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Heterogeneous households' intertemporal characteristics and the aggregation problem

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Summary

Aggregation in exchange markets is studied by imposing restrictions on the distribution of the households' intertemporal characteristics. Approximate bounds for the derivatives of market demand that depend upon specific measure of sensitivity of the distribution of households' intertemporal choices to changes in the real interest rate are provided. This has strong consequences for the prevalence, in the aggregate, of gross substitutability between current and future consumption.

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1. Introduction

Recently, the "distributional" approach to the modelling of market demand which focuses on assumptions on the distribution of household characteristics rather than individual rationality has made great strides. Strong structural properties of market demand, which are not satisfied at the household level, are proved to emerge when aggregation is not arbitrary. Furthermore, in this approach individual rationality plays a minor role. So that with Kirman (1991) we no longer know "Whom or what does the representative individual represent?"

First, smoothness of market demand was obtained for non-convex preferences in Dierker *et al.* (1984) for conveniently dispersed distributions of preferences and wealth. The interested reader is referred to Trockel (1984) for a survey on market demand in large economies with non-convex preferences and for instances in which aggregation has a smoothing effect.

The problem of the monotonicity of market demand was then

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attacked following two steps. The first and decisive step was taken by Hildenbrand (1983) and followed by Härdle *et al.* (1991). The gist of their argument is to put restrictions on the shape of the distribution of income. The conditional distribution of income among households that have the same demand function is assumed to have a continuous and non-increasing density function. However, distributions of the log-normal type are observed. Chiappori (1985), Grodal and Hildenbrand (1992) and Marhuenda (1995) relaxed this assumption to other types of density function. The structuring impact of non-arbitrary aggregation is decisively demonstrated. However, these works led to the conclusion that realistic assumptions on the conditional distributions of income—that is, distributions of the log-normal type—cannot by themselves give an account of the Law of Demand.

A second line of argument has been put forth, using distinct methodologies by Grandmont (1992), Hildenbrand (1994) and Kneip (1993, 1995). The gist of this second approach is to impose new restrictions on the shape of the distribution of individual demand functions. This is obtained through restrictions on the distribution of household characteristics that affect demand, independently of the current price system and current income. In Grandmont (1992) the use of a very particular linear structure on the space of demand functions is essential. The author proves a very nice property of the aggregate demand: market demand has a dominant diagonal Jacobian matrix. A decisive proof of the fruitful use of distributional requirements for a general framework has been given in Hildenbrand (1994). The author does not restrict the analysis to a specific structure of the space of demand functions but rather focuses on the distribution of household demand vectors generated for each price system. His distributional requirement consists of restricting the way this distribution evolves with income and generates through aggregation an average positive income effect. The Law of Demand is then derived from the traditional Slutsky decomposition. The interest of the approach in Kneip (1995) is then to get back the two previous results in the same framework. Restrictions are made in terms of the sensitivity of the distribution of household choices to changes in income or prices. The result of an almost diagonal Jacobian of market demand is obtained through restrictions on the degree of sensitivity of the distribution to changes in prices. The average positive income effect, in turn, is obtained through restrictions on the degree of sensitivity of the distribution to changes in income.

There are still two unsatisfactory features in the aforementioned viewpoint. First, the consumption decision is not apprehended within the theory of intertemporal choice. Hence, the intertemporal characteristics of households which are determinant in the consumption decision are not explicitly taken into account. It is true that Kneip suggested that an intertemporal version of his model should be easily derived. However, once households are assumed to have perfect foresight, the reckoning used to extend this model to an intertemporal framework is equivalent to introducing dated commodities. Thus, Kneip's initial distributional assumption, which can be interpreted as the outcome of restrictions on the conditional distributions of (static) preferences of individuals with the same income, is easily transposed to the conditional distributions of intertemporal preferences.† Then, one easily deduces the monotonicity of the current market demand function. In contrast, in the model under consideration disaving is not allowed. This yields a more complex intertemporal set-up, by introducing into the individual intertemporal decision program an additional budget constraint specific to the intertemporal choice theory. The second unsatisfactory feature is that income is exogenous. This means, in particular, that current income is independent of current prices. Note that this has been allowed in the static model of Grandmont (1992). However, the dependence of future income on inflation has not yet been taken into account. In our model we shall allow for such a dependence by defining the household global income as the nominal value of its initial endowments in the current and future consumption commodities. This seriously affects the macroeconomic regularities induced by distributional requirements. For example, once the price-dependence of income is allowed, the monotonicity of market demand is no longer ensured by the assumptions considered in Kneip's model.

The first goal of this paper is to deduce the monotonicity of market demand from restrictions on the distribution of household intertemporal characteristics. In contrast to the literature, this is carried out in a set-up where the intertemporal feature of the model is not an alternative to introducing dated commodities. Furthermore, the price-dependence of income is allowed in this intertemporal set-up.

The second goal of the paper is to define assumptions on the distribution of household intertemporal characteristics in a framework directly tractable by macroeconomists, namely the overlapping generations model of a pure exchange economy. The motivation is to try to contribute to a more systematic use of distributional assumptions in economics. Despite the fact that the representative agent assumption is now well known, in most circumstances, to be a very bad approximation of the aggregation of individual behaviours and to be misleading, its use is still almost systematic in macroeconomic models. The reason for this is that no other tractable tool has been offered. Thus, we conclude the

† The income on which the distributions of preferences are conditioned is now the two-period income.

paper by giving a direct implication of the macroeconomic regularity ensured by our assumptions. These requirements rule out the emergence of endogenous deterministic business cycles in a pure exchange economy with perfect foresight. The linearity of the model is well known to ensure uniqueness of long-run (deterministic) equilibria in the economy under consideration and the global stability of the unique stationary temporary equilibrium in the backward perfect foresight dynamics. The advantage of our approach in contrast with the Keynesian representative agent models is to deduce this linearity from our distributional requirement. In turn, rationality plays a minor role; no requirements are imposed on individual functions other than homogeneity and the satisfaction of the intertemporal budget constraints. This allows, in particular, for large degrees of relative risk aversion of old households.†

The paper is organized as follows: in Section 2 we present the model considered by Grandmont (1985), into which we introduce, in Section 3, heterogeneous consumers. The restrictions we introduce on the distribution of household intertemporal characteristics are of two types. First, we decompose the population into subpopulations. All households of a given subpopulation are characterized by the fact that they all start to save at the same real interest rate, $\bar{\theta}$. The first requirement concerns the marginal distribution of $\bar{\theta}$, which is assumed to be sufficiently dispersed in the population in the spirit of Dierker *et al.* (1984): the weight of a given subpopulation is assumed to be negligible. The divergence of individual demand functions can be the outcome of both heterogeneous preferences and heterogeneous initial endowments. The second requirement concerns the conditional distribution of intertemporal household characteristics in a given subpopulation; it is adapted from a distributional property of one-period demand functions (Kneip, 1995). The idea is to restrict the degree of sensitivity of the distribution of households' budget share vectors (current and future consumption) to changes of the real interest rate. This restriction is obtained when households react with sufficient heterogeneity to changes in the real interest rate—if they react at all; it admits as a degenerate case a population where all households possess intertemporal preferences of the

† While in Keynesian representative agent models a large multiplicity of endogenous business cycles is deduced from *a priori* restrictions on the degree of concavity of the utility function of the representative agent. In particular, the consumer should have a higher relative risk aversion when old than when young. This result was obtained, for example, by Benhabib and Day (1982) and Grandmont (1985) in a deterministic set-up. The same phenomenon may arise when traders predict that stochastic fluctuations will occur, see Azariadis (1981), Azariadis and Guesnerie (1986), Chiappori and Guesnerie (1991) and Grandmont (1986, 1989).

Cobb–Douglas type. Furthermore, we provide an example of a class of populations that satisfy our requirement. In this example households diverge in their intertemporal preferences and their initial endowments so that individual demand functions can be parameterized in the spirit of Grandmont (1992).† Introducing restrictions on the degree of flatness of the distribution of the parameters yields then to our distributional requirement. In Section 4 we give approximate evaluations of the partial derivatives of market demand with respect to the real interest rate, denoted θ, the precision of which increases as the degree of sensitivity of the distribution of budget share vectors decreases. We prove that for a sufficiently small degree of sensitivity, market demand for the two periods satisfies the gross substitutability property on a compact set of θ, the size of which goes up as the degree of sensitivity decreases. Becker (1962) proves this without assuming any form of individual rationality other than homogeneity and (intertemporal) budget constraints. The striking point is that this result is valid, in particular, for a population that is built starting from a household with an increasing demand function and introducing heterogeneous households following the aforementioned parameterization. The implications of this structuring impact of aggregation are discussed in Section 5, where the unique monetary stationary temporary equilibrium is proved to be globally stable; that is, no endogenous business cycles with perfect foresight can emerge in this economy.

2. Behavioral assumptions

Although one of the goals of the present work is to develop concepts and methods that can be applied to a larger class of situations, the analysis will proceed by studying a particular example: the overlapping generations model of a pure exchange economy, with a constant population and without bequests, in which households live two periods only. The model involves one perishable consumption good and money. Young consumers have the opportunity to save part of their income in each period by holding non-negative money balances (disaving is not allowed).

The mean money stock in the economy will be assumed to be constant over time and will be denoted $\bar{m}>0$. At each date there are competitive spot markets for the consumption good and for money. Money is chosen as numéraire. Markets are assumed to clear, in the Walrasian sense, at every date.

† Hence, in contrast to the models of Kneip and Grandmont, households diverge both in their initial endowments and their preferences (which are here intertemporal).

Assumptions on household behaviour will be limited as the main focus of this paper is on the distribution of households' types in the population. The consumer intertemporal characteristics are as follows. Consumption c_τ in each period τ of his life ($\tau = 1, 2$) must be non-negative. It is assumed that the agent has a non-negative endowment in the perishable commodity in each period of his life $\omega_\tau \geqslant 0$.

Agent i's problem, when young, is to choose his current consumption $c_1^i \geqslant 0$, his demand for nominal money m^i and to plan for future consumption $c_2^i \geqslant 0$ subject to the current and expected budget constraint:

$$pc_1^i + m^i = p\omega_1^i$$
$$p^e c_2^i = m^i + p^e \omega_2^i.$$

We shall assume that the household demand $c_\tau^i - \omega_\tau^i$ depends only on $\theta = (p/p^e)$ or equivalently on its expected real interest rate $\theta - 1$, and is denoted $z_\tau^i(\theta)$ for $\tau = 1, 2$ (absence of money illusion). Denote by $\bar{\theta}^i$ the price ratio, i.e. $\bar{\theta}^i - 1$ is the critical interest rate at which the household i starts to save money for future consumption. z_1^i and z_2^i are respectively the current and expected excess demands of i. These functions are required to satisfy the following assumptions:

ASSUMPTION 1: *(i)* $z_1^i(\theta)$ *and* $z_2^i(\theta)$ *are continuous on* $(0, +\infty)$. $\forall\ \theta > 0$,

$$\theta z_1^i(\theta) + z_2^i(\theta) \equiv 0. \tag{1}$$

Moreover, $z_1^i(\theta) = z_2^i(\theta) = 0$ *whenever* $\theta \leqslant \bar{\theta}^i$, *and* $-\omega_1^i < z_1^i(\theta) < 0$, $z_2^i(\theta) > 0$ *whenever* $\theta > \bar{\theta}^i$;

(ii) The restrictions of the excess demand functions to the interval $(\bar{\theta}^i, +\infty)$ *are continuously differentiable. Furthermore, for* θ *in a neighbourhood of* $\bar{\theta}^i$ *with* $\theta > \bar{\theta}^i$, *we have* $z_1^{i\prime}(\theta) < 0$.

The household i's demand for money $m_d^i(p, p^e)$ is then given by

$$m_d^i(p, p^e) \equiv -pz_1^i(\theta) \equiv p^e z_2^i(\theta). \tag{2}$$

This individual demand may be interpreted as the outcome of a standard utility maximization program. For a separable utility function $U_1^i(c_1^i) + U_2^i(c_2^i)$,† as in Grandmont (1985), we have $\bar{\theta}^i = (U_1^{i\prime}(\omega_1^i)/U_2^{i\prime}(\omega_2^i))$. Furthermore, $\forall\ \theta > \bar{\theta}^i$,

† The functions U_1^i and U_2^i are assumed to be continuous on $[0, +\infty)$, twice continuously differentiable on $(0, +\infty)$ with $U_\tau^{i\prime}(c_\tau) > 0$, $\lim_{c_\tau \to 0} U_\tau^{i\prime}(c_\tau) = +\infty$, $U_\tau^{i\prime\prime}(c_\tau) < 0$.

$$z_1^{i\prime}(\theta) = \frac{U_2^{i\prime}(z_2^i(\theta)+\omega_2^i) + z_2^i(\theta)U_2^{i\prime\prime}(z_2^i(\theta)+\omega_2^i)}{\Delta}$$
$$z_2^{i\prime}(\theta) = -\frac{U_1^{i\prime}(z_1^i(\theta)+\omega_1^i) + z_1^i(\theta)U_1^{i\prime\prime}(z_1^i(\theta)+\omega_1^i)}{\Delta} \tag{3}$$

where

$$\Delta = U_1^{i\prime\prime}(z_1^i(\theta)+\omega_1) + \theta^2 U_2^{i\prime\prime}(z_2^i(\theta)+\omega_2)<0.$$

In particular, $z_2^{i\prime}(\theta)>0,\ \forall\ \theta>\bar{\theta}^i$. Moreover, $\lim_{\theta\to+\infty} z_2^i(\theta)=+\infty$. In this case, the consequences on the individual decisions of a change of θ are the following. A change of θ generates both an intertemporal effect as well as an income (or wealth) effect. For future consumption, these two effects work in the same direction and $z_2^{i\prime}(\theta)>0,\ \forall\ \theta>\bar{\theta}^i$. On the other hand, for current consumption the induced variation on $z_1^i(\theta)$ is ambiguous. From equation (1) we know that $-(z_2^{i\prime}(\bar{\theta}^i)/z_1^{i\prime}(\bar{\theta}^i))=\bar{\theta}^i$. Thus, $z_1^{i\prime}(\bar{\theta}^i)<0$ and by continuity for θ close to $\bar{\theta}^i$, $z_1^{i\prime}(\theta)<0$. However, for larger values of θ the sign of $z_1^{i\prime}(\theta)$ is *a priori* indeterminate, since income and substitution effects are working in the opposite direction. From equation (3), this sign is dependent on the value taken by the Arrow–Pratt relative degree of risk aversion

$$R_\tau^i(c_\tau) = -\frac{c_\tau U_\tau^{i\prime\prime}(c_\tau)}{U_\tau^{i\prime}(c_\tau)}$$

which is well defined whenever $c_\tau>0$. This measure is considered, although there is no uncertainty in the model. First, it is a useful tool to measure the degree of concavity of a trader's utility function. Secondly, this allows for comparison with the representative agent version of the model under consideration, where linearity results have been given in terms of this measure. In a representative–agent model, the monotonicity of market demand is deduced from the monotonicity of the individual demand through restrictions on the value of the relative degree of risk aversion of the old household. If, for example, $R_2^i(c_2)\leqslant 1$ for all c_2, then $z_1^{i\prime}(\theta)<0,\ \forall\ \theta>\bar{\theta}^i$. The purpose of this paper is to avoid such arbitrary restrictions at the household level.

3. The distribution of household characteristics

We denote by α the vector of all exogenous characteristics that affect household demand independently of the real interest rate and define its type, namely its endowments, its preferences, but also socio-economic and demographic factors such as the family size, the age of the household head, . . . , etc.

As an illustration, when individual demand is the outcome of a standard optimization program, this vector includes the initial endowments in the perishable commodity ω_τ for $\tau = 1, 2$, and the other arguments are the factors that determine the household preferences. Hence, the utility of household i and its initial endowments in period τ can be written, for $\tau = 1, 2$:

$$U_\tau^i = U_\tau(c_\tau, \alpha_\tau^i)$$
$$\omega_\tau^i = \omega_\tau(\alpha_\tau^i)$$

and $\alpha = (\alpha_1, \alpha_2) = (\alpha_{11}, \alpha_{12}, \ldots, \alpha_{21}, \alpha_{22}, \ldots)$.

Let $\mathscr{A}$ denote the space of household types α. Household type i is represented by the element α^i in the set $\mathscr{A}$. Hence, given the current price p and the expected price p^e, the current excess demand of household i, its expected demand and its demand for money can be written, respectively:

$$z_1^i(\theta) = z_1(\theta, \alpha^i)$$
$$z_2^i(\theta) = z_2(\theta, \alpha^i)$$
$$m_d^i(p, p^e) = m_d(p, p^e, \alpha^i)$$

and its critical interest rate is written $\bar{\theta}^i = \bar{\theta}(\alpha^i)$.

A large and heterogeneous population of young household types in period t is represented by a distribution μ_t of α on $\mathscr{A}$.† The large and heterogeneous population of old households in period t is therefore represented by the distribution μ_{t-1} of α on $\mathscr{A}$. We require that in each period there is one old household and one young household of each type, hence we have:

ASSUMPTION 2: *the distribution of household types is time-independent, i.e.* $\forall\ t,\ \mu_t = \mu_{t-1}$.

For any period of time, given the current price and the expected price (identical for all household types), young households possess the following mean current excess demand for the consumption commodity:

$$Z_1(\theta, \mu) = \int_{\mathscr{A}} z_1(\theta, \alpha) d\mu. \tag{4}$$

† The distribution μ is a probability measure on the smallest σ-algebra of $\mathscr{A}$ containing all sets of the form $A = \{\alpha \in \mathscr{A} \mid z_1(\theta, \alpha) \in I_\theta \text{ for all } \theta\}$ with $\{I_\theta\}_{\theta \in \mathbb{R}_+}$ being an arbitrary sequence of Borel subsets of $[-\tilde{\omega}_1, 0]$, where $\tilde{\omega}_1 > 0$.

Their mean future excess demand for the commodity and their mean current excess demand for money follow then from equations (1) and (2).

$$Z_2(\theta, \mu) \equiv -\theta Z_1(\theta, \mu) \tag{5}$$

$$M_d(p, p^e, \mu) \equiv -pZ_1(\theta, \mu). \tag{6}$$

Thus, mean demands depend in addition to (p, p^e) on the distribution μ of household characteristics.

We shall decompose the population into subpopulations. All households of a given subpopulation are characterized by an identical critical interest rate $\bar{\theta}$. We refer to this subpopulation as the $\bar{\theta}$-subpopulation. Denote by Θ the set of $\bar{\theta}$. Let $\mu/\bar{\theta}$ denote the conditional distribution of household characteristics given $\bar{\theta}$ and $\mu^{\bar{\theta}}$ the marginal distribution of $\bar{\theta}$. The set of household characteristics is rewritten $\mathscr{A} = \mathscr{B} \times \Theta$, so that $\mu/\bar{\theta}$ and $\mu^{\bar{\theta}}$ are distributions on $\mathscr{B}$ and Θ respectively. The integral in equation (4) can be understood in two steps: first, for every $\bar{\theta}$, one integrates over β, and then one integrates over $\bar{\theta}$. Define the conditional mean excess demand for current consumption by

$$\bar{z}_1(\theta, \mu/\bar{\theta}) = \int_{\mathscr{B}} z_1(\theta, \beta, \bar{\theta}) d\mu/\bar{\theta}. \tag{7}$$

Thus, we obtain

$$Z_1(\theta, \mu) = \int_{\Theta} \bar{z}_1(\theta, \mu/\bar{\theta}) d\mu^{\bar{\theta}}. \tag{8}$$

Later on, we shall be concerned with the case where the mean demand for money is strictly positive when price p is constant over time, i.e. when $\theta = 1$. This corresponds to the Samuelson case according to the terminology of Gale (1973). Formally, we introduce the following assumption.

ASSUMPTION 3: *denote by* $\bar{\theta}_{\inf} = \inf_{\alpha \in \mathscr{A}} \bar{\theta}(\alpha)$. *The set of household types such that* $\bar{\theta}<1$ *is of strictly positive measure, i.e.* $\mu^{\bar{\theta}}((\bar{\theta}_{\inf}, 1))>0$.

We shall also require the following assumption.

ASSUMPTION 4: $\forall\ \theta \varepsilon \Theta$, for $\mu/\bar{\theta}$ almost all $\beta \in \mathscr{B}$,

$$\omega_1(\beta, \bar{\theta}) \leqslant \tilde{\omega}_1 < +\infty$$
$$\omega_2(\beta, \bar{\theta}) \leqslant \tilde{\omega}_2 < +\infty.$$

To introduce further assumptions on the distribution of household characteristics we concentrate on individual budget share functions. For a given current excess demand function z_1 the corresponding budget share function s_1: $\mathbb{R}_+ \to I \subset [0, 1]$ is given by:

$$s_1(\theta, \alpha) = \frac{\theta(z_1(\theta, \alpha) + \omega_1(\alpha))}{\theta\omega_1(\alpha) + \omega_2(\alpha)}.$$

Symmetrically,

$$s_2(\theta, \alpha) = \frac{z_2(\theta, \alpha) + \omega_2(\alpha)}{\theta\omega_1(\alpha) + \omega_2(\alpha)}.$$

To formalize our distributional requirements we focus on the distribution of households' choices in a given $\bar{\theta}$-subpopulation. More precisely, we consider the distribution $\upsilon_{\theta,\bar{\theta}}$ of households' current budget shares on $I \subset [0, 1]$, generated by the conditional distribution $\mu/\bar{\theta}$ of β at each $\theta > \bar{\theta}$.

The main thrust of our approach will be to study how the qualitative and quantitative properties of the conditional distributions $d\mu/\bar{\theta}$ affect aggregate demand, and more precisely its partial derivative with respect to θ. Clearly, we have to assume that the distribution of household characteristics is regular enough to ensure differentiability in the aggregate. This is the purpose of the decomposition of the population introduced above and of the following requirement.

ASSUMPTION 5: *(i) for every continuous function* υ: $I \to \mathbb{R}$ *the integral* $\int_I \upsilon(s)\upsilon_{\theta,\bar{\theta}}(ds)$ *is differentiable with respect to* θ.

(ii) $\forall\, \theta \in \Theta$ *such that* $\theta > \bar{\theta}_{\inf}$, $\mu^{\bar{\theta}}(\theta) = 0$.

The restriction (ii) is inspired by the price-dispersed preferences introduced by Dierker *et al.* (1984). Note that here this dispersion of $\bar{\theta}$ in the population may be interpreted as generated by a joint dispersion of preferences and initial endowments. Effectively, if household demand is interpreted as the outcome of a standard (separable) utility maximization program, one has

$$\bar{\theta}^i = \frac{U_1^{i\prime}(\omega_1^i)}{U_2^{i\prime}(\omega_2^i)}.$$

Hence, in contrast to the static models considered in the literature, by introducing a non-disaving constraint specific to intertemporal choice theory one has to restrict the distribution of household initial endowments, included in the distribution of household intertemporal characteristics of the model.

We shall now restrict the conditional distributions of household characteristics $\mu/\bar{\theta}$. First, in a given $\bar{\theta}$-subpopulation the set of household characteristics is assumed to be sufficiently large. Some "type of price-dispersion" is introduced on household demand functions. Formally, we have the following assumption:

ASSUMPTION 6: *for a given $\bar{\theta}\in\Theta$, the support of the conditional distribution $\mu/\bar{\theta}$ is large enough such that $\forall\ \beta\in\mathscr{B}$ there exists a non-empty neighbourhood*† *of* 1 *such that for all λ in this neighbourhood with $\lambda\theta>\bar{\theta}$, there exists $\gamma\in\mathscr{B}$ such that $\forall\ \theta>\bar{\theta}$.*

$$s_1(\theta, \gamma, \bar{\theta}) = s_1(\lambda\theta, \beta, \bar{\theta}).$$

To interpret this assumption let us write $\lambda = \lambda_1/\lambda_2$. The demand of household γ is deduced from the demand of household β by stretching the axis corresponding to the consumption of period τ by a factor λ_τ. This is the affine transformation introduced by Mas-Colell and Neuefind (1977) and Dierker *et al.* (1984). This assumption is referred to as a requirement of "interest-rate dispersed household characteristics". Note, however, that the "dispersion" is all relative since when, for example, $s_1(\cdot, \beta, \bar{\theta})$ is independent of the real interest rate, then $\gamma = \beta$.

Let us now give an intuitive explanation of the main restriction on the distribution $\mu/\bar{\theta}$. At the limit the argument would go as follows; given a large $\bar{\theta}$-subpopulation, households must be fairly equally spread on the feasible set $\mathscr{B}$, where dispersion on this set has to be understood in the sense of the above *interest-rate dispersion*. More precisely, none of the households' type β must have significantly more chance to appear than any other of its "interest rate transformed" type. Intuitively some connection can be made between this requirement and the "flat distribution of parameters" parameterizing individual demands introduced by Grandmont (1992). We shall come back to this point in subsection 3.1.2. The crucial assumption will be that for all θ the distribution $\upsilon_{\theta,\bar{\theta}}$ tends to vary slowly with θ. In other words, the distributions $\upsilon_{\theta,\bar{\theta}}$ and $\upsilon_{\lambda\theta,\bar{\theta}}$ tend to be close to each other for λ close to 1.

ASSUMPTION 7: *let $\rho_{\theta,\bar{\theta}}$ be the density function with respect to the Lebesgue measure of the distribution $\upsilon_{\theta,\bar{\theta}}$ on $I\subseteq[0, 1]$. For all $\theta>\bar{\theta}$, the partial derivative $\partial_\lambda \rho_{\lambda\theta,\bar{\theta}}(s)\mid_{\lambda=1}$ exists and is continuous in s. Moreover, the distribution is "structurally insensitive" to changes in θ, i.e. the parameter*

† Note that the type of dispersion required is less restrictive than the one considered in Kneip (1995) in the sense that the assumption holds for any λ in a neighbourhood of 1—which might be arbitrarily small—and not for any $\lambda>0$.

$$h(\upsilon_{\bar{\theta}}) = \sup_{\theta > \bar{\theta}} \int_I | \partial_\lambda \rho_{\lambda\theta,\bar{\theta}}(s) |_{\lambda=1} | ds$$

is small.

Quantitative restrictions on $h(\upsilon)$ will be introduced later on. The parameter $h(\upsilon_{\bar{\theta}})$ is a coefficient of sensitivity of the distribution υ_θ to a variation of θ. As $h(\upsilon_{\bar{\theta}})$ tends to zero the distribution becomes invariant to changes of θ. It can also be interpreted as a degree of "flatness" of the distribution of households over $\mathscr{B}$, for the structure on $\mathscr{B}$ defined by Assumption 6. As $h(\upsilon_{\bar{\theta}})$ tends to zero the distribution $\mu/\bar{\theta}$ is more "flat" over $\mathscr{B}$ following the "interest-rate dispersion". This assumption is referred to as a "structural insensitivity" of the distribution of household choices to changes in θ, or equivalently to changes in the real interest rate $\theta - 1$. This is adapted from the work of Kneip (1995) who requires some insensitivity of the distribution of household current choices to price changes and income changes. Note that, in contrast to Kneip, who introduces the insensitivity requirement for any price and any income, Assumptions 6 and 7 are only required on a subset of the set of real interest rates, namely $(\bar{\theta}, +\infty)$. For $\theta \leqslant \bar{\theta}$, all households of the $\bar{\theta}$-subpopulation have rather close demand behaviour since they all consume in the first period their first period endowment (which might nevertheless differ from one agent to the other). Furthermore, since our definition of household demand is distinct, the interpretation of our distributional requirement diverges from the one of Kneip's assumptions, as illustrated in the examples below. Furthermore, the macroeconomic implications are also distinct as shown in the next section.

It is important to observe that a small coefficient of sensitivity $h(\upsilon_{\bar{\theta}})$ is obtained when budget share functions are *heterogeneous in terms of their reaction to a change in* θ or equivalently to a change in the real interest rate $\theta - 1$. Effectively, $h(\upsilon_{\bar{\theta}})$ small means that the probability that the budget share vector belongs to some given interval J varies slowly with θ. Hence, after a change in θ the frequency of budget share vectors which leave J is compensated by the frequency of budget share vectors which enter J. In particular, if after a variation of θ some households increase their budget share of current consumption, there will always exist households that decrease their budget share of current consumption. To require such compensation phenomena implies that households are supposed to be sufficiently heterogeneous (see Kneip, 1995).

Note, however, that a small $h(\upsilon_{\bar{\theta}})$ can also be induced by budget share functions which are not heterogeneous but are independent of the real interest rate at the *household level,* as in the degenerate case of individual demands of the Cobb–Douglas type. However, as long as some households do deviate after a change in θ there should exist some compensation phenomena such that the *distribution as a whole is not affected.*

3.1. EXAMPLES OF $\bar{\theta}$-SUBPOPULATIONS WITH SMALL $h(\upsilon_{\bar{\theta}})$

3.1.1. *Households more irrational than in Becker (1962)*

Consider a population where in each $\bar{\theta}$-subpopulation all households select their current budget share at random on [0, 1]. This is the demand behaviour considered in Becker (1962), which is the outcome of a household rationality limited to the household's one-period budget constraint. In our intertemporal set-up, the household rationality behind this demand behaviour is not even the satisfaction of the household budget constraints, which are more demanding. When varying θ, any path of realizations of the random selection procedure creates a first-period budget share function. This procedure induces a second-period budget share function under the constraint that for any θ the sum of the two budget shares should be equal to one. This defines individual budget share functions that depend on "random household characteristics" including household initial endowments in the two periods. For all θ, the corresponding distribution $\upsilon_{\theta,\bar{\theta}}$ on I is the uniform distribution on I. Hence, $\forall\ \lambda>0$, $\upsilon_{\lambda\theta,\bar{\theta}}=\upsilon_{\theta,\bar{\theta}}$ and $h_{\bar{\theta}}(\upsilon)=0$.

3.1.2. *Intertemporal parametric model of demand "à la Grandmont"*

The following example gives us an interpretation of our requirement in terms of the joint dispersion of household intertemporal preferences and initial endowments. Consider a population where each $\bar{\theta}$-subpopulation follows a parametric model of demand. The parameterization is adapted from the parameterization of one-period individual demand functions introduced by Grandmont (1992) to our intertemporal set-up. Nevertheless, in contrast to Grandmont, we parameterize not only household preferences (which here are intertemporal) but also household initial endowments. Consider the initial endowments $(\omega_1, \omega_2)\gg 0$ and the separable utility function $U(\cdot,\cdot)=U_1(\ )+U_2(\ )$,† (ω_1, ω_2) and $U(\cdot,\cdot)$ describe the "generic household" of a given $\bar{\theta}$-subpopulation. Household β is described by the intertemporal utility function $U(\cdot,\cdot,\beta)$ and the initial endowments $(\omega_1(\beta), \omega_2(\beta))$ which

† The functions U_1 and U_2 are assumed to be continuous on $[0, +\infty)$, twice continuously differentiable on $(0, +\infty)$ with $U_\tau'(c_\tau)>0$, $\lim_{c_\tau\to 0} U_\tau'(c_\tau)=+\infty$, $U_\tau''(c_\tau)<0$.

are respectively obtained from $U(\cdot,\cdot)$ and (ω_1, ω_2). These characteristics of β would correspond with the ones of the generic household if the current unit of measurement of the perishable commodity had not been multiplied by $\exp \beta_1$ and the future unit of measurement by $\exp \beta_2$. That is, $z_1(\theta, \beta)$, $z_2(\theta, \beta)$ and $m_d(p, p^e, \beta)$ are obtained by maximizing $U(c_1, c_2, \beta)$ with respect to $0 \leqslant c_1 \leqslant \omega_1(\beta)$ and $c_2 \geqslant 0$ subject to $pc_1 + p^e c_2 \leqslant p\omega_1(\beta) + p^e\omega_2(\beta)$, where

$$U(c_1, c_2, \beta) = U_1(c_1, \beta_1) + U_2(c_2, \beta_2) = U_1\left(\frac{c_1}{\exp \beta_1}\right) + U_2\left(\frac{c_2}{\exp \beta_2}\right)$$

$$(\omega_1(\beta), \omega_2(\beta)) = (\omega_1 \exp \beta_1, \exp \beta_2).$$

One obtains for the excess demand functions and the demand for money

$$z_1(\theta, \beta) = e^{\beta_1} z_1\left(\frac{e^{\beta_1}}{e^{\beta_2}}\,\theta\right)$$

$$z_2(\theta, \beta) = e^{\beta_2} z_1\left(\frac{e^{\beta_1}}{e^{\beta_2}}\,\theta\right)$$

$$m_d(p, p^e, \beta) = m_d(e^{\beta_1}p, e^{\beta_2}p^e). \tag{9}$$

The current budget share function is therefore given by†

$$s_1(\theta, \beta) = s_1\left(\frac{e^{\beta_1}}{e^{\beta_2}}\,\theta\right).$$

Note that

$$\forall\ \beta \in \mathbb{R}^2,\ \bar{\theta}(\beta) = \frac{U_1'(\omega_1(\beta), \omega_2(\beta), \beta)}{U_2'(\omega_1(\beta), \omega_2(\beta), \beta)} = \frac{U_1'\left(\dfrac{\omega_1(\beta)}{e^{\beta_1}}, \dfrac{\omega_2(\beta)}{e^{\beta_2}}\right)}{U_2'\left(\dfrac{\omega_1(\beta)}{e^{\beta_1}}, \dfrac{\omega_2(\beta)}{e^{\beta_2}}\right)} = \frac{U_2'(\omega_1, \omega_2)}{U_2'(\omega_1, \omega_2)} = \bar{\theta}.$$

† The transformations of the unit of measurement of the perishable commodity considered are now exponential versions of the affine transformations considered by Mas-Colell and Neuefind (1977) and Dierker *et al.* (1984).

Let us assume that the distribution $\mu/\bar{\theta}$ of β on $\mathbb{R}^2$ is concentrated over the subset $B=(-\infty, \tilde{\beta}_1)\times(-\infty, \tilde{\beta}_2)$, i.e. the complement of B is a negligible set, $\mu/\bar{\theta}(B)=0$. We will assume in addition that $\mu/\bar{\theta}$ admits on B a differentiable density function $\eta(\cdot)$ in β. This will generate at each $\theta>\bar{\theta}$ a distribution $\upsilon_{\theta,\bar{\theta}}$ of the budget shares $s_1(\theta, \beta)$ on $[0, 1]$.

One can easily check that any requirement of Assumptions 3 and 5 holds or can be assumed without loss of generality. Furthermore, by definition of the parameterization, $\forall\ \beta\in B$, there exists $\gamma\in B$ such that $\forall\ \theta>\bar{\theta}$ and $\forall\ \lambda$ sufficiently close to 1 with $\lambda\theta>\bar{\theta}$ and $\lambda\neq 1$,

$$s_1(\theta, \gamma, \bar{\theta})=s_1(\lambda\theta, \beta, \bar{\theta})$$

where, for $(\lambda_1, \lambda_2)\in\mathbb{R}^2_+$ with $\lambda_1/\lambda_2=\lambda$,

$$\begin{cases}\gamma_1=\beta_1+\ln\lambda_1=\tau_{\lambda,1}(\beta)\\ \gamma_2=\beta_2+\ln\lambda_2=\tau_{\lambda,2}(\beta)\end{cases}. \tag{10}$$

Hence, Assumption 6 holds. Following Grandmont (1992), we introduce the requirement of a "flat" parameter distribution, i.e. a small value of $\max_{\tau=1,2}\int|\partial_{\beta_\tau}\eta(\beta)|\,d\beta$. This degree of flatness of the parameter distribution is directly connected with our coefficient of sensitivity $h(\upsilon_{\bar{\theta}})$. More precisely, it is shown in the Appendix that

$$h(\upsilon_{\bar{\theta}})\leqslant 2\max_{\tau=1,2}\int|\partial_{\beta_\tau}\eta(\beta)|\,d\beta. \tag{11}$$

Hence, a high degree of flatness of the parameter distribution generates a low degree of sensitivity of the distribution of current budget shares $\upsilon_{\theta,\bar{\theta}}$ to changes of θ. In other words, Assumption 7 holds as long as

$$\sum_{r=1}^{2}\int_B|\partial_{\beta_\tau}\eta(\beta)|\,d\beta$$

is small.† Note that in contrast to Grandmont (1992) households diverge in their initial endowments. It is important to observe that the above requirements do not impose any *a priori* severe restrictions on the set of intertemporal characteristics. This allows,

† In fact we even get that $\sup_\theta\int_I|\partial_\lambda\rho_{\lambda\theta,\bar{\theta}}(z)|_{\lambda=1}|\,dz\leqslant 2\max_{\tau=1,2}\int|\partial_{\beta_\tau}\eta(\beta)|\,d\beta$ and Assumptions 5 and 6 hold for any θ, which is more than required.

in particular, for subpopulations where all old households possess a high degree of relative risk aversion. Effectively, for a separable utility function of the generic household with the relative risk aversion $R_2(c_2)$ $\forall\, c_2>0$, household β possesses the following degree of relative risk aversion, $\forall\, c_2>0$:

$$R_2(c_2, \beta) = R_2\left(\frac{c_2}{e^{\beta_2}}\right).$$

This implies that if $R_2(c_2)>1$, $\forall\, c_2>0$, then $\forall\, \beta\in B$, $R_2(c_2, \beta)>1$, $\forall\, c_2>0$.

It is important to point out that what is required is a sufficient heterogeneity of the budget share functions *in terms of their reaction to a change in* θ if households react to such a change. A degenerate case is obtained when the generic household of a given subpopulation is described by a Cobb–Douglas intertemporal utility function, i.e. $U(c_1, c_2) = a \ln c_1 + (1-a) \ln c_2$ with $0 \leqslant a \leqslant 1$. In this case all households of the subpopulation are described by the same budget share function, $\forall\, \beta\in B$, $s_1(\theta, \beta) = a$, which is independent of θ. Trivially, $h(\upsilon_{\bar\theta}) = 0$.

4. Structural properties of market demand

The purpose of this section is to assess the extent to which market demand is "well behaved" as a function of the degree of sensitivity of the distribution to movements in θ. To implement this program, we shall design approximate quantitative evaluations of the derivative of market demand with respect to θ, which depend on the degrees of sensitivity of the distributions $\upsilon_{\theta,\bar\theta}$ to a change in θ. The following lemma shows that a small degree of sensitivity $h(\upsilon_{\bar\theta})$ implies a small derivative of conditional budget share functions (for current consumption). This provides the basis for the following discussion.

LEMMA 1: *under Assumptions 1 to 6. The function* $\bar{s}_1$: $(\bar\theta, +\infty)\to[0, 1]$ *defined by*

$$\bar{s}_1(\theta, \mu/\bar\theta) = \int_{\mathscr{B}} s_1(\theta, \beta, \bar\theta)\, d\mu/\bar\theta$$

has derivatives such that

$$\theta\, |\, \partial_\theta \bar{s}_1(\theta, \bar\theta)\, | \leqslant \frac{h(\upsilon_{\bar\theta})}{2}.$$

PROOF: under Assumption 5 $\bar{s}_1(\theta, \bar{\theta})$ has partial derivatives with respect to θ, $\forall\ \theta > \bar{\theta}$. Furthermore, $\forall\ \theta > \bar{\theta}$

$$\left| \theta \partial_\theta \int_{\mathscr{B}} s_1(\theta, \beta, \bar{\theta}) d\mu/\bar{\theta} \right| = \left| \partial_\lambda \int_{\mathscr{B}} s_1(\theta, \beta, \bar{\theta}) d\mu/\bar{\theta} \Big|_{\lambda=1} \right|$$
$$= \left| \partial_\lambda \int_I s \rho_{\lambda\theta,\bar{\theta}}(s) ds \Big|_{\lambda=1} \right|$$
$$= \left| \int_I s \partial_\lambda \rho_{\lambda\theta,\bar{\theta}}(s) \Big|_{\lambda=1} ds \right|$$
$$\leqslant \frac{1}{2} \int_I \mid \partial_\lambda \rho_{\lambda\theta,\bar{\theta}}(s) \mid_{\lambda=1} \mid ds = \frac{h(\upsilon_{\bar{\theta}})}{2}.$$

The last inequality follows from the fact that $s \in I \subset [0, 1]$ and from $\int_I \partial_\lambda \rho_{\lambda\theta,\bar{\theta}}(s) ds = 0$.

From this lemma, one easily deduces that:

$$\left| \partial_\theta \bar{z}_1(\theta, \mu/\bar{\theta}) + \int_{\mathscr{B}} \frac{\omega_2(\beta, \bar{\theta})}{\theta^2} s_1(\theta, \beta, \bar{\theta}) d\mu/\bar{\theta} \right| \leqslant \frac{(\theta \tilde{\omega}_1 + \tilde{\omega}_2) h(\upsilon_{\bar{\theta}})}{2\theta}. \tag{12}$$

An important feature of this result is that it is "additive" in the sense that if one puts together different $\bar{\theta}$, then we may set the bound for the mixture as the mean of the elementary bounds. For a given θ_0 denote $\Theta^+(\theta_0) = \{\bar{\theta} \in \Theta \mid \bar{\theta} > \theta_0\}$ and $\Theta^-(\theta_0) = \{\bar{\theta} \in \Theta \mid \bar{\theta} < \theta_0\}$. We have $\forall\ \bar{\theta} \in \Theta^+(\theta_0)$, $\forall\ \beta \in \mathscr{B}$, $z_1(\theta_0, \beta, \bar{\theta}) = 0$ and $\partial_\theta z_1(\theta_0, \beta, \bar{\theta}) = 0$. Furthermore, $\mu^{\bar{\theta}}(\theta_0) = 0$, i.e. the set of household types where $z_1(\theta_0, \beta, \bar{\theta})$ has no derivative with respect to θ at θ_0 is of measure zero. Hence, it is not difficult to check that under Assumptions 1 to 6, market excess demand has partial derivatives with respect to θ at θ_0. Furthermore, the bounds for these partial derivatives can be obtained as in equation (12) by replacing $h(\upsilon_{\bar{\theta}})$ by $h(\upsilon)$, where $h(\upsilon) = \int_\Theta h(\upsilon_{\bar{\theta}}) d\mu^{\bar{\theta}}$. Denote $\mathscr{K}(\theta_0) = \int_{\mathscr{B} \times \Theta^-(\theta_0)} \omega_2(\beta, \bar{\theta}) s_1(\theta_0, \beta, \bar{\theta}) d\mu$ and $\bar{\theta}_{\inf} = \inf_{\bar{\theta} \in \Theta} \bar{\theta}$.

PROPOSITION 2: *under Assumptions 1 to 7 market excess demand* $Z_1(\theta, \mu)$ *has partial derivatives with respect to* θ *such that* $\forall\ \theta \leqslant \bar{\theta}_{\inf}$, $\partial_\theta Z_1(\theta, \mu) = 0$ *and* $\forall\ \theta > \bar{\theta}_{\inf}$,

$$\left| \partial_\theta Z_1(\theta_0, \mu) + \frac{K(\theta_0)}{\theta_0^2} \right| \leqslant \frac{(\theta_0 \tilde{\omega}_1 + \tilde{\omega}_2) h(\upsilon)}{2\theta_0^2}. \tag{13}$$

Let us introduce the following desirability assumption:

ASSUMPTION 8: $\forall\ \theta>\bar{\theta}_{\text{inf}},\ K(\theta)>\varepsilon$.

Then, one easily deduces the following result.

PROPOSITION 3: *under Assumptions 1 to 8 and the assumption that* $\forall\ \theta>\bar{\theta}_{\text{inf}},\ K(\theta)<+\infty$,

- *(i)* $\lim_{\theta\to+\infty} \partial_\theta Z_1(\theta, \mu)=0$.
- *(ii) Let* $\theta(h(\upsilon))=\sup\{\theta>\bar{\theta}_{\text{inf}} \mid (2\varepsilon/(\theta\tilde{\omega}_1+\tilde{\omega}_2))>h(\upsilon)\}$. For $h(\upsilon)<(2\varepsilon/(\theta_{\text{inf}}\tilde{\omega}_1+\tilde{\omega}_2))$ *we have* $\theta(h(\upsilon))>\bar{\theta}_{\text{inf}}$ *and* $\forall\ \theta\in(\bar{\theta}_{\text{inf}}, \theta(h(\upsilon)))=I(h(\upsilon))$, $\partial_\theta Z_1(\theta, \mu)<0$, *where in addition* $\theta(h(\upsilon))\cdot 1$ *when* $h(\upsilon)<(2\varepsilon/(\tilde{\omega}_1+\tilde{\omega}_2))$.
- *(iii) The open interval* $I(h(\upsilon))$ can be made to include any a priori *given upper closed interval in* $(\bar{\theta}_{\text{inf}}, +\infty)$ *by making* $h(\upsilon)$ *small enough, i.e.* $\lim_{h(\upsilon)\to 0} \theta(h(\upsilon))=+\infty$.

Hence, reducing the degree of sensitivity $h(\upsilon)$ has a structuring impact on market demand, by making the mean budget share function sufficiently independent of the real interest rate, so that one gets a strictly decreasing function Z_1 of θ on $I(h(\upsilon))$. In addition, the range of the interest rate $\theta-1$, for which Z_1 is strictly decreasing becomes larger and larger as $h(\upsilon)$ becomes smaller. It is important to observe that this is obtained for an individual rationality limited to Assumption 1. This is obtained, in particular, if one builds a population starting with a household that possesses "bad characteristics" such as an increasing demand function z_1 of θ, with $R_2(c_2)>1,\ \forall\ c_2>0$, and introducing heterogenous households following the example of subsection 3.2.1 by taking this household as the generic household. One can then assume that households diverge in their preferences and initial endowments so that the degree of sensitivity $h(\upsilon(\bar{\theta}))$ remains sufficiently small. Following this procedure, we get an example where all our assumptions hold; hence, the market demand $Z_1(\cdot)$ is monotone, while in each $\bar{\theta}$-subpopulation all old households possess a high degree of relative risk aversion, $\forall\ \beta,\ R_2(c_2, \beta)>1,\ \forall\ c_2>0$. This has to be confronted with the results of the representative agent model, where the monotonicity of $Z_1(\cdot)$ is explained by a low degree of relative risk aversion of the representative old household. It is important to observe that the monotonicity of current market demand would not have been ensured by the distributional requirements of Kneip (1995) when applied to an economy with dated commodities with price dependence of income. The reason for that is that the price dependence of income gives rise to an additive demand variation after a change in the real interest rate. Furthermore, Kneip's distributional requirements do not restrict this additive effect in a determinant way to allow for any conclusion about the global impact on market demand of a change in θ.

An immediate implication of Proposition 3 is that the monotonicity of Z_2, which is traditionally deduced at the household level from utility maximization, is now deduced at the aggregate level from our distributional requirements. Indeed, from household budget identities, it follows that

$$\partial_\theta Z_2(\theta) \equiv -Z_1(\theta) - \theta\partial_\theta Z_1(\theta) \quad \forall\ \theta > \bar{\theta}_{\text{inf}}. \tag{14}$$

Thus, under the assumptions of Proposition 3, we have $\partial_\theta Z_2(\theta) > 0$, $\forall\ \theta \in I(h(\upsilon))$. Hence, our distributional requirements ensure the gross substitutability between current and future consumption. At the limit market demand for current and future consumption is close to being of the Cobb–Douglas type.

5. An application : the non-emergence of endogenous business cycles

In this section, we give an important economic implication of our distributional requirements that underlines the structuring impact of heterogeneity. We turn to the dynamics of a sequence of temporary competitive equilibria with perfect foresight. Equilibrium at date t for the good and for money is given by

$$Z_1\left(\frac{p_t}{p^e_{t+1}}, \mu\right) + \frac{\bar{m}}{p_t} = 0 \tag{15}$$

$$M^d(p_t, p^e_{t+1}, \mu) = \bar{m} \tag{16}$$

in which p_t is the current price and p^e_{t+1} the price expected to prevail at the next date by all households. Under perfect foresight, as old household mean demand is equivalent to what they planned to do when young, the system can be rewritten as,

$$Z_1(\theta_t, \mu) + Z_2(\theta_{t-1}, \mu) = 0 \tag{17}$$

$$p_{t+1} Z_2(\theta_t, \mu) = \bar{m} \tag{18}$$

where $\theta_t = (p_t / p_{t+1}) > \bar{\theta}_{\text{inf}}$ for all t, since $Z_2(\theta_t, \mu) = (\bar{m}/p_{t+1}) > 0$. The well-known property of this system is that it dichotomizes. Hence, we can focus on the dynamics of θ_t. Equation (17) induces well-defined (but fictitious) backward perfect foresight (b.p.f.) dynamics. Let us introduce the following requirement:

ASSUMPTION 9: *the restriction of Z_2 to the interval* $[\bar{\theta}_{\inf}, +\infty)$ *is increasing and maps that interval onto* $[0, +\infty)$.†

The function Z_2 has therefore an inverse and thus equation (17) gives rise to the very simple difference equation,

$$\theta_{t-1} = Z_2^{-1}(-Z_1(\theta_t, \mu), \mu) \equiv G(\theta_t). \tag{19}$$

The b.p.f. map G on real interest rates is continuously differentiable and maps $[\bar{\theta}_{\inf}, +\infty)$ onto $[\bar{\theta}_{\inf}, \bar{\bar{\theta}})$ with $\bar{\bar{\theta}} = Z_2^{-1}(\omega_1)$. Then a periodic equilibrium with perfect foresight $(\theta_1^*, \ldots, \theta_k^*)$ can be related to the cycle $(\theta_k^*, \ldots, \theta_1^*)$ (in which time has been reversed) of the b.p.f. map G such that $\theta_i^* > \bar{\theta}_{\inf}$ for all i.

A trajectory that appears as a plausible solution of the dynamics with perfect foresight is a periodic orbit, since, intuitively for a repetitive enough environment, we would expect households to have perfect foresight. Finding a periodic monetary equilibrium is equivalent to finding a periodic orbit of the b.p.f. dynamics $\theta_{t-1} = G(\theta_t)$ with $\theta_t > \bar{\theta}_{\inf}$, $\forall\, t$. The existence and multiplicity of such cycles with period $k \geqslant 2$ have been proved, for example, by Grandmont (1985) in a representative agent version of the model under consideration. These cycles emerge whenever there is an important conflict between the intertemporal substitution effect and the wealth effect that result from a variation of the real interest rate. This conflict is the outcome of a high risk aversion of the representative old household, in comparison with the risk aversion of the representative young household. However, once heterogeneity of household characteristics is allowed, we know from Proposition 3 that whatever type of individual risk aversion, the conflict between the intertemporal effect and the wealth effect is solved in the aggregate on $I(h(\upsilon))$.‡ An interesting implication on the dynamics is that periodic monetary equilibria can no longer emerge for any initial interest rate such that $\theta_0 \in I(h(\upsilon))$, where we recall that $I(h(\upsilon))$ eventually covers the whole set $(\bar{\theta}_{\inf}, +\infty)$ as $h(\upsilon)$ converges to 0.

The map G can be rewritten,

$$G(\theta) = Z_2^{-1}(-Z_1(\theta, \mu), \mu) = Z_2^{-1}\left(\frac{Z_2(\theta, \mu)}{\theta}, \mu\right).$$

† This property is ensured, for example, if household demand is the outcome of a traditional utility maximization program. It is also induced by our distributional requirements when pushed to the limit, i.e. for $h(\upsilon) = 0$.

‡ This is obtained, in particular, in a population where all households possess large degrees of relative risk aversion.

The properties of G are given by in the following Lemma (see Grandmont, 1985).

LEMMA 4: *assume 1, 3 and 9. Then G maps the interval* $[\bar{\theta}_{\inf}, +\infty)$ *into* $[\bar{\theta}_{\inf}, \tilde{\theta})$ *in which* $\tilde{\theta}=Z_2^{-1}(\omega_1, \mu)$, *and is continuously differentiable. Moreover, (i)* $G(\bar{\theta}_{\inf})=\bar{\theta}_{\inf}$, $G(1)=1$, $G(\theta)>\theta$ *whenever* $\bar{\theta}_{\inf}<\theta<1$ *and* $G(\theta)<\theta$ *whenever* $\theta>1$*; (ii)* $G'(\theta)=1/\bar{\theta}_{\inf}>1$ *and* $G'(1)<1$.

THEOREM 5: *under the assumptions of Proposition 3 and Assumption 9 and for a degree of sensitivity* $h(\upsilon)$ *small enough, i.e.* $h(\upsilon)<(2\varepsilon/(\tilde{\omega}_1+\tilde{\omega}_2))$, *the map G has no cycles (apart from the stationary equilibrium, cycle of period one) for any initial value* $\theta_0\in I(h(\upsilon))$. *Moreover, the unique stationary monetary equilibrium is G-stable on* $I(h(\upsilon))$, *i.e.* $\lim_{j\to+\infty} G^j(\theta)=1$ *for every* $\theta\in I(h(\upsilon))$.

PROOF : we have

$$G'(\theta)=-\frac{\partial_\theta Z_1(\theta, \mu)}{\partial_\theta Z_2(\theta, \mu)}$$

where for any $\theta>\bar{\theta}_{\inf}$, $\partial_\theta Z_2(\theta, \mu)>0$ and $\forall\ \theta\in I(h(\upsilon))$, $\partial_\theta Z_1(\theta, \mu)<0$. Thus, $\forall\ \theta\in \mathrm{I}(\mathrm{h}(\upsilon))$, $G'(\theta)>0$. For $h(\upsilon)<(2\varepsilon/(\tilde{\omega}_1+\tilde{\omega}_2))$, we know from Proposition 3 that $\theta(h(\upsilon))>1$. Thus, from Lemma 4, $\forall\ \theta_0\neq 1$ and $\theta_0\neq\bar{\theta}_{\inf}$, if $\theta_0<1$, $\theta_0<G(\theta_0)<G(1)=1$, and if $1<\theta_0<\theta(h(\upsilon))$, $\theta_0>G(\theta_0)>G(1)=1$. By applying the same argument to the iterates $G^j(\theta_0)$ one finds that $G^j(\theta_0)$ converges monotonically to 1 when j tends to $+\infty$.

Once the degree of sensitivity of the distribution is fixed, the conclusion of Theorem 5 is valid on a compact set of interest rates. Thus, it does not say anything about initial values of the interest rate such that $\theta_0>\theta(h(\upsilon))$. Therefore, it might still appear that if the initial value of the interest rate is very high the economy will evolve along a periodic cycle where the interest rate remains at very high values. However, the result of Theorem 5 can be extended to any value of θ_0 by requiring that all old households have a high risk aversion for high values of θ. This assumption when imposed for every level of the interest rate (in a representative agent model) is well known in the literature to guarantee that the map G has no cycles with a period $k\geqslant 2$. The advantage of our approach is that, under our distributional requirement, it is enough to introduce this restriction for high values of the interest rate, which seems more legitimate. Old households probably feel more insecure when the interest rate reaches high values.

THEOREM 6: *under the assumptions of Theorem 5 and if* $\forall\ \alpha\in\mathscr{A}$, $\sup_{\theta>\theta(h(\upsilon))} R_2(c_2(\theta), \alpha)\leqslant 1$, *then for* $h(\upsilon)<(2\varepsilon/(\tilde{\omega}_1+\tilde{\omega}_2))$ *the conclusions of Theorem 5 are valid for any initial value* $\theta_0>\bar{\theta}_{\inf}$.

PROOF : the requirement that $\forall\ \alpha\in\mathscr{A}$, $\sup_{\theta>\theta(h(\upsilon))} R_2(c_2(\theta),\ \alpha)\leqslant 1$ is a necessary and sufficient condition to have that for every household β in the $\bar{\theta}$-subpopulation, $\partial_\theta z_1(\theta,\ \beta,\ \bar{\theta})<0,\ \forall\ \theta>\bar{\theta}$. The result $\forall\ \alpha=(\beta,\ \bar{\theta})\in\mathscr{A},\ \partial_\theta z_1(\theta,\ \beta,\ \bar{\theta})<0,\ \forall\ \theta>\bar{\theta}_{\inf}$ is obtained if and only if

$$R_2(\omega_2+z_2(\theta,\ \alpha),\ \alpha)<\frac{\omega_2+z_2(\theta,\ \alpha)}{z_2(\theta)}\quad \forall\ \alpha\in\mathscr{A}$$

This can be verified immediately by looking at the expression of $\partial_\theta z_1(\theta,\ \alpha)$ in equation (3). Then $\forall\ \alpha\in\mathscr{A}$, if $R(c_2(\theta))\leqslant 1$ for all $\theta>\theta(h(\upsilon))$, the left-hand side of the above inequality never exceeds 1, while the right-hand side is always greater than 1 whenever $\theta>\theta(h(\upsilon))$, which shows this first result. This implies therefore that, under the assumptions of Lemma 3, market demand has partial derivatives with respect to θ and $\partial_\theta Z_1(\theta,\ \mu)<0,\ \forall\ \theta>\bar{\theta}_{\inf}$. Hence, following the same line of argument as in the proof of Theorem 5 we find that the same conclusions hold for all $\theta_0>\bar{\theta}_{\inf}$.

6. Concluding remarks

The monotonicity of current market demand has been deduced from restrictions on the distribution of household intertemporal characteristics in a set-up where the price-dependence of income is allowed. Assumptions on the distribution of household intertemporal characteristics have been defined in a framework directly tractable by macroeconomists. Furthermore, we gave an interesting implication of the macroeconomic regularity ensured by our distributional assumptions which solve the indeterminacy of long-run equilibria. The model that we have analysed in this paper is too rudimentary to draw general conclusions about the link between the heterogeneity of household characteristics and the emergence of endogenous cycles.† However, these conclusions were beyond the scope of this paper. Our hope is that this paper could contribute to a more systematic use of distributional assumptions in macroeconomic models.

The next step in studying the impact of heterogeneity on the modelization of market demand could be to introduce heterogeneous price expectations. If the requirement of perfect foresight is perfectly legitimate in a repetitive environment, in particular at a stationary state or along any periodic orbit, it is more severe outside such trajectories. Price expectations could be formalized

† For this purpose, a more general study of the impact of distributional requirements on the prevalence of business cycles is called for. In particular, one should study whether distributional requirements could rule out non-linearities arising from particular features of the production technology.

as functions of current and past prices. Again, heterogeneity of individual functions could be used to give an account of structural properties of the aggregate price expectation function, when the set of feasible individual expectation functions remains as wide as possible. Depending on these structural properties, some properties of the market demand function could then be established as well as some relationship between stability with learning and stability of the perfect foresight dynamics. First attempts in this direction have been made by Maret (1995*a,b*). In Maret (1995*a*) the monotonicity of market demand is deduced from distributional requirements of household demand functions combined with distributional requirements of household price expectation rules. In Maret (1995*b*) a specific heterogeneity of the price expectation functions between households is shown to reverse the stability property of the unique monetary stationary state. A deeper examination of these issues would be useful.

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Appendix

In this section we prove that equation (11) is true. We have first to point out that,

$$h(\upsilon_{\bar\theta}) = \sup_{\theta > \bar\theta} \int_I | \, \partial_\lambda \rho_{\lambda\theta,\bar\theta}(z) \, |_{\lambda=1} \, | \, dz$$

$$= \sup_{\theta > \bar\theta} \int_I | \, \chi_I(z) \partial_\lambda \rho_{\lambda\theta,\bar\theta}(z) \, |_{\lambda=1} \, | \, dz$$

where $\chi_I(\cdot)$ is the characteristic function of the interval I. Thus,

$$h(\upsilon_{\bar{\theta}}) = \sup_{\theta > \bar{\theta}} \int_C | \partial_\lambda (\chi_I(s_1(\lambda\theta, \beta))\eta(\beta)) | \, d\beta \Big|_{\lambda=1}$$

$$= \sup_{\theta > \bar{\theta}} \int_C | \partial_\lambda (\chi_I(s_1(\theta, \tau_\lambda(\beta)))\eta(\beta)) | \, d\beta \Big|_{\lambda=1}$$

$$= \sup_{\theta > \bar{\theta}} \int_{\tau_\lambda{}^{-1}(c) \cap C} | \partial_\lambda (\chi_I(s_1(\theta, \beta)(\eta(\tau_\lambda^{-1}(\beta))$$

$$\times \det(\partial_\beta \tau_\lambda^{-1}(\beta))) | \, d\beta \Big|_{\lambda=1}$$

$$= \sup_{\theta > \bar{\theta}} \int_C | \partial_\lambda (\eta(\tau_\lambda^{-1}(\beta)) \det(\partial_\beta \tau_\lambda^{-1}(\beta))) |_{\lambda=1} | \, d\beta$$

where τ_λ is defined by equation (10). Thus,

$$\det(\partial_\beta \tau_\lambda^{-1}(\beta)) = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$$

and

$$\left| \partial_\lambda (\eta(\tau_\lambda^{-1}(\beta))) \Big|_{\lambda=1} \right| = | \partial_{\beta_1} \eta(\beta) | .$$

Finally,

$$h(\upsilon_{\bar{\theta}}) = \int_C | \partial_{\beta_1} \eta(\beta) | \, d\beta \leqslant \int_C \sum_{r=1}^{2} | \partial_{\beta_\tau} \eta(\beta) | \, d\beta.$$