_i}{\ell_j^*} = 0, \qquad i,j\in\{h,s,u\},\ i\neq j. \quad (14)$$

It is well known that when the technology is separable in outputs, the cost function can be written as

$$c(w,q,y_d,y_x,t) = B_y\big(w,q,g_y(y_d,y_x),t\big), \quad (15)$$

where the aggegator g_y is the same as in (6). This structure is equivalent to

$$\partial\left(\frac{\partial c/\partial y_d}{\partial c/\partial y_x}\right)\Big/\partial w_i = \partial\left(\frac{\partial c/\partial y_d}{\partial c/\partial y_x}\right)\Big/\partial q_j = \partial\left(\frac{\partial c/\partial y_d}{\partial c/\partial y_x}\right)\Big/\partial t = 0, \quad i\in\{h,s,u\}, j\in\{k,m\}, \quad (16)$$

or in terms of elasticities,

$$\begin{aligned} \frac{\varepsilon(z_i^*,y_d)}{\varepsilon(c,y_d)} &= \frac{\varepsilon(z_i^*,y_x)}{\varepsilon(c,y_x)}, \qquad i\in\{h,s,u,k,m\},\\ \frac{y_d\,\partial^2 c/\partial t\,\partial y_d}{\varepsilon(c,y_d)} &= \frac{y_x\,\partial^2 c/\partial t\,\partial y_x}{\varepsilon(c,y_x)}. \end{aligned} \quad (17)$$

Restrictions (17) mean that domestic production and exports have a similar impact on a given input demand, because the (normalized) elasticity of input z_i^* with respect to domestic output and exports are identical. Moreover, the impacts of y_d and y_x on productivity, measured by $\partial c/\partial t$, must be similar.

The technology is homothetically separable in outputs when, in addition to (17), g_y is linearly homogeneous in y. The following result, proven in Appendix A, is useful for identifying that structure from the cost function c.

Proposition 1. The technology f is homothetically separable in outputs iff conditions (17) and

$$\varepsilon(\partial c/\partial y_d, y_d) + \varepsilon(\partial c/\partial y_d, y_x) = \varepsilon(\partial c/\partial y_x, y_d) + \varepsilon(\partial c/\partial y_x, y_x) \quad (18)$$

are satisfied.

This result is used for testing homothetic separability in outputs. Notice that condition (18) does not restrict the level of overall returns to scale.

In the case where the cost function is additively separable in outputs, we have

$$c(w,q,y_d,y_x,t) = B_y\big(w,q,g_d(y_d)+g_x(y_x),t\big), \quad (19)$$

which is satisfied iff in addition to (17), Sono's independence criteria is also fulfilled (see Blackorby et al. 1978, p. 159),

$$\frac{\partial^2\left(\ln\frac{\partial c/\partial y_d}{\partial c/\partial y_x}\right)}{\partial y_x\,\partial y_d} = 0. \quad (20)$$

In (19), the function $g_y = g_d + g_x$ is not necessarily homothetic in y. Because most aggregators available for empirical investigation are both homogeneous and additive, it makes also sense to test whether (17), (18), and (20) are simultaneously satisfied, which corresponds to homothetic additive separability.

Moreover, price and quantity indices commonly published by statistical institutes are weighted sums of elementary prices and quantities. For instance, Laspeyres indices take the form $g_y(y) = \delta_d y_d + \delta_x y_x$, where δ_d and δ_x are constant weights. This situation can be termed *linear separability in outputs*.

Proposition 2. The technology f is linearly separable in outputs iff conditions (17) and

$$\varepsilon(\partial c/\partial y_d, y_o) = \varepsilon(\partial c/\partial y_x, y_o), \qquad o = d,x, \quad (21)$$

are satisfied.

It is immediate to see that (21) implies (18). Even more restrictive, the requirement

$$\frac{\partial c}{\partial y_d} = \frac{\partial c}{\partial y_x} \tag{22}$$

allows us to further simplify the cost function, which then becomes

$$c(w, q, y_d, y_x, t) = B_y(w, q, y_d + y_x, t). \tag{23}$$

In this case, domestic and exported outputs are strictly identical goods from a technological standpoint. Marginal variations of y_d and y_x have the same impact on a given input demand. For later reference, it is useful to note that although (22) consists of only one equality restriction, it is more stringent than the eight equality restrictions (17) and (21).

For the labor inputs that, in contrast to outputs, are optimally adjusted, it is generally assumed that the aggregator g_ℓ is linearly homogeneous. Under this condition, separability of f in labor inputs is equivalent to separability of the cost function in wages. This point was discussed in detail by Diewert and Wales (1995), for example. Indeed, it is then possible to write

$$\begin{aligned} c(w, q, y, t) &= \min_{\ell, v}\{w^\top \ell + q^\top v : G_\ell(g_\ell(\ell), v, y, t) = 0\} \\ &= \min_{\ell}\{w^\top \ell + A_\ell(g_\ell(\ell), q, y, t)\} \\ &= \min_{\ell, L}\{w^\top \ell + A_\ell(g_\ell(\ell), q, y, t) : g_\ell(\ell) = L\}, \end{aligned} \tag{24}$$

where

$$A_\ell(g_\ell(\ell), q, y, t) = \min_{v}\{q^\top v : G_\ell(g_\ell(\ell), v, y, t) = 0\} \tag{25}$$

denotes a restricted cost function that is *separable* in labor inputs. When the labor input aggregator g_ℓ is linearly homogeneous, it is possible to further simplify c,

$$\begin{aligned} c(w, q, y, t) &= \min_{\mathbf{L}}\{g_w(w)\mathbf{L} + A_\ell(\mathbf{L}, q, y, t)\} \\ &= B_\ell(\mathbf{W}, q, y, t), \end{aligned} \tag{26}$$

where

$$\mathbf{W} = g_w(w) \equiv \min_{\ell}\{w^\top \ell : g_\ell(\ell) = 1\}$$

can be interpreted as an aggregate wage corresponding to one unit of aggregate labor $\mathbf{L}$. The unrestricted cost function B_ℓ depends only on the aggregate wage level $\mathbf{W}$. Briefly, when the technology is homothetically separable in labor inputs, the cost function c is homothetically separable in labor input prices (and conversely). Testing for homothetic separability can be achieved on the basis of the following restrictions:

$$\partial\left(\frac{\partial c/\partial w_i}{\partial c/\partial w_j}\right)\Big/\partial r = 0 \tag{27}$$

for $i, j \in \{h, s, u\}$ and $r \in \{q_k, q_m, y_d, y_x, t\}$. In terms of elasticities, homothetic separability is equivalent to

$$\varepsilon(\ell_i^*, r) \equiv \frac{\partial \ell_i^*}{\partial r}\frac{r}{\ell_i^*} = \frac{\partial \ell_j^*}{\partial r}\frac{r}{\ell_j^*} \equiv \varepsilon(\ell_j^*, r). \tag{28}$$

The elasticities of any distinct labor inputs with respect to a given explanatory variable are identical. Labor demands for different skills must react identically with respect to changes of any of the explanatory variables q_k, q_m, y_d, y_x, or t. Ten independent equalities are involved in (28).

In most empirical contributions, homotheticity of g_ℓ is taken for granted. To avoid this additional restriction on the function g_ℓ, it is, of course, possible to rely on the restricted cost function

$$a(\ell, q, y, t) = \min_{v}\{q^\top v : f(\ell, v, y, t) = 0\}, \tag{29}$$

which is equal to A_ℓ of (25) iff

$$\partial\left(\frac{\partial a/\partial \ell_i}{\partial a/\partial \ell_j}\right)\Big/\partial r = 0 \tag{30}$$

for $i, j \in \{h, s, u\}$ and $r \in \{q_k, q_m, y_d, y_x, t\}$. The following result, proven in Appendix A, gives the corresponding restrictions on the unrestricted labor demand elasticities.

Proposition 3. Under the assumption of cost minimization, the following statements are equivalent:

a. The technology f is separable in labor inputs.
b. The matrix

$$D_\ell \equiv \begin{bmatrix} \dfrac{\partial \ell^*}{\partial q^\top} & \dfrac{\partial \ell^*}{\partial y^\top} & \dfrac{\partial \ell^*}{\partial t} \end{bmatrix}$$

has at most rank 1, or, equivalently, the matrix

$$E_\ell \equiv \begin{bmatrix} \varepsilon(\ell_h^*, q_k) & \varepsilon(\ell_h^*, q_m) & \varepsilon(\ell_h^*, y_d) & \varepsilon(\ell_h^*, y_x) & \varepsilon(\ell_h^*, t) \\ \varepsilon(\ell_s^*, q_k) & \varepsilon(\ell_s^*, q_m) & \varepsilon(\ell_s^*, y_d) & \varepsilon(\ell_s^*, y_x) & \varepsilon(\ell_s^*, t) \\ \varepsilon(\ell_u^*, q_k) & \varepsilon(\ell_u^*, q_m) & \varepsilon(\ell_u^*, y_d) & \varepsilon(\ell_u^*, y_x) & \varepsilon(\ell_u^*, t) \end{bmatrix}$$

has at most rank 1.

c. For any $i, j \in \{h, s, u\}$ and $r \in \{q_k, q_m, y_d, y_x, t\}$,

$$\varepsilon(\ell_i^*, r)\sum_{n=1}^{5}\varepsilon(\ell_j^*, r_n) = \varepsilon(\ell_j^*, r)\sum_{n=1}^{5}\varepsilon(\ell_i^*, r_n). \tag{31}$$

The conditions (31) of Proposition 3 also restrict the way in which prices, output growth, and the time trend influence the demands for different qualifications of workers. Like (28), this results imposes some similarity on how different labor demands react to changes in the same explanatory variable. Proposition 3, however, allows one to test for separability in labor inputs that are optimally allocated, without assuming homogeneity of the aggregator. It can be seen that this result is implied by homothetic separability in ℓ: When the cost function is $B_\ell(g_w(w), q, y, t)$, the Hessian matrix with respect to w and (q, y, t) indeed has rank 1. Point c gives the necessary and sufficient conditions for separability in ℓ in terms of the elasticities of scale and substitution. Because of the occurrence of sums over n in the left and right sides of (31), fewer (independent) restrictions are involved in (31) than in (28). A careful count indicates that 7 out of 15 equalities in (31) are dependent, leaving 8 independent restrictions in condition c. The requirements (31) are not only less numerous, but also weaker than those in (28) prevailing under homothetic separability. Indeed, homogeneity of g_ℓ also implies that $\sum_n \varepsilon(\ell_j^*, r_n) = \sum_n \varepsilon(\ell_i^*, r_n)$, making (31) coincide with (28) (see Koebel 2001 for details).

Concerning additive separability in labor inputs, the following result is useful for testing that structure.

Proposition 4. The technology f is additively separable in labor inputs iff conditions (31) and

$$\begin{aligned}&\varepsilon(\ell_s^*; w_u)[\varepsilon(\ell_h^*; w_s) + \varepsilon(\ell_h^*; w_h)]\\&\quad = \varepsilon(\ell_h^*; w_u)[\varepsilon(\ell_s^*; w_h) + \varepsilon(\ell_s^*; w_s)],\\&\varepsilon(\ell_u^*; w_h)[\varepsilon(\ell_s^*; w_u) + \varepsilon(\ell_s^*; w_s)]\\&\quad = \varepsilon(\ell_s^*; w_h)[\varepsilon(\ell_u^*; w_s) + \varepsilon(\ell_u^*; w_u)]\end{aligned} \qquad (32)$$

are satisfied.

The two requirements (32) of Proposition 4 restrict the admissible values of own and cross-price labor demand elasticities. Note that when labor inputs cannot be substituted for one other, that is, when all elasticities are equal to 0, (32) is satisfied. Therefore, a technology that is additively separable in labor input is more general than a technology that is nonjoint in (y, v).

Because labor inputs are optimally allocated, it does not make much sense to test whether the different qualifications of labor are identical from a technological standpoint. Indeed, such a technology would lead the production unit to use a single labor input, the cheapest one. This is not seen at any observation of our sample.

Briefly, from this discussion on functional structure, it can be seen that separability conditions imposes two different kinds of restrictions on labor demands. Whereas separability imposes restrictions between the goods belonging to the separable group and those outside the group, homothetic and additive separability restrict the relationships both within aggregated goods and between aggregated and nonaggregated goods.

3. THE EMPIRICAL MODEL

A version of Berndt and Khaled's (1979) Box–Cox cost function, extended to nest both the normalized quadratic and the translog functional forms by Koebel, Falk, and Laisney (2003), is considered here. Let $p = (w^\top, q^\top)^\top$ be the price vector. The multiple-output cost function is given by

$$c(p, y, t; \beta, \gamma; \mu, \theta) = \begin{cases} q^\top\mu\big(\gamma_2 C(P, Y, t; \beta) + 1\big)^{1/\gamma_2} & \text{for } \gamma_2 \neq 0 \\ q^\top\mu \exp(C(P, Y, t; \beta)) & \text{for } \gamma_2 = 0, \end{cases} \qquad (33)$$

where, in a first stage,

$$\begin{aligned}C(P, Y, t; \beta) = {}&\beta_C + B_P P + B_Y Y + \beta_{Ct} t\\&+ \frac{1}{2} P^\top B_{PP} P + \frac{1}{2} Y^\top B_{YY} Y + \frac{1}{2}\beta_{tt} t^2\\&+ P^\top B_{PY} Y + P^\top B_{Pt} t + Y^\top B_{Yt} t.\end{aligned} \qquad (34)$$

(This definition of C is amended later.) Some restrictions are placed on the parameters for the Hessian of the cost function to be symmetric in p and y and for parsimony,

$$\begin{aligned}&\iota_5^\top B_P = 1, \qquad B_{PP} = B_{PP}^\top, \qquad B_{YY} = B_{YY}^\top,\\&\iota_5^\top B_{PP} = 0, \qquad \iota_5^\top B_{PY} = \iota_5^\top B_{Pt} = 0,\end{aligned} \qquad (35)$$

where ι_5 denotes a (5×1)-vector of 1's.

The components P_i and Y_j of the vectors P and Y are Box–Cox-like transformations of the corresponding variables p_i and y_j,

$$P_i = b_i(p) = \begin{cases} \dfrac{(p_i/(q^\top\theta))^{\gamma_1}}{\gamma_1} & \text{for } \gamma_1 \neq 0 \\ \ln\big(p_i/(q^\top\theta)\big) & \text{for } \gamma_1 = 0, \end{cases} \quad i = k, h, s, u, m, \qquad (36)$$

and

$$Y_j = b_j(y_j) = \begin{cases} \dfrac{y_j^{\gamma_1}}{\gamma_1} & \text{for } \gamma_1 \neq 0 \\ \ln y_j & \text{for } \gamma_1 = 0, \end{cases} \qquad j = d, x. \qquad (37)$$

The parameters $\alpha = (\beta^\top, \gamma^\top)^\top$ must be estimated. The (5×1)-vectors μ and θ compose fixed weights, defined later, so that $q^\top\mu$ and $q^\top\theta$ are kinds of Laspeyres indices for nonlabor costs. Note that $q^\top\theta$ is used to ensure that the functions b_i and C are homogeneous of degree 0 in prices, and that the multiplicative term $q^\top\mu$ appearing in (33) then guarantees linear homogeneity in prices of c.

The two parameters γ_1 and γ_2 capture the way in which variables y_j, p_j, and c are changed by the Box–Cox transformations b_i. For the special case in which $\gamma_1 = 1$ and $\gamma_2 = 1$, the normalized quadratic functional form is obtained, whereas for $\gamma_1 \to 0$ and $\gamma_2 \to 0$, (33) coincides with the translog. Other usual functional forms nested within (33) have been discussed by Koebel et al. (2003).

4. ON THE SEPARABILITY–INFLEXIBILITY DEBATE

Now that a parametric specification has been chosen, it is interesting to study which restrictions on the parameters of the cost function correspond to the aforementioned restrictions on the elasticities of scale and substitution. A cost function is said to be (locally) flexible when it is able to approximate the level as well as all first- and second-order derivatives of any twice-continuously differentiable cost function at a given point. This concept, first defined by Diewert (1974), initiated numerous contributions in the fields of production and consumer analysis. It is not difficult to see that the Box–Cox specification of this article, yielding some well-known locally flexible specifications as special cases, is itself flexible. Applying theorem 6 of Blackorby and Diewert (1979) allows us to conclude that the (unknown) technology dual to the Box–Cox cost function is flexible as well.

The main problem with the functional form (33)–(34) was noted by Blackorby et al. (1977, 1978): Once separability is imposed globally, locally flexible functional forms often lose their flexibility property. In that context, it may be that separability is rejected, not because it is de facto invalid, but just because the restricted function is no longer able to approximate an arbitrary separable function.

Let us consider the case of weak separability in outputs to illustrate this drawback. The cost function c is weakly separable in y iff (16) holds, a condition that can be written in the Box–Cox case as

$$\begin{aligned}&\beta_{jx}(\beta_d + \beta_{dx} Y_x + \beta_{dd} Y_d + P^\top B_{Pd} + \beta_{td} t)\\&\quad = \beta_{jd}(\beta_x + \beta_{xx} Y_x + \beta_{xd} Y_d + P^\top B_{Px} + \beta_{tx} t)\end{aligned} \qquad (38)$$

for $j = h, s, u, k, m, t$. These equalities are satisfied iff

$$\begin{aligned} \beta_d \beta_{jx} - \beta_x \beta_{jd} &= 0, \\ \beta_{jd} \beta_{ix} - \beta_{jx} \beta_{id} &= 0, \end{aligned} \tag{39}$$

for $j \in \{h, s, u, k, m, t\}$ and $i \in \{h, s, u, k, m, t, d, x\}$. For instance, when, for any $j \in \{h, s, u, k, m, t\}$, the equalities

$$\beta_{jx} = \beta_{jd} = 0, \tag{40}$$

which imply (39), are fulfilled, C becomes

$$\widetilde{C}(P, \mathbf{Y}, t; \beta) = \beta_C + B_P P + \boldsymbol{\beta}_{\mathbf{Y}} \mathbf{Y} + \beta_t t + \frac{1}{2} P^\top B_{PP} P + \beta_{tt} t^2 + P^\top B_{Pt} t, \tag{41}$$

with

$$\boldsymbol{\beta}_{\mathbf{Y}} \mathbf{Y} = B_Y Y + \frac{1}{2} Y^\top B_{YY} Y. \tag{42}$$

Here boldface characters denote aggregate variables and aggregate parameters. The specification (41) is no longer flexible in $(P, \mathbf{Y}, t)$; the first derivative of $\widetilde{C}$ with respect to P for instance is independent of $\mathbf{Y}$. As a consequence, the imposition of global separability conditions on local flexible specifications destroys flexibility (see Blackorby et al. 1978, sec. 8.3, for a thorough discussion).

To avoid the inflexibility drawback, Diewert and Wales (1995) proposed a functional specification that comprises more parameters than (33) and is still locally flexible after imposition of global separability [i.e., separability over the whole set of possible values of (p, y, t)]. In the context of output aggregation, the strategy of Diewert and Wales would lead to specify C as

$$\begin{aligned} C_{\mathbf{Y}}(P, Y, \mathbf{Y}, t; \beta) &= \beta_C + B_P P + \beta_t t \\ &\quad + \frac{1}{2} P^\top B_{PP} P + \beta_{tt} t^2 + P^\top B_{Pt} t \\ &\quad + \boldsymbol{\beta}_{\mathbf{Y}} \mathbf{Y} + \frac{1}{2} \boldsymbol{\beta}_{\mathbf{YY}} \mathbf{Y}^2 + P^\top \mathbf{B}_{P\mathbf{Y}} \mathbf{Y} + \boldsymbol{\beta}_{\mathbf{Y}t} \mathbf{Y} t \\ &\quad + P^\top B_{PY} Y + Y^\top B_{Yt} t, \end{aligned} \tag{43}$$

where $\mathbf{Y}$ is as defined as in (42). The first three lines of $C_{\mathbf{Y}}$ correspond to a function of $(P, \mathbf{Y}, t)$ that is both flexible in $(P, \mathbf{Y}, t)$ and separable in y, whereas the last line is added to allow $C_{\mathbf{Y}}$ to be nonseparable in y. When this last line vanishes, the cost function is clearly separable in y (but not conversely, as we show later).

Because aggregation of output and labor inputs is of interest here, we consider the following specification in the sequel instead of (43), with $P = (W^\top, Q^\top)^\top$:

$$\begin{aligned} &C_{\mathbf{WY}}(P, Y, \mathbf{W}, \mathbf{Y}, t; \beta) \\ &\quad = \beta_C + B_Q Q + \beta_t t + \frac{1}{2} Q^\top B_{QQ} Q + Q^\top B_{Qt} t + \frac{1}{2} \beta_{tt} t^2 \\ &\qquad + \boldsymbol{\beta}_{\mathbf{Y}} \mathbf{Y} + \frac{1}{2} \boldsymbol{\beta}_{\mathbf{YY}} \mathbf{Y}^2 + Q^\top \mathbf{B}_{Q\mathbf{Y}} \mathbf{Y} + \boldsymbol{\beta}_{\mathbf{Y}t} \mathbf{Y} t \\ &\qquad + \boldsymbol{\beta}_{\mathbf{W}} \mathbf{W} + \frac{1}{2} \boldsymbol{\beta}_{\mathbf{WW}} \mathbf{W}^2 + \mathbf{W} \mathbf{B}_{\mathbf{W}Q} Q + \boldsymbol{\beta}_{\mathbf{W}t} \mathbf{W} t + \boldsymbol{\beta}_{\mathbf{WY}} \mathbf{WY} \\ &\qquad + \mathbf{W} \mathbf{B}_{\mathbf{W}Y} Y + Q^\top B_{QY} Y + Y^\top B_{Yt} t \\ &\qquad + W^\top \mathbf{B}_{W\mathbf{Y}} \mathbf{Y} + W^\top B_{WY} Y + W^\top B_{WQ} Q + W^\top B_{Wt} t. \end{aligned} \tag{44}$$

The aggregate output, $\mathbf{Y} \in \mathbb{R}$, is as defined in (42), and the aggregate wage $\mathbf{W} \in \mathbb{R}$ is given by

$$\boldsymbol{\beta}_{\mathbf{W}} \mathbf{W} = B_W W + \frac{1}{2} W^\top B_{WW} W. \tag{45}$$

For the purpose of identification, the parameters $\boldsymbol{\beta}_{\mathbf{Y}}$ and $\boldsymbol{\beta}_{\mathbf{W}}$ can be normalized,

$$\boldsymbol{\beta}_{\mathbf{Y}} = \boldsymbol{\beta}_{\mathbf{W}} = 1. \tag{46}$$

Some other parameter restrictions are imposed for the sake of parsimony: (35) and $\iota_5^\top \mathbf{B}_{P\mathbf{Y}} = 0$. In the first three lines of (44), the variables W and Y are aggregated into $\mathbf{W}$ and $\mathbf{Y}$. Because this expression is quadratic in $(Q, \mathbf{W}, \mathbf{Y}, t)$, it follows that $C_{\mathbf{WY}}$ is locally flexible in these variables. The two last lines of (44) introduce the additional disaggregate variables Y and W and thereby allow the cost function to be nonseparable in y and w.

Specification (44) is interesting, because it nests specifications (34), (41), and (43). Note that when $B_{\mathbf{W}Y} = 0$, $\mathbf{B}_{Q\mathbf{Y}} = 0$, and $\boldsymbol{\beta}_{\mathbf{Y}t} = \boldsymbol{\beta}_{\mathbf{WY}} = 0$, the cost function is separable in y under the usual conditions (39). Furthermore, the condition

$$\mathbf{B}_{\mathbf{W}Y} = 0, \qquad B_{QY} = 0, \qquad B_{Yt} = 0, \qquad \text{and} \qquad B_{WY} = 0 \tag{47}$$

implies separability in y, but is not equivalent to it [a weaker set of sufficient conditions for output separability comprises $B_{\mathbf{W}Y} = 0$ and (40)]. For this reason, output separability tests relying on (47), like those formulated by Diewert and Wales (1995), are prone to reject the null hypothesis too often. Similarly,

$$\mathbf{B}_{W\mathbf{Y}} = B_{Wt} = 0 \qquad \text{and} \qquad B_{WY} = B_{WQ} = 0 \tag{48}$$

implies global separability in w, but not conversely.

In theory, it is possible to give the necessary and sufficient *parametric* conditions for a technology to be *globally* separable in output and homothetically separable in labor inputs. These restrictions would be more stringent than the restrictions (17) and (28), which are of local nature. However, the derivation of these parametric conditions is messy and devoid of economic meaning. This observation gives some value added to the characterization in terms of elasticities provided in Section 2, the validity of which does not depend on parametric specification of the cost function.

The interesting result relative to the specification (33), (44) in comparison to (33)–(34), follows from Diewert and Wales (1995).

Proposition 5. The cost function c defined by (33), (35), (42), (44), and (45) is

(a) flexible in (w, q, y, t),
(b) flexible in $(w, q, \mathbf{Y}, t)$ when c is globally separable in y,
(c) flexible in $(\mathbf{W}, q, y, t)$ when c is globally separable in w, and
(d) flexible in $(\mathbf{W}, q, \mathbf{Y}, t)$ when c is globally separable in w and in y.

Proposition 5 means that the cost function is flexible in (w, q, y, t) and that this flexibility property is still satisfied when the technology is globally separable in outputs [points (b) and (d)] and/or homothetically separable in labor inputs [points (c) and (d)]; see Appendix A for a proof. Proposition 5 justifies using (44) for testing separability in outputs and/or homothetic separability in labor inputs globally, without losing flexibility. This result is used in the empirical part of the article to see whether the loss of flexibility of (33)–(34) once separability is imposed can explain why separability is often rejected.

5. DATA AND REGRESSION

The data used for the empirical investigation are two-digit industry data for West German manufacturing. Data are available for $N = 24$ industries covering the period 1976–1994 ($T = 19$). The subscripts $n = 1, \dots, N$ and $t = 1, \dots, T$ denote industry and time. Over this period, total manufacturing exports in constant prices grew at an average annual rate of 3.3%, whereas domestic production grew only by 1.1%. Turning to the labor inputs, the employment of university graduates, ℓ_h (measured as full-time equivalents), increased at an average annual rate of 3.8%. In contrast, the number of unskilled workers, ℓ_u, decreased by 3.3% per year. The number of skilled workers grew by 0.6% average. Relative wages do not appear to have changed much during the period. In the aggregate (over all industries and years considered in this article), exports represented roughly 27% of the total level of production. Moreover the share of exports in production increased over time, from 23% in 1976 to 31% in 1994.

To account for heterogeneous technologies, we allow some parameters, β_C and B_P, to be industry-specific. In earlier work on similar data, Koebel et al. (2003) found that the data-transformation parameters γ also varied across groups of industries; thus here we allow these parameters to vary across industries (hence the notation γ_{1n} and γ_{2n}). This means that the functional form of the cost function is allowed to differ across the industries; it may, for example, be translog for some industries (those with $\gamma_{1n} \to 0$ and $\gamma_{2n} \to 0$), whereas for others, a normalized quadratic specification ($\gamma_{1n} = \gamma_{2n} = 1$) is more adequate.

The optimal demand functions, z^*, are related to the cost function by Shephard's lemma, which is applied here to form the input-output coefficients considered in the regression

$$\frac{z_{nt}}{\mathbf{y}_{nt}^{+}} = \frac{z^*(p_{nt}, y_{nt}, t; \beta_n, \gamma_n; \mu_n, \theta_n)}{\mathbf{y}_{nt}^{+}} + \upsilon_{nt}. \tag{49}$$

To make the assumption of homoscedastic residuals more plausible, we consider the input-output ratio $z_{nt}/\mathbf{y}_{nt}^{+}$ (with total output given by $\mathbf{y}_{nt}^{+} \equiv y_{d,nt} + y_{x,nt}$), instead of the absolute demand levels z_{nt} as regressands.

In the specification (33), the parameters θ and μ could, in principle, be estimated. It can be seen by adapting the argument of Diewert and Wales (1987) that the flexibility of the foregoing specification does not depend on a particular choice for θ and μ. For that reason, they have often been fixed, to limit the overall number of parameters. These weights are usually defined as a function of inputs and costs, a practice that introduces some correlation between θ and the residual term υ_{nt}. To avoid this difficulty, we specify

$$\theta_n \equiv \frac{\mu_n}{\frac{1}{N-1}\sum_{i \neq n} q_{i,91}^{\top} v_{i,91}}$$

and

$$\mu_n \equiv \frac{1}{N-1}\sum_{i \neq n} v_{i,91}.$$

Because all prices are normalized to 1 in 1991, it follows that $q_{n,91}^{\top}\theta_n = 1$.

The residual vector υ_{nt} ($J \times 1$) is assumed to be independent of the regressors. It also satisfies $\mathrm{E}[\upsilon_{nt}] = 0$, $\mathrm{E}[\upsilon_{nt}\upsilon_{nt}^{\top}] = \Psi$ and also may be subject to first-order serial correlation,

$$\upsilon_{nt} = R\upsilon_{n,t-1} + \zeta_{nt}, \qquad \lim_{j \to +\infty} R^j = 0, \tag{50}$$

with $\mathrm{E}[\zeta_{nt}] = 0$, $\mathrm{E}[\upsilon_{n,t-1}\zeta_{nt}^{\top}] = 0$, $\mathrm{E}[\zeta_{nt}\zeta_{nt}^{\top}] = \Psi - R\Psi R^{\top}$, and $\mathrm{E}[\zeta_{nt}\zeta_{is}^{\top}] = 0$ for all $n \neq i$ or $t \neq s$.

The variance matrix of υ ($JTN \times 1$) is $V[\upsilon\upsilon^{\top}] = I_N \otimes \Phi$, where Φ ($JT \times JT$) is given by

$$\Phi = \begin{pmatrix} \Psi & \Psi R^{\top} & \cdots & \Psi R^{\top T-1} \\ R\Psi & \Psi & \cdots & \Psi R^{\top T-2} \\ \vdots & \vdots & & \vdots \\ R^{T-1}\Psi & R^{T-2}\Psi & \cdots & \Psi \end{pmatrix}. \tag{51}$$

When $R = 0$, Φ simplifies to the conventional SUR variance matrix $I_{NT} \otimes \Psi$. Preliminary estimates of the conventional SUR residuals were used to build a consistent estimate $\widehat{\Phi}$ of Φ by equaling the elements of Φ to their sample analog. In all cases, the matrix $\widehat{\Phi}$ was found to be positive definite. As was shown by Koebel (2004), this ensures that $\widehat{R}$ is a convergent matrix (in the sense that $\lim_{j \to +\infty} \widehat{R}^j = 0$).

6. EMPIRICAL RESULTS

6.1 First Model

The first model considered consists of (49), which is obtained from (33)–(37). The complete model comprises 151 free α_n parameters (among which $5 \times 24 = 120$ industry-specific parameters) and $2 \times 24 = 48$ γ_n-parameters that must be estimated on the basis of $5 \times 19 \times 24 = 2{,}280$ observations. Because there are more than 10 observations per parameter, the incidental parameter problem should not be substantial here. This specification of the demand functions includes not only the usual additive fixed effects, but also industry-specific Box–Cox parameters, so that the marginal impacts of output on input demands are in no way restricted to be identical from one industry to the other. This extension is compatible with heterogeneous outputs across industries, in contrast to cost functions linear in outputs.

The estimates of the variable transforming parameters, γ_n, are plotted on Figure 1. In most cases the estimates are included between 0 and 1; only two values for $\widehat{\gamma}_{2n}$ lie outside the 0–1 interval. All estimates are significantly different from 0. It can be seen that the estimates of $\widehat{\gamma}_{2n}$ are smaller (and less dispersed) than those of $\widehat{\gamma}_{1n}$. The assumption that $\gamma_{1n} = \gamma_{2n}$, leading to a Berndt and Khaled (1979) type of specification, is rejected by the data. Although there is a cloud of points around the mean

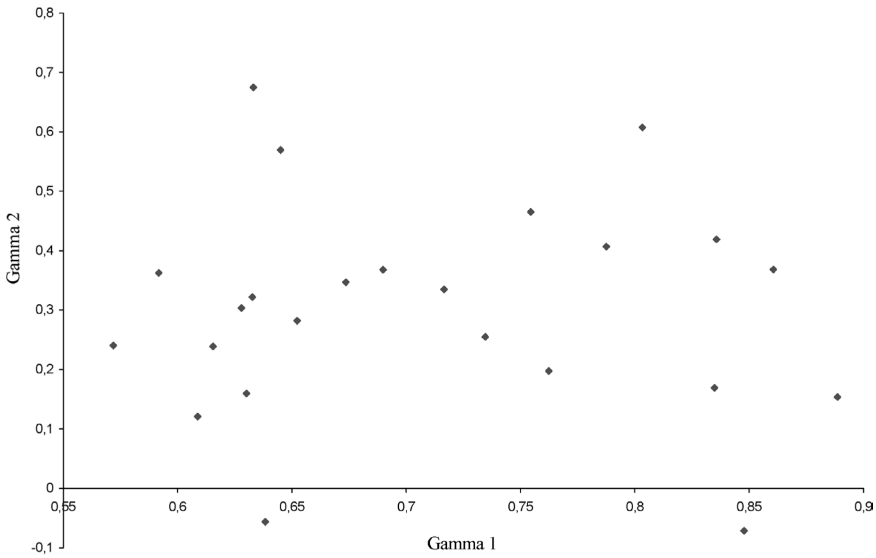

Figure 1. Scatterplot of $\widehat{\gamma}_{1n}$ and $\widehat{\gamma}_{2n}$.

point, $\overline{\gamma} = (.71, .30)$, a test for the hypothesis that the functional family is the same across industries is rejected at the 1% threshold. Several usual functional specifications that are nested within the Box–Cox are all rejected (see Koebel et al. 2003).

The data were found to exhibit strong serial correlation. Only one estimate of the five eigenvalues of matrix R was real-valued. In all cases, however, the modulus of the estimated eigenvalues are <1 (between .45 and .80). Thus a necessary condition for the matrix $\widehat{\Phi}$ to be positive definite is satisfied, and it turns out that, in fact, $\widehat{\Phi}$ is definite positive.

The estimated values of the elasticities of substitution and scale are reported in Table 1. The own-price elasticities appear on the main diagonal of the upper part, and are all negative. The off-diagonal entries correspond to the cross-price elasticities and indicate that high-skilled and skilled labor are substitutes, as are skilled and unskilled labor. High-skilled and unskilled labor are complements, however. It can be seen from Table 1 that the prices q_k and q_m have a quite different impact on the different qualifications of labor. It turns out that the more skilled the labor, the more complementary it is with capital, $\epsilon(\ell_h^*, q_k) \le \epsilon(\ell_s^*, q_k) \le \epsilon(\ell_u^*, q_k) \le 0$.

The scale elasticities are reported in the lower part of Table 1. Although all inputs are nondecreasing in y_d and in y_x, the impacts of domestic and exported production on inputs are quite different. With regard to labor inputs, it can be seen that exports are intensive in education, in the sense that $\epsilon(\ell_h^*, y_x) \ge \epsilon(\ell_s^*, y_x) \ge \epsilon(\ell_u^*, y_x)$. The increase in exports in Germany thereby provides a partial explanation for the observed shift in the structure of labor demand in favor of more educated workers. The last line of Table 1 gives the impact of the time trend on input demands. The impact of t on heterogeneous labor inputs is also quite different. Note that these strong dissimilarities in input demand reactions to changes in q_k, q_m, y_d, y_x, and t are inconsistent with the conditions favorable to labor aggregation, which require that the impact of the different explanatory variables be "similar" across educational levels. Thus these empirical results foreshadow the difficulties of aggregating labor inputs.

Table 1. Elasticities of the Disaggregate Model

$\epsilon(j^*, i)$	ℓ_h	ℓ_s	ℓ_u	v_k	v_m
w_h	**−.07(−.8)**	.03(3.6)	−.10(−5.3)	−.03(−4.6)	.01(6.2)
w_s	.41(2.4)	**−.34(−10.5)**	.20(6.0)	−.08(−3.0)	.05(6.3)
w_u	−.88(−4.8)	.13(4.3)	**−.42(−6.2)**	−.02(−.8)	.05(4.5)
q_k	−.26(−4.9)	−.05(−3.6)	−.02(−1.9)	**−.18(−5.2)**	.04(6.7)
q_m	.72(7.6)	.24(6.3)	.36(6.3)	.30(9.2)	**−.16(−5.3)**
y_d	.80(9.5)	.54(12.7)	.60(9.7)	.58(6.1)	.70(23.8)
y_x	.41(9.0)	.06(4.3)	−.01(−.3)	.14(5.4)	.18(13.6)
t	.035(8.8)	.003(3.5)	−.031(−6.6)	.001(.4)	.003(4.5)

NOTE: Median value of the elasticities evaluated for the 1985 data, with estimated t values in parentheses.

There is also evidence that domestic and exported production have different impacts on a given input; in all cases, $\epsilon(x_j^*, y_d) \ge \epsilon(x_j^*, y_x)$ in Table 1. This observation does not necessarily contradict output aggregation; when $c(w, q, y_d, y_x, t) = B_y(w, q, y_d + y_x, t)$, for instance, it follows that $\epsilon(x_j^*, y_d) \ge \epsilon(x_j^*, y_x)$ as soon as $y_d \ge y_x$, although outputs are perfectly identical. To study the possibilities of aggregating outputs, the more formal hypotheses, given in Section 2, must be tested.

Results of the tests for functional structure are summarized in Table 2. The different hypotheses tested are listed in the first column. Column 2 recapitulates the formal expressions of the corresponding equality restrictions presented in Section 2. Column 3 gives the number of independent restrictions tested at a given observation point (n, t). Because the test statistics depend on the observations (w, q, y_d, y_x, t), they take different values over the sample. To avoid dependence on a arbitrarily chosen reference point, we compute the hypothesis test for each observation point in turn, and report both the mean value of the statistic and the percentage of significant violations in columns 4 and 5 of Table 2.

The main result is that all types of restrictions are strongly rejected. Separability in outputs is rejected for 84.9% and separability in labor input is rejected for 95.4% of the observations. The rejection rate increases for the more restrictive versions of homothetic and additive separability. For labor inputs, homothetic separability is rejected for all 456 observation points.

Table 2. Tests for Functional Structure, Model 1

	Tested restrictions	Independent restrictions	Mean Wald test[a]	% observations with H_0 rejected[b]
Outputs				
Nonjointness	(12)	1	$31.8_{(20.4)}$	86.0
Separability	(17)	6	$41.2_{(17.1)}$	84.9
Homothetic separability	(17), (18)	7	$43.1_{(15.1)}$	90.1
Additive separability	(17), (20)	7	$43.4_{(13.6)}$	93.0
Linear separability	(17), (21)	8	$112.6_{(69.7)}$	90.5
Identical outputs	(22)	1	$14.9_{(14.7)}$	62.5
Labor inputs				
Nonjointness	(14)	3	$59.5_{(23.3)}$	100
Separability	(31)	8	$51.0_{(22.6)}$	95.4
Homothetic separability	(28)	10	$374.5_{(180.0)}$	100
Additive separability	(31), (32)	10	$54.8_{(20.4)}$	96.9

[a] Mean value of the Wald test, with calculated standard deviation in parentheses.
[b] There are 456 observations at which the Wald test for a given null hypothesis is calculated.

This is a logical consequence of the fact that the impacts of prices, outputs, and time differ greatly across the three types of labor inputs (Table 1). Comparable separability structures are more strongly rejected when involving labor inputs, as is the case with respect to output.

Notice that the outcomes of the tests are sometimes inconsistent with the theory; whereas the null of identical outputs is rejected for 62.5% of the observations, output separability is invalidated in 84.9% of the cases. This finding is paradoxical, because identical output implies output separability, but not conversely. This contradiction might be due to the fact that the weaker assumption of separability involves six equality restrictions (17), whereas the stronger requirement of output identity involves only one restriction (22). This paradox is related to the lack of invariance of the Wald test to nonlinear transformations. Similarly, the assumption of additive separability is rejected for more observation points than the more stringent linear separability hypothesis.

The validity of the null assumptions was also tested globally. This is not a straightforward task, because the test statistics, which depend on observations and estimated parameters, take different values over the sample. The details of the procedure used to test the null globally are given in Appendix B. The different types of restrictions are globally rejected at any reasonable threshold.

For the cases of separability in outputs and homothetic separability in labor inputs, some parametric restrictions *implying* global separability are known [see (39)]. The Wald statistics corresponding to these parametric hypotheses are found to be smaller than those corresponding to the former global test. This contradiction also suggests that two theoretically equivalent sets of hypotheses can be rejected at different thresholds, in terms of the number and the form of the restrictions used to characterize them. This question has been addressed by several researchers (see, e.g., Phillips and Park 1988); however, the issue of how to best formulate a Wald test for a given set of nonlinear restrictions in empirical research remains mostly unsolved.

6.2 Second Model

In the second model, the parameter vector $(\alpha_n^\top, \alpha^\top, \gamma_{1n}, \gamma_{2n})^\top$ is estimated from the system of five regressions (49), which is now obtained from (33), (34), (35), and (44). The complete model comprises the 13 additional free parameters α. For obtaining the optimal input demand functions z^* through Shephard's lemma, the dependence of the aggregate wage $\mathbf{W}$ on the elementary wages w must be taken into account.

For this extended model, several local maxima were detected, and convergence was often very difficult to obtain. (An important number of iterations is required, and convergence is strongly dependent on the choice of the starting values.) This might be related to the high colinearity between the elementary and the aggregate variables, both of which appear as regressors in (44). In the sequel, the estimates corresponding to the highest log-likelihood are retained. The log-likelihood of this extended model is significantly higher than that of the previous model.

The different elasticity estimates are reported in Table 3. Although model 1 is rejected against model 2 at any reasonable significance level, a comparison of the estimates with those reported in Table 2 shows that only few results are quantitatively different. There are only two contradictions between both models, in the sense that an insignificant estimate becomes significant or vice versa. Estimates $\epsilon(\ell_h^*, w_h)$ and $\epsilon(\ell_u^*, y_x)$ both seem economically more plausible in model 2.

As in the preceding section, it is possible to test whether the different aggregation conditions of Section 2 are satisfied. The corresponding results for local separability are reported in Table 4. Given the results of Proposition 5, one would expect that the separability tests are no longer joint tests for separability and functional form adequacy, and thus that the separability restrictions should be rejected less often than occurred with the

Table 3. Elasticities of Model 2

$\epsilon(j^*, i)$	ℓ_h	ℓ_s	ℓ_u	v_k	v_m
w_h	$\mathbf{-.26_{(-4.7)}}$	$.06_{(5.7)}$	$-.10_{(-4.9)}$	$-.02_{(-3.7)}$	$.01_{(5.3)}$
w_s	$.79_{(6.0)}$	$\mathbf{-.49_{(-8.4)}}$	$.16_{(2.6)}$	$-.08_{(-3.7)}$	$.08_{(7.7)}$
w_u	$-.95_{(-5.4)}$	$.09_{(2.4)}$	$\mathbf{-.40_{(-3.1)}}$	$-.02_{(-.6)}$	$.05_{(4.5)}$
q_k	$-.16_{(-3.3)}$	$-.05_{(-2.9)}$	$-.02_{(-.8)}$	$\mathbf{-.14_{(-3.6)}}$	$.02_{(2.5)}$
q_m	$.59_{(6.9)}$	$.36_{(6.5)}$	$.36_{(5.2)}$	$.27_{(4.4)}$	$\mathbf{-.17_{(-6.4)}}$
y_d	$.75_{(15.7)}$	$.58_{(7.1)}$	$.70_{(11.3)}$	$.56_{(11.4)}$	$.70_{(10.7)}$
y_x	$.23_{(6.1)}$	$.08_{(5.5)}$	$.09_{(7.3)}$	$.15_{(10.0)}$	$.15_{(10.6)}$
t	$.033_{(11.3)}$	$.004_{(3.1)}$	$-.033_{(-14.9)}$	$-.002_{(-.9)}$	$.003_{(3.3)}$

NOTE: Median value of the elasticities evaluated for the 1985 data, with estimated t values in parentheses.

Table 4. Tests for Functional Structure, Model 2

	Tested restrictions	*Independent restrictions*	*Mean Wald test*[a]	*% observations with H_0 rejected*[b]
Outputs				
Nonjointness	(12)	1	$34.2_{(19.7)}$	90.0
Separability	(17)	6	$35.7_{(10.3)}$	93.4
Homothetic separability	(17), (18)	7	$45.1_{(22.3)}$	95.4
Additive separability	(17), (20)	7	$36.8_{(9.3)}$	97.8
Linear separability	(17), (21)	8	$137.4_{(68.0)}$	99.6
Identical outputs	(22)	1	$18.4_{(23.2)}$	62.5
Labor inputs				
Nonjointness	(14)	3	$44.6_{(13.2)}$	100
Separability	(31)	8	$61.7_{(27.4)}$	98.5
Homothetic separability	(28)	10	$367.0_{(176.4)}$	100
Additive separability	(31), (32)	10	$65.1_{(27.0)}$	98.7

[a]Mean value of the Wald test, with calculated standard deviation in parentheses.
[b]There are 456 observations at which the Wald test for a given null hypothesis is calculated.

first model. Nonetheless, the outcomes of the tests for the different functional structures are relatively similar to those obtained for the separability inflexible cost function. The mean Wald statistics reported in Table 4 are all relatively similar to those of Table 2. However, the percentage of observations for which the null hypothesis is rejected is never smaller in Table 4 than in Table 2. Only for the test of identical outputs are both percentages the same. This evidence suggests that the strong rejection of the null hypotheses obtained with model 1, is not due to the loss of flexibility under the null of separability.

As in the first model, here identical output is less strongly rejected than the more restrictive functional structures. The test of this assumption yields the smallest mean Wald statistic (18.4), which leads to fewer rejections than any other functional structure. The persistence of this paradoxical result demonstrates that it is not just a consequence of choosing a more restricted functional form.

For the purpose of comparison, we also computed the Diewert and Wales type of separability tests. The values of the Wald test for restrictions (47), (48), and both (47) and (48) are 69.6, 1,554.5, and 1,663.7, leading us to reject the null in all cases. As expected, these Wald test statistics are greater than the mean Wald statistic reported in Tables 2 and 4, due in part to the fact that the validity of the functional restriction is now tested globally (over all observations), not locally (at each observation point in turn).

The global validity of some functional structures is also tested using the procedure given in Appendix B. Surprisingly, the (nonreported) Wald test for the global validity of the functional structure is then much higher than the parametric test of the corresponding restrictions (47) and (48). This additional paradox should also motivate further research on the empirical performance of Wald tests. The foregoing results seem to suggest that the Wald statistic for a *given* assumption increases when the assumption is formulated with a greater number of equality restrictions. Unfortunately, tests based on the likelihood or on the regression residuals seem to not be easily implementable in this context. Because the restrictions to be tested are nonparametric (even in parametric models), there is no obvious relationship between the restrictions and the model parameters.

7. CONCLUSION

The necessary and sufficient conditions for aggregating (fixed) outputs and (flexible) labor inputs have been derived and used for testing the empirical validity of several exact aggregate representations. The homothetic separability assumption has been relaxed to allow for aggregates that are not homogeneous. The separability assumption has also been strengthen to allow for aggregates to be additive, like those usually provided by statistical offices.

The empirical findings regarding the possibility of providing an exact aggregate representation of the technology are mainly pessimistic. Although there is some evidence for the local validity of some restrictions, they are globally rejected. The impacts of wages, prices, outputs, and time on the different qualifications of labor are too different to allow aggregation of the three types of labor into a scalar measure. Similarly, domestic and exported outputs are rather different commodities, each produced from different technological requirements.

A more optimistic conclusion can be drawn from the frequent rejection of restrictions on functional structures. The fact that exact aggregation is statistically invalidated does not necessarily mean that aggregation cannot be achieved, but may be interpreted as evidence that *exact* aggregation is too demanding and that its requirements should be weakened. This is the *approximate* approach developed by Lewbel (1996) and Koebel (2002). In a companion work Koebel and Laisney (2005), we illustrate how approximate aggregation can be achieved, using the same data as here to better illustrate the difference in the two approaches.

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APPENDIX A: PROOF OF THE PROPOSITIONS

Proof of Proposition 1

Homothetic separability ⇒ (17) and (18). Because homothetic separability in y implies separability in y, it is clear that (17) must hold. Because g_y is homothetic, it can be written as $g_y(y) = H(h(y))$, where h is linearly homogeneous. Thus

$$\frac{\partial g_y}{\partial y_d}y_d + \frac{\partial g_y}{\partial y_x}y_x = H'\frac{\partial h(y)}{\partial y_d}y_d + H'\frac{\partial h(y)}{\partial y_x}y_x = H'h(y).$$

Using this expression and the fact that c is separable in y, that is,

$$c(w, q, y, t) = B_y(w, q, g_y(y), t),$$

leads to

$$\frac{\partial c}{\partial y_d}y_d + \frac{\partial c}{\partial y_x}y_x = \frac{\partial B_y}{\partial \mathbf{Y}}H'h(y), \tag{A.1}$$

with $\mathbf{Y} = g_y(y)$. Differentiating (A.1) with respect to y_d gives

$$\begin{aligned}
&\frac{\partial^2 c}{\partial y_d^2}y_d + \frac{\partial c}{\partial y_d} + \frac{\partial^2 c}{\partial y_d\,\partial y_x}y_x \\
&\quad = \frac{\partial^2 B_y}{\partial \mathbf{Y}^2}(H')^2 h(y)\frac{\partial h}{\partial y_d} + \frac{\partial B_y}{\partial \mathbf{Y}}H''\frac{\partial h}{\partial y_d}h(y) + \frac{\partial B_y}{\partial \mathbf{Y}}H'\frac{\partial h}{\partial y_d} \\
&\Leftrightarrow \quad \frac{\partial^2 c}{\partial y_d^2}y_d + \frac{\partial^2 c}{\partial y_d\,\partial y_x}y_x \\
&\qquad = \frac{\partial^2 B_y}{\partial \mathbf{Y}^2}(H')^2 h(y)\frac{\partial h}{\partial y_d} + \frac{\partial B_y}{\partial \mathbf{Y}}H''\frac{\partial h}{\partial y_d}h(y),
\end{aligned} \tag{A.2}$$

where the last line is obtained using $\partial c/\partial y_d = (\partial B_y/\partial \mathbf{Y})(\partial g_y/\partial y_d)$. Because this last expression is positive, it is possible to normalize (A.2) by $\partial c/\partial y_d$ to obtain

$$\frac{\partial^2 c}{\partial y_d^2}\frac{y_d}{\partial c/\partial y_d} + \frac{\partial^2 c}{\partial y_d\,\partial y_x}\frac{y_x}{\partial c/\partial y_d} = \left(\frac{\partial^2 B_y/\partial \mathbf{Y}^2}{\partial B_y/\partial \mathbf{Y}}H' + \frac{H''}{H'}\right)h(y).$$

Similarly, differentiating (A.1) with respect to y_x leads to

$$\frac{\partial^2 c}{\partial y_d\,\partial y_x}\frac{y_d}{\partial c/\partial y_x} + \frac{\partial^2 c}{\partial y_x^2}\frac{y_x}{\partial c/\partial y_x} = \left(\frac{\partial^2 B_y/\partial \mathbf{Y}^2}{\partial B_y/\partial \mathbf{Y}}H' + \frac{H''}{H'}\right)h(y),$$

and hence (18).

Conversely, it is well known that (17) implies separability. It remains to show that (18) implies homogeneity. Because c is separable in y, the equality

$$\sum_{o=d,x}\frac{y_o}{\partial c/\partial y_d}\frac{\partial^2 c}{\partial y_d\,\partial y_o} = \sum_{o=d,x}\frac{y_o}{\partial c/\partial y_x}\frac{\partial^2 c}{\partial y_x\,\partial y_o}$$

becomes

$$\begin{aligned}
&\frac{\frac{\partial^2 B_y}{\partial \mathbf{Y}^2}\left(\frac{\partial g_y}{\partial y_d}\right)^2 + \frac{\partial B_y}{\partial \mathbf{Y}}\frac{\partial^2 g_y}{\partial y_d\,\partial y_d}}{(\partial B_y/\partial \mathbf{Y})(\partial g_y/\partial y_d)}y_d + \frac{\frac{\partial^2 B_y}{\partial \mathbf{Y}^2}\frac{\partial g_y}{\partial y_d}\frac{\partial g_y}{\partial y_x} + \frac{\partial B_y}{\partial \mathbf{Y}}\frac{\partial^2 g_y}{\partial y_d\,\partial y_x}}{(\partial B_y/\partial \mathbf{Y})(\partial g_y/\partial y_d)}y_x \\
&\quad = \frac{\frac{\partial^2 B_y}{\partial \mathbf{Y}^2}\frac{\partial g_y}{\partial y_d}\frac{\partial g_y}{\partial y_x} + \frac{\partial B_y}{\partial \mathbf{Y}}\frac{\partial^2 g_y}{\partial y_d\,\partial y_x}}{(\partial B_y/\partial \mathbf{Y})(\partial g_y/\partial y_x)}y_d + \frac{\frac{\partial^2 B_y}{\partial \mathbf{Y}^2}\left(\frac{\partial g_y}{\partial y_x}\right)^2 + \frac{\partial B_y}{\partial \mathbf{Y}}\frac{\partial^2 g_y}{\partial y_x\,\partial y_x}}{(\partial B_y/\partial \mathbf{Y})(\partial g_y/\partial y_x)}y_x,
\end{aligned}$$

which can be simplified to

$$\begin{aligned}
&\frac{\partial^2 g_y}{\partial y_d\,\partial y_d}\frac{\partial g_y}{\partial y_x}y_d + \frac{\partial^2 g_y}{\partial y_d\,\partial y_x}\frac{\partial g_y}{\partial y_x}y_x \\
&\quad = \frac{\partial^2 g_y}{\partial y_d\,\partial y_x}\frac{\partial g_y}{\partial y_d}y_d + \frac{\partial^2 g_y}{\partial y_x\,\partial y_x}\frac{\partial g_y}{\partial y_d}y_x \\
&\Leftrightarrow \quad \left(\frac{\frac{\partial^2 g_y}{\partial y_d\,\partial y_d}}{\partial g_y/\partial y_d} - \frac{\frac{\partial^2 g_y}{\partial y_d\,\partial y_x}}{\partial g_y/\partial y_x}\right)y_d \\
&\qquad = \left(\frac{\frac{\partial^2 g_y}{\partial y_x\,\partial y_x}}{\partial g_y/\partial y_x} - \frac{\frac{\partial^2 g_y}{\partial y_d\,\partial y_x}}{\partial g_y/\partial y_d}\right)y_x \\
&\Leftrightarrow \quad \frac{\partial \ln\left(\frac{\partial g_y/\partial y_d}{\partial g_y/\partial y_x}\right)}{\partial y_d}y_d = \frac{\partial \ln\left(\frac{\partial g_y/\partial y_x}{\partial g_y/\partial y_d}\right)}{\partial y_x}y_x.
\end{aligned} \tag{A.3}$$

Consequently, the function $(\partial g_y/\partial y_d)/(\partial g_y/\partial y_x)$ is homogeneous of degree 0 in y, and therefore g_y is homothetic by Lau's (1970) lemma 1.

Proof of Proposition 2

The condition

$$c(w, q, y_d, y_x, t) = B_y(w, q, \delta_d y_d + \delta_x y_x, t)$$

is equivalent to

$$\frac{\partial c/\partial y_d}{\partial c/\partial y_x} = \frac{\partial g_y/\partial y_d}{\partial g_y/\partial y_x} = \frac{\delta_d}{\delta_x},$$

which in turn is equivalent to (17) and (21).

Lemma A.1. There exist two continuously differentiable real-valued function g_ℓ and A_ℓ, such that

$$a(\ell, q, y, t) \equiv \min_v\{q^\top v : f(\ell, v, y, t) = 0\} = A_\ell(g_\ell(\ell), q, y, t) \tag{A.4}$$

iff

$$\operatorname{rank}\left[\frac{\partial^2 a}{\partial \ell\,\partial(q^\top, y^\top, t)}\right] \le 1. \tag{A.5}$$

Proof of Lemma A.1. Let $\mathbf{L} = g_\ell(\ell)$; then condition (A.4) implies that

$$\begin{aligned}
\frac{\partial^2 a}{\partial \ell\,\partial(q^\top, y^\top, t)} &= \partial\left(\frac{\partial A_\ell}{\partial \mathbf{L}}\frac{\partial g_\ell}{\partial \ell}\right)\Big/\partial(q^\top, y^\top, t) \\
&= \frac{\partial g_\ell}{\partial \ell}\frac{\partial^2 A_\ell}{\partial \mathbf{L}\,\partial(q^\top, y^\top, t)},
\end{aligned}$$

with rank clearly 1 or 0. Conversely, (A.5) means that any 2×2 submatrix of $\partial^2 a/\partial \ell\,\partial(q^\top, y^\top, t)$ has zero determinant, in which case

$$\left(\frac{\partial a/\partial \ell_i}{\partial a/\partial \ell_j}\right)\Big/\partial(q^\top, y^\top, t) = 0,$$

implying, as shown by Leontief (1947), the existence of a function g_ℓ such that (A.4) holds.

Proof of Proposition 3

Different and much longer proofs of a related result have been given by Blackorby, Davidson, and Schworm (1991) and Koebel (2001). Because under cost minimization,

$$f(\ell, v, y, t) = G_\ell(g_\ell(\ell), v, y, t) \quad \Leftrightarrow \quad a(\ell, q, y, t) = A_\ell(g_\ell(\ell), q, y, t),$$

the proof of the left-side equality can rely on that of the right-side equality. The first-order conditions

$$\frac{\partial a}{\partial \ell}\big(\ell^*(w, q, y, t), q, y, t\big) = w, \tag{A.6}$$

corresponding to minimization of a with respect to ℓ, together with

$$c(w, q, y, t) = w^\top \ell^*(w, q, y, t) + a\big(\ell^*(w, q, y, t), q, y, t\big), \tag{A.7}$$

lead to

$$\frac{\partial c}{\partial w} = \ell^*(w, q, y, t), \qquad \frac{\partial^2 a}{\partial \ell\, \partial \ell^\top} \frac{\partial^2 c}{\partial w\, \partial w^\top} = I_d, \tag{A.8}$$

where I_d denotes the identity matrix. From this last line, it can be seen that $\partial^2 a/\partial \ell\, \partial \ell^\top$ is the inverse of $\partial^2 c/\partial w\, \partial w^\top$. Differentiation of $\partial a/\partial \ell$ with respect to $(q^\top, y^\top, t)$ then yields

$$\frac{\partial^2 a}{\partial \ell\, \partial \ell^\top} \frac{\partial^2 c}{\partial w\, \partial (q^\top, y^\top, t)} = -\frac{\partial^2 a}{\partial \ell\, \partial (q^\top, y^\top, t)} \tag{A.9}$$

and

$$\frac{\partial^2 c}{\partial w\, \partial (q^\top, y^\top, t)} = -\left[\frac{\partial^2 a}{\partial \ell\, \partial \ell^\top}\right]^{-1} \frac{\partial^2 a}{\partial \ell\, \partial (q^\top, y^\top, t)}. \tag{A.10}$$

a $\Leftrightarrow$ b: By (A.10) $\operatorname{rank} \partial^2 c/\partial w\, \partial (q^\top, y^\top, t) = \operatorname{rank} \partial^2 a/\partial \ell\, \partial (q^\top, y^\top, t)$. The conclusion follows from Lemma 1.

b $\Leftrightarrow$ c follows directly from the definition of the elasticities.

Proof of Proposition 4

Because there are three types of labor inputs, the Leontief conditions for additive separability in ℓ can be applied, stating that a is additive in ℓ iff (30) is satisfied and

$$\partial\left(\frac{\partial a/\partial \ell_h}{\partial a/\partial \ell_s}\right)\Big/\partial \ell_u = \partial\left(\frac{\partial a/\partial \ell_s}{\partial a/\partial \ell_u}\right)\Big/\partial \ell_h = \partial\left(\frac{\partial a/\partial \ell_u}{\partial a/\partial \ell_h}\right)\Big/\partial \ell_s = 0, \tag{A.11}$$

or equivalently (because one of the three equalities is redundant),

$$\frac{1}{w_h} \frac{\partial^2 a}{\partial \ell_h\, \partial \ell_u} = \frac{1}{w_s} \frac{\partial^2 a}{\partial \ell_s\, \partial \ell_u}, \qquad \frac{1}{w_s} \frac{\partial^2 a}{\partial \ell_h\, \partial \ell_s} = \frac{1}{w_u} \frac{\partial^2 a}{\partial \ell_h\, \partial \ell_u}. \tag{A.12}$$

The conditions (30) and (A.11) can be equivalently given in terms of function c. From Proposition 3, (30) is equivalent to (31). From (A.8), and the formula of the inverse of a matrix, restrictions (A.12) become

$$\frac{1}{w_h} \det\begin{bmatrix} \frac{\partial^2 c}{\partial w_h\, \partial w_s} & \frac{\partial^2 c}{\partial w_s\, \partial w_s} \\ \frac{\partial^2 c}{\partial w_h\, \partial w_u} & \frac{\partial^2 c}{\partial w_s\, \partial w_u} \end{bmatrix} = -\frac{1}{w_s} \det\begin{bmatrix} \frac{\partial^2 c}{\partial w_h\, \partial w_h} & \frac{\partial^2 c}{\partial w_h\, \partial w_s} \\ \frac{\partial^2 c}{\partial w_h\, \partial w_u} & \frac{\partial^2 c}{\partial w_s\, \partial w_u} \end{bmatrix}$$

and

$$-\frac{1}{w_s} \det\begin{bmatrix} \frac{\partial^2 c}{\partial w_h\, \partial w_s} & \frac{\partial^2 c}{\partial w_s\, \partial w_u} \\ \frac{\partial^2 c}{\partial w_h\, \partial w_u} & \frac{\partial^2 c}{\partial w_u\, \partial w_u} \end{bmatrix} = \frac{1}{w_u} \det\begin{bmatrix} \frac{\partial^2 c}{\partial w_h\, \partial w_s} & \frac{\partial^2 c}{\partial w_s\, \partial w_s} \\ \frac{\partial^2 c}{\partial w_h\, \partial w_u} & \frac{\partial^2 c}{\partial w_s\, \partial w_u} \end{bmatrix},$$

leading to (32).

Proof of Proposition 5

Diewert and Wales (1995, prop. 2) proved point a for the normalized quadratic functional form. Adapting their proof to the somewhat different context of the Box–Cox function shows that c is flexible in (w, q, y, t). For point b, set

$$B_{QY} = 0, \qquad B_{WY} = 0, \qquad B_{Yt} = 0, \qquad \text{and} \qquad \mathbf{B}_{\mathbf{W}Y} = 0 \tag{A.13}$$

(which imply separability in y) in the expression of $\mathbf{C}_{\mathbf{WY}}$. Under (A.13), $\mathbf{C}_{\mathbf{WY}}$ is still quadratic in $(W, Q, \mathbf{Y}, t)$. Thus the first- and second-order derivatives of $\mathbf{C}_{\mathbf{WY}}$ can locally approximate those of an arbitrary function of $(w, q, \mathbf{Y}, t)$ (and hence the local flexibility). Points c and d are obtained similarly.

APPENDIX B: A GLOBAL TEST

For testing the overall validity of the different null hypotheses, we first need to stack up the local test statistics $\widehat{s}_{nt} \equiv s(p_{nt}, y_{nt}, t, \widehat{\alpha}_n)$ over all n and t to obtain $\widehat{s} \equiv (\widehat{s}_{11}, \ldots, \widehat{s}_{NT})^\top$. In the local tests reported in Tables 2 and 4, the hypothesis $s_{nt}(\alpha_n) = 0$ [with an abuse of notation for $s(p_{nt}, y_{nt}, t, \alpha_n) = 0$] is tested for any n and t. Asymptotically, $\widehat{s}_{nt} \overset{a}{\sim} \mathrm{N}(s_{nt}, (\partial s_{nt}^\top/\partial \alpha_n)\Omega_n(\partial s_{nt}/\partial \alpha_n^\top))$, with $\mathrm{V}(\alpha_n) = \Omega_n$. However, computing the variance of $\widehat{s}_{nt}$ using the delta method is a relatively time-consuming task. (It took up to 500 seconds to calculate the local test at one observation point, using TSP version 4.5 on a Pentium III personal computer.) For testing whether $s(\alpha) \equiv (s_{11}(\alpha_1), \ldots, s_{NT}(\alpha_N))^\top = 0$, an estimate $\widehat{\Upsilon}$ of the variance matrix Υ of $\widehat{s}$ is needed. In the present case, the dimension of the matrix Υ is between (456×456) and $(4{,}560 \times 4{,}560)$, depending on the functional structure being tested. Approximating the matrix Υ using the delta method [yielding $\Upsilon_\Delta = (\partial s^\top/\partial \alpha)\Omega(\partial s/\partial \alpha^\top)$] would require months of calculation. To avoid this drawback, we instead rely on a resampling method to obtain an estimate of Υ.

Because $D(\widehat{\alpha}_{NT} - \alpha) \overset{d}{\to} \mathcal{N}(0, \Omega)$, where Ω is finite and positive definite and D is a diagonal matrix of scaling factors (depending on N and T), it follows that $\mathcal{N}(D\alpha, \Omega)$ is the limiting density of $\{D\widehat{\alpha}_{NT}, D\widehat{\Omega}_{NT}D\}$, where $\widehat{\Omega}_{NT}$ is the estimated variance matrix of $\widehat{\alpha}_{NT}$. Thus the density $\mathcal{N}(\widehat{\alpha}_{NT}, \widehat{\Omega}_{NT})$ can be used to draw a sample of $\widehat{\alpha}^i$, $i = 1, \ldots, I$, for which the value of $s^i(p_{nt}, y_{nt}, t, \widehat{\alpha}_n^i) \equiv \widehat{s}_{nt}^i$ is calculated for each observation point n and t. The vector $\widehat{s}^i$ is obtained by stacking up $\widehat{s}_{nt}^i$ over n and t. Then it is easy to calculate the empirical mean, $\bar{s}_I = I^{-1}\sum_i \widehat{s}^i$, and variance matrix, $\widehat{\Upsilon}_I = I^{-1}\sum_i(\widehat{s}^i - \bar{s}_I)(\widehat{s}^i - \bar{s}_I)^\top$, of $\widehat{s}^i$. By the law of large numbers, this matrix $\widehat{\Upsilon}_I$ converges in probability to Υ. These are the steps on which we rely to calculate the Wald statistic, $\widehat{W} = \widehat{s}^\top \widehat{\Upsilon}_I^- \widehat{s}$, for testing the (global) null hypothesis $s = 0$. In our experimentations, we chose to set I either to 10,000 or to 30,000, values for which the outcomes of the Wald test were no longer changing very much with increasing I.

A comparison of the estimates of the diagonal terms of Υ_Δ (the variances obtained using the delta method and used in the local test of functional structure) with those of $\widehat{\Upsilon}_I$ reveals a relatively small difference, about 5% on average. However, for some observations, the gap between the two estimates of the variance is about 50%. Because for I high enough, $\widehat{\Upsilon}_I$ provides a better approximation to Υ than $\widehat{\Upsilon}_\Delta$, we have a preference for the results based on $\widehat{\Upsilon}_I$. Nonetheless the percentages of observations for which the null is rejected are very similar regardless of the method used.

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