« Monetary hyperinflations, speculative hyperinflations and modelling the use of money »

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Monetary hyperinflations, speculative hyperinflations and modelling the use of money

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Abstract
The aim of this paper is to clarify the failure of the Cagan model with perfect foresight and to draw new axes for investigation of monetary hyperinflation analysis. Firstly, the paper evaluates the relevancy of the Cagan ad-hoc model with perfect foresight as a theoretical framework for investigating hyperinflation processes. We show that deficits can never generate monetary hyperinflations, confirming the results of Buiter (1987). The only hyperinflationary processes that can occur are speculative hyperinflations. Secondly, the paper assesses consistency of hyperinflationary paths with the optimizing behaviour of representative agents within two perfect foresight inflationary finance frameworks modelling the use of money as a medium of exchange. In the context of a money-in-the-utility framework, the results obtained in the Cagan ad-hoc model with perfect foresight are founded and confirmed. This implies restricting the use of the latter model only to speculative hyperinflations analysis. In the context of a transaction costs based model, we show that deficits can generate monetary hyperinflations. Moreover, speculative hyperinflations remain possible. This result is in sharp contrast to that of the money-in-the-utility framework and implies a demand for money different from the Cagan form.

JEL classification: E31, E41

1 Introduction

The modelling of hyperinflation starts with the seminal work of Cagan (1956) within the New Quantity Theory of Money. In his original contribution Cagan defines hyperinflation as a speeding up inflation process where the inflation rates exceed fifty percent monthly. He provides the explanation of hyperinflation as being the result of an excessive public budget deficit financed by money creation or seigniorage revenues. In this framework relying on the specification of an ad-hoc demand for money, hyperinflation is viewed as a pure monetary phenomenon where both inflation rates and money growth rates accelerate and explode. This model deals with monetary hyperinflation. However, Cagan original modelling with adaptive inflation expectations doesn’t succeed to produce monetary hyperinflations as monetary growth isn’t modelled yet. Evans and Yarrow (1981) and Bruno and Fischer (1987) complete Cagan’s model by modelling the money supply process through the government budget constraint. Nevertheless, Cagan’s model approach is seriously challenged with the introduction of rational expectations when Buiter (1987) shows that the model with perfect foresight is unable to produce any hyperinflation. The failure of the Cagan model to generate hyperinflations under rational expectations has created some troubles in the literature. Evans (1995), for instance, states a strict association between the correct running of the model and adaptive expectations. Blanchard and Fischer (1990) or Walsh (2003) present the Cagan model only under the adaptive expectations assumption.
Evans and Yarrow (1981) and Bernholz and Gersbach (1992) find out that the crucial condition for generating hyperinflation is that real money balances should not decrease more than inflation increases with high rates of inflation. As Bruno (1989) points out restoring the correct running of the model requires the inclusion of a sufficiently large friction in the adjustment of some nominal variable like expected inflation, money holdings or the exchange rate. Thus, assuming sufficiently slow adaptive expectations, as in Evans and Yarrow (1981) or Bruno and Fischer (1990), learning as in Marcet and Nicolini (2003), a crawling peg rule for the exchange rate as in Bruno (1989), or a sufficiently slow adaptive adjustment on the money market as in Kiguel (1989) can guarantee the occurrence of monetary hyperinflation in the Cagan model. However, even if one can find arguments in favour of the use of adaptive expectations during hyperinflationary episodes as Bruno and Fischer (1990) or Cukierman (1988) do for instance, it is hard to justify the persistent presence of behaviours involving either systematic forecast mistakes or maladjustments that are costly for the agents in a hyperinflationary context. The problem is that perfect foresight and instantaneous adjustment on the money market don’t allow Cagan model to highlight monetary hyperinflation.

The challenge of this problem is not only theoretical. There is a large empirical literature on hyperinflationary episodes from Cagan (1956), Sargent and Wallace (1973) or Sargent (1977) to Petrovic and Vujosevic (1996), Choudry (1998), Bogetic et al (1999), Petrovic and Mladenovic (2000), Slavova (2003), or Georgoutsos and Kouretas (2004) among the most recent works for instance. Most of these empirical investigations rely on the Cagan model with rational expectations and instantaneous money market clearing. This issue could cast doubt on hyperinflation empirical studies that have a priori adopted the Cagan framework with perfect foresight. Interestingly, in most hyperinflationary episodes studied in the literature Cagan model seems to characterize well the dynamics of hyperinflation.

The aim of this paper is two-fold. First, we explore the Cagan ad-hoc model with perfect foresight to assess its relevancy as a theoretical framework for investigating hyperinflation processes. Second, we resort to the more economically satisfactory case when the demand for money is derived from first principles. The aim is to assess the consistency of hyperinflationary paths with the optimizing behaviour of representative agents within a perfect foresight inflationary finance framework modelling the use of money as a medium of exchange. Therefore, the main objective of the paper is to clarify the failure of the Cagan model with perfect foresight and to draw new axes for investigation of monetary hyperinflation analysis.

We confirm the results of Buiter about the failure of the Cagan ad-hoc model with perfect foresight by showing that deficits can never generate monetary hyperinflations. However, we show that the Cagan ad-hoc model can be saved for the analysis of hyperinflation as it remains relevant to generate speculative hyperinflations or hyperinflationary bubbles, hyperinflations not driven by money growth fundamentals. Resorting to first principles in the context of an optimizing model with money-in-the-utility-function, this paper brings foundations to the Cagan ad-hoc model, founding the Cagan money demand and confirming that deficits can’t generate monetary hyperinflation. Moreover, we show that the only hyperinflation process that can be generated is speculative hyperinflation. In the context of an optimizing model with transaction costs, we show that deficits can generate monetary hyperinflation in a perfect foresight framework. This result is in sharp contrast to that of the money-in-the-utility framework in which Buiter results were founded and confirmed, and implies a demand for money different from the Cagan form. This suggests saving the Cagan model for the analysis of monetary hyperinflation with perfect foresight by removing the
Cagan money demand. Moreover, we show that speculative hyperinflation remains a strong possibility.

The main contribution of this paper is to provide a distinction of hyperinflationary processes between monetary hyperinflations and speculative hyperinflations within the Cagan ad-hoc model and within the first principles context of optimizing monetary models. This is something new since the optimizing monetary models literature, from Brock (1974, 1975, 1978), Kingston (1982), Obstfeld and Rogoff (1983, 1986) to Vazquez (1998), Barbosa and da Cunha (2003) or Gutierrez and Vazquez (2004) for instance, implicitly or explicitly always deals with speculative hyperinflations. This distinction may help to save the Cagan ad-hoc model with perfect foresight for the analysis of speculative hyperinflation and to provide new guidance for studying monetary hyperinflation within a perfect foresight framework derived from first principles.

The paper is organized as follows. Section 2 explores the failure of the Cagan ad-hoc model with perfect foresight. Section 3 resorts to the first principles in the context of a money-in-the-utility optimizing model. Section 4 resorts to an alternative way of modelling the role of money as a medium of exchange using a transaction costs model. Section 5 presents the conclusions.

2 The Cagan ad-hoc model paradox

The demand for real cash balances is the fundamental equation of the Cagan approach of modelling monetary hyperinflation. During hyperinflations real variables such as the real rate of interest and real output may be reasonably treated as constant since all the action involves money and prices. Therefore, since the demand for real cash balances depends on the nominal interest rate, it will depend only on the expected rate of inflation. The traditional approach builds upon an ad-hoc semi-logarithmic demand for real per-capita cash balances $m_i^d$, known as Cagan money demand:

\[
    m_i^d = e^{\gamma - \alpha \pi_e^i}.
\]

In this equation $\pi_e^i$ represents the expected rate of inflation, $\alpha$ and $\gamma$ are parameters. The role of constant $\gamma$ is assumed to describe the influence of real income and real interest rate, which are supposed to be constant in this framework. The positive constant $\alpha$, the semi-elasticity of money demand, describes the decreasing demand for real cash balances with respect to expected rate of inflation.

Rational expectations prevail implying, in this framework without uncertainty, perfect foresight:

\[
    \pi_e^i = \pi_i,
\]

Where $\pi_i$ is the actual rate of inflation.

Instant clearing prevails on the money market:
\[ m_t \equiv \frac{M_t}{P_t} = m^d_t. \]  

(3)

where \( M_t \) is per-capita money supply and \( P_t \) the general price level.

The process explaining the money supply completes the model. In this kind of inflationary finance model a constant per-capita share of government’s deficit \( d_t \) is assumed to be financed by issuing high-powered money that is through seigniorage revenues. Formally,

\[ d_t = \frac{M_t}{P_t}. \]  

(4)

As usual a dot on a variable denotes its time derivative. Denoting per-capita real cash balances by \( m \), equation (4) may be rewritten in the following way:

\[ d = \dot{m} + \pi m \]  

(4)’

Combining (1) (2) and (3) and omitting time index for simplicity leads to

\[ m = e^{\gamma - \alpha \pi} \]  

(5)

Extracting the expression for \( \pi \) from the latter equation and substituting it into (4)’ gives, after rearranging, the law of motion of real per-capita cash balances for this economy:

\[ \dot{m} = d + \alpha^{-1} (\log m - \gamma)m \]  

(6)

The dynamics of real money balances of this economy are described by the latter autonomous differential equation in \( m \) and can be conveniently represented in the phase diagram depicted in Figure 1.
The function given by (6), representing the law of motion of real par-capita money balances, is defined for \( m \in [0, +\infty] \) and its limits on boundaries are:

\[
\lim_{m \to 0} (\dot{m}) = d, \quad \lim_{m \to +\infty} (\dot{m}) = +\infty.
\]  

(7)

The derivative of the function represented by (6) with respect to \( m \) and its sign are given by:

\[
\frac{d\dot{m}}{dm} = \alpha^{-1} (1 - \gamma + \log m) \begin{cases} 
  > & \text{if } m = e^{\gamma - 1} \\
  < & \text{if } m < e^{\gamma - 1} \\
  < & \text{if } m > e^{\gamma - 1} 
\end{cases}
\]  

(8)

As the second derivative, \( \frac{d^2\dot{m}}{dm^2} = \frac{1}{\alpha m} \), is strictly positive, the curve \( \dot{m} \) is strictly convex with a minimum at \( m = e^{\gamma - 1} \). The size of the government deficit determines the relative position of the phase curve in the diagram: a higher \( d \) shifts the curve upwards. Then, the existence of steady states depends on the sign of the value of \( \dot{m} \) at its minimum \( m = e^{\gamma - 1} \)

\[
\dot{m}(e^{\gamma - 1}) \begin{cases} 
  < & \text{if } d < d^* \\
  > & \text{if } d > d^* 
\end{cases}
\]  

(9)

Based on these calculations monetary dynamics of this economy can be represented on Figure 1 which exhibits two well known key aspects of the Cagan ad-hoc model. First, dual steady states co-exist for any level of government deficit \( d \) below \( d^* \) (curve \( \dot{m}_L \)) and the absence of any steady state for any \( d \) above \( d^* \) (curve \( \dot{m}_H \)). Second, the existence of a high inflation trap when the government deficit is not excessive (\( d < d^* \)) since the high-inflation steady state, \( m_B \), is stable and the low-inflation steady state, \( m_A \), is unstable.

These results come from the existence of a Laffer curve for the steady state seigniorage revenues. In the case of the Cagan demand for money the curve representing seigniorage revenues is a bell-shaped curve. That’s why it is called a “Laffer curve” for the seigniorage revenues. Therefore, there is a maximal level of real deficit, \( \overline{d} \), consistent with a stationary inflation rate, on one hand, and the possibility for the government to finance the same real budget deficit with two different stationary inflation rates, one called “low” and the other “high”, as long as the real budget deficit remains lower than \( \overline{d} \), on the other hand. For the economy it means that there is the possibility of two different steady-state equilibria, one of “low inflation” and the other of “high inflation”. There is duality of steady-state equilibrium as long as the budget deficit remains lower than \( \overline{d} \).

Assessing \( \overline{d} \) can be done in the following way. Calling \( R \) the steady-state amount of seigniorage revenues, the value of \( \overline{d} \) is the maximal value of \( R \). This value is given by:
\[
\dd = \max \{ R(\pi) \} = R^* = \max \{ \pi \cdot e^{r-\alpha \pi} \} = \alpha^{-1} e^{r-1}, 
\]

and is exactly the value \( d^* \) showed previously: \( \dd = d^* \). The value of the stationary inflation rate corresponding to this maximum of seigniorage revenues is then:

\[
\pi^* = \arg \max_{\pi} \{ \pi e^{r-\alpha \pi} \} = \alpha^{-1}. 
\]

Figure 2 shows the “Laffer curve” for the stationary seigniorage revenues. The issue of duality of the steady-state equilibrium clearly appears and so does the issue of financing a real budget deficit higher than \( d^* \) like \( d_1 \) for instance. As can be seen on figure 2, there isn’t any possibility to finance such a higher deficit with a stationary inflation rate. Therefore, this leaves the place for a non stationary way. In his seminal paper Cagan (1956) proposed the explanation of monetary hyperinflation as being the result of an excessive public deficit financed through money creation.

**Proposition 1:** The Cagan ad-hoc model with perfect foresight is unable to neither produce nor explain any monetary hyperinflation.

**Proof:** The curve \( \dot{m}_2 \) depicted in Figure 1 represents the dynamics of real money balances when \( d > d^* \). As the value of \( \dot{m}_2 \) at its minimum \( m = e^{r-1} \) is strictly positive, according to the variations of the function representing \( \dot{m}_2 \) and its limits on boundaries it is clearly established that the curve \( \dot{m}_2 \) is entirely above the horizontal axis. Then, \( \dot{m}_2 > 0 \) for all \( m > 0 \): the paths followed by the economy when the public deficit is excessive are paths of continuously increasing real per-capita money balances associated with continuously decreasing inflation rates according to (1). Therefore, there aren’t any steady states and an explosive non stationary process takes place. However, unlike the standard explanation brought by Cagan (1956) in its seminal paper, this explosive process caused by an excessive public deficit is not a monetary hyperinflation but a “hyperdeflation” as shown by Buiter (1987) in a similar framework with linear money demand and perfect foresight. This is the “hyperdeflation paradox”.
It should be stressed that this model is unable to neither produce nor explain any monetary hyperinflation. In the case of a relatively small budget deficit \((d < d^*)\), the trajectories to the left of the low inflation steady state leading to the high inflation steady state should not be considered as hyperinflation. Along these paths the inflation rates are rising but are converging to a stable one and there is no place for any explosive inflation paths. As Buiter (1987) pointed out describing these paths as hyperinflation is “akin to describing a mild summer breeze as a hurricane”.

To show this we use the dynamics of real money balances given by (6) to calculate the dynamics of the rate of inflation. Substituting the value of \(\pi\) from (5) into (6) leads to the law of motion of inflation rate:

\[
\frac{\dot{\pi}}{\pi} = \alpha^{-1} \left(1 - \frac{d}{\pi e^{\gamma - \alpha \pi}} \right)
\]  

(12)

The variations of the curve describing \(\frac{\dot{\pi}}{\pi}\) are given by

\[
\frac{d\left(\frac{\dot{\pi}}{\pi}\right)}{d\pi} = \frac{de^{\gamma - \alpha \pi}}{\left(\pi e^{\gamma - \alpha \pi}\right)^2} \left(1 - \alpha \pi\right) > 0 \quad \text{if} \quad \pi > \alpha^{-1}
\]

(13)

As the value of \(\frac{\dot{\pi}}{\pi}\) at its maximum in \(\pi = \alpha^{-1}\) is positive \((\alpha^{-1} \frac{d^* - d}{d^*})\) for small deficit case, it is easy to see that the transition paths from the low inflation steady state to the high inflation steady state start with an accelerating inflation and, after having reached \(\pi = \alpha^{-1}\), end up with decelerating inflation before reaching a stationary inflation rate. Therefore, no monetary hyperinflation occurs in the model. This completes the proof.

**Proposition 2:** The only hyperinflation process that the Cagan ad-hoc model with perfect foresight may produce is speculative hyperinflation.

**Proof:** Considering the case of a balanced public budget that is a zero government deficit \((d = 0)\) leads to the following degenerated law of motion for real money balances:

\[
\dot{m} = \alpha^{-1} \left(\log m - \gamma\right) m.
\]  

(6')

The curve \(\dot{m}\) representing the latter monetary dynamics is depicted in Figure 1. It is easily obtained from the properties of the law of motion (6). There is a unique unstable steady state \((m = e^\gamma)\). All the paths starting to the left of the unique steady state lead to a zero level of real per-capita cash balances and an infinite rate of inflation and price level. According to (5) and (12), along these paths the inflation rate is continuously increasing at a constant rate without any money supply growth since the money stock is constant (see equation (4)). So hyperinflation processes may develop without any fundamentals: these paths are rational hyperinflationary bubbles or speculative hyperinflations. This completes the proof.
This section established that the ad-hoc model with perfect foresight is seriously flawed for
the study of monetary hyperinflation according to the “hyperdeflation paradox”, and that it
might be used only to investigate speculative hyperinflation. These issues give rise to a kind
of paradox since the Cagan model has been originally designed to produce and explain
monetary hyperinflation and not speculative hyperinflation. The Cagan model is an ad-hoc
model because it is built upon an ad-hoc demand for real per-capita cash balances. In order to
tell more about these problems it is necessary to look at the economically more satisfactory
case when the demand for money is derived from first principles. Therefore, the next two
sections develop two standard optimizing monetary frameworks to model the use of money as
a medium of exchange in a context where money growth relies on the government need for
seigniorage revenues.

3 Money in the utility function and Cagan money demand

The two optimizing monetary models considered in this paper assume a continuous time
model where the economy consists of a large number of identical infinitely-lived forward
looking households endowed with perfect foresight. Population is constant and its size is
normalized to unity for convenience. There is no uncertainty. Each household has a non-
produced constant endowment $y > 0$ of the non-storable consumption good per unit of time.

In the money-in-the-utility-function model the role of money as a medium of exchange is
assumed to be captured by introducing real money balances into the household utility

The optimization problem that the representative household faces at time 0 is given by

$$\max_{c,m} \int_0^\infty U(c_t, m_t) e^{-\delta t} dt,$$  \hspace{1cm} (14)

subject to

$$c_t + m_t = y - \tau - \pi m_t,$$  \hspace{1cm} (15)

where $U(c, m)$ is known as the instantaneous utility function assumed to be increasing and
concave in both $c$ and $m$, $c_t$ is the household’s consumption at time $t$, $m_t$ is his holdings of real
monetary balances, $\delta$ is the rate of time preference, $\pi$, the inflation rate, and $\tau$ is a lump-sum
tax assumed to be constant.

The first-order conditions for this problem are made up of the following respective Euler
equation and transversality condition:

$$U_{cc}(c_t, m_t)c_t + U_{cm}(c_t, m_t)m_t = -U_{mc}(c_t, m_t) + U_c(c_t, m_t)(\pi + \delta),$$  \hspace{1cm} (16)

$$\lim_{t \to \infty} \left[U_c(c_t, m_t) e^{-\delta t} m_t \right] = 0.$$  \hspace{1cm} (17)

The equilibrium condition in the goods market is
\[ y = c_i + g \], \quad (18)\]

where \( g \) is the constant government expenditure. We assume that the utility function \( U \) is additive separable in \( c \) and \( m \) such that

\[ U(c_i, m_i) = u(c_i) + v(m_i), \quad (19) \]

where \( u \) is increasing and concave in \( c \), and \( v \) is increasing and concave in \( m \). Using (18) and (19), and normalizing the constant value of \( u_c(y - g) \) to unity for convenience, the first-order conditions can be re-written as

\[ v'(m_i) = (\pi_c + \delta), \quad (16)' \]

\[ \lim_{t \to \infty} \left[ e^{-\delta t} m_i \right] = 0. \quad (17)' \]

Substituting the value of \( \pi \) extracted from (5) into equation (16)' and then integrating provides microeconomic foundations of the Cagan money demand since we get, similarly as Kingston (1982) or Calvo and Leiderman (1992)

\[ v(m) = \alpha^{-1}(1 + \gamma + \alpha \delta - \log m)m \quad \text{for all} \quad 0 < m < e^{\gamma + \alpha \delta}. \quad (20) \]

The latter expression of \( v \) shows that Cagan money demand can be derived from the representative household preferences.

We assume that the constant per capita government budget deficit \( d \) is financed by issuing high-powered money

\[ d = g - \tau = \frac{M_t}{P_t} = \frac{\dot{M}_t}{M_t} m_i. \quad (21) \]

Using the latter expression, the definition of real per-capita money balances and the value of \( \pi \) extracted from Euler equation (16)' we have

\[ \dot{m} = \left( \frac{\dot{M}_t}{M_t} - \pi \right) m = d - (v'(m) - \delta)m \quad (22) \]

Equation (22) provides a complete characterization of real per-capita money balances dynamics. Using the expression of \( v \) given by (20) founding the Cagan money demand we get

\[ \dot{m} = d + \alpha^{-1}(\log m - \gamma)m \quad \text{for all} \quad 0 < m < e^{\gamma + \alpha \delta}. \quad (23) \]

which provides exactly the same real money balances dynamic properties as in the Cagan ad-hoc model with perfect foresight presented in section 2. However, the crucial difference is that real per-capita money balances dynamics given by (23) are derived from the optimizing behaviour of the representative household described by (14) and (15) and not from an ad-hoc
framework. Therefore, we may use conveniently Figure 1 for representing the different dynamic properties of the autonomous differential equation (23).

**Proposition 3:** (i) All hyperdeflationary paths corresponding to the absence of steady states and an excessive government budget deficit ($d > d^*$) can be ruled out. (ii) All hyperdeflationary paths starting to the right of the low inflation steady state in the case of a small government deficit ($0 \leq d < d^*$) can be ruled out.

**Proof:** (i) As the qualitative dynamic properties of differential equation (23) are the same as those of the one obtained within the Cagan ad-hoc model, we have shown in section 2 that an excessive government budget deficit ($d > d^*$) will shift the curve describing $\hat{m}$ above the horizontal axis ($\hat{m}_1$). Along the paths described by curve $\hat{m}_1$, the real per-capita money balances are continuously increasing without boundaries (“hyperdeflation” paradox). These paths can be easily ruled out because they violate the transversality condition (17)'. Along these hyperdeflationary paths $\lim_{t \to \infty} e^{-\delta t} m(t) > 0$ because, whereas as $t \to \infty$, $e^{-\delta t} \to 0$ at the decreasing rate of $\delta$, $m(t) \to \infty$ at an ever increasing rate $(\lim_{t \to \infty} \frac{m(t)}{m(t)} = \infty)$.

This feature is sufficient to preclude these hyperdeflationary paths because it is not optimal for households to follow them.

(ii) The continuum of paths starting to the right of the low-inflation steady state in the case of a small government budget deficit ($0 \leq d < d^*$) are hyperdeflationary paths since $0 \hat{m} > \hat{m}$ for all $\alpha > m_A$. Therefore, according to the transversality condition (17)' they can be ruled out. This completes the proof.

**Proposition 4:** In this money-in-the-utility framework:

(i) no monetary hyperinflation can be generated,

(ii) the only hyperinflationary paths that can be generated are speculative hyperinflations.

**Proof:** As the qualitative dynamic properties of differential equation (23) are the same as those of the one obtained within the Cagan ad-hoc model, Proposition 1 proves (i) and Proposition 2 proves (ii). It should be stressed that hyperinflationary bubbles paths starting for any $m < e^\alpha$ (occurring in the case of a balanced government budget) are fully consistent with the rational behaviour of the representative household and cannot be ruled out as Obstfeld and Rogoff (1983) have shown. In the more general case of the separable additive utility function $U(c, m) = u(c) + v(m)$, it is possible to show from (22), as done by Brock (1978) and Obstfeld and Rogoff (1983), that ruling out hyperinflationary bubbles paths would require the “super-Inada” condition $\lim_{m \to 0} mv'(m) > 0$. Scheinkman (1980) related this condition to the essentiality of money i.e. the fact that “money is very necessary to the system”. Obstfeld and Rogoff (1983) showed that ruling out hyperinflationary paths would impose severe requirements on the preferences since it would require that $\lim_{m \to 0} v(m) = -\infty$. Therefore, even in the general case of $U(c, m) = u(c) + v(m)$ speculative hyperinflations paths are consistent with the rational household behaviour. This completes the proof.

This section establishes that the ad-hoc model with perfect foresight can be derived from a money-in-the-utility optimizing framework. Since hyperdeflationary paths are clearly ruled
out by violating the transversality condition the model can only support cases of small government budget deficits \((d < d^*)\). More importantly, section 3 strongly confirms that the Cagan model with perfect foresight cannot be used to investigate monetary hyperinflation and that it is only accurate to investigate speculative hyperinflation. Next section, however, shows that it is possible to highlight monetary hyperinflation within a perfect foresight optimizing monetary model where transaction role of money is explicitly modelled by a transaction cost technology.

4 Transaction costs, demand for money and monetary hyperinflation

In this section the role of money as a medium of exchange is captured explicitly by modelling a transaction technology showing that the use of money balances decreases the transaction costs. We use a very similar framework as that provided by Vazquez (1998). The transaction technology is characterized by the fact that transaction costs increase linearly with the volume of consumption, but decrease (at a decreasing rate) with increasing holdings of real per-capita money balances. Using a generalization of the transaction costs implied by the Baumol-Tobin model (see Feenstra, 1986), these are modelled by a multiplicative separable transaction technology given by

\[
T(c, m_t) = c_t f(m_t), \quad (24)
\]

where \(f(m)\) is an isoelastic function such that \(f'(m) < 0\) and \(f''(m) > 0\).

Since output \(y\) and government expenditure \(g\) are constants by assumption, the goods market equilibrium can be written as

\[
y = c_t + c_t f(m_t) + g. \quad (25)
\]

Along a hyperinflationary path where inflation rates explode and real per-capita money balances vanish, transaction costs must always increase since \(f''(m) > 0\) and \(T\) is a linear function in \(c\). Then, according to goods market equilibrium condition (25), an upper bound \(\bar{T}\) for transaction costs must be considered as private resources are limited and per-capita consumption is non negative. This upper bound \(\bar{T}\) for transaction costs represents the threshold at which representative households may switch to a system of barter in order to avoid the excessive costs of transactions involving the use of money. \(\bar{T}\) is defined by the level of transaction costs such that the households are indifferent between trading involving the use of money and barter trading because the two systems provide the same amount of per-capita consumption \(\bar{c}\).

Representing the level of real per-capita money balances associated with the upper bound of transaction costs \(\bar{T}\) by \(\bar{m}\) and using goods market equilibrium condition (25), the following relationship can be established

\[
\bar{m} = f^{-1}\left(\frac{y - g - \bar{c}}{\bar{c}}\right). \quad (26)
\]
We use the same notations as in section 3. The representative household, knowing the lower bound for per-capita consumption \( \bar{c} \), is assumed to solve at time 0 the following optimization problem

\[
\max_{c_t, m_t} \int_0^\infty e^{-\delta t} u(c_t) dt,
\]

subject to

\[
m_t = y - c_t - c_t f(m_t) - \tau - \pi_t m_t,
\]

\[
c_t \geq \bar{c}.
\]

This is a dynamic optimization problem with a bounded control. The solution to this problem is obtained (Kamien and Schwartz, 1991) by setting the Lagrangian which is built by appending the feasibility constraint (29) to the Hamiltonian

\[
H = e^{-\delta t} \left[ u(c_t) + \lambda_t \left( y - c_t - c_t f(m_t) - \tau - \pi_t m_t \right) + \mu_t \left( c_t - \bar{c} \right) \right]
\]

where \( \mu_t \) is the Lagrangian multiplier. Assuming a logarithmic utility function \( u(c) = \log c \), the necessary and sufficient conditions for an optimum are

\[
1 - \frac{\lambda_t}{c_t} \left( 1 + f(m_t) \right) + \mu_t = 0,
\]

\[
\lambda_t = \lambda_t \left( \delta + \pi_t + c_t f'(m_t) \right),
\]

\[
\mu_t \geq 0, \quad \mu_t \left( c_t - \bar{c} \right) = 0,
\]

\[
\lim_{t \to +\infty} \left[ e^{-\delta t} \lambda_t m_t \right] = 0.
\]

By denoting \( \bar{T} \) the time when per-capita consumption reaches the lower bound \( \bar{c} \), that is, the economy switches to a barter system, we have from (33) and (31)

\[
\lambda_t \left( c_t + c_t f(m_t) \right) = 1 \quad \text{for} \quad t < \bar{T}
\]

Moreover, from the goods equilibrium condition (25) we have

\[
c_t + c_t f(m_t) = y - g.
\]

(25)’

Substituting (25)’ into (35) we get

\[
\lambda_t \left( y - g \right) = 1 \quad \text{for} \quad t < \bar{T}.
\]

(35)’
Since \( y \) and \( g \) are constants, the latter implies that the co-state variable \( \lambda \) is also a constant. Therefore, we have from (32)

\[ 0 = \lambda \left( \delta + \pi_t + c_i f'(m_i) \right). \quad (36) \]

Substituting the value of \( c_i \) extracted from (35) into the latter equation and rearranging we obtain

\[ \delta + \pi_t = -\frac{f'(m_i)}{\lambda (1 + f(m_i))}. \quad (37) \]

The latter condition shows a relationship between inflation and real per-capita money balances as in Cagan’s model.

**Proposition 5:** The current framework of transaction technology micro-modelling provides through equation (37) new micro-foundations for the demand for money in which real per-capita money balances and inflation are inversely related as in Cagan’s model.

**Proof:** Condition (37) establishes a relationship between real per-capita money balances and inflation. Let us introduce function \( \Psi(m) = \frac{f'(m)}{1 + f(m)} \). The first derivative of the function \( \Psi \) is given by

\[ \Psi'(m) = \frac{(f'(m))^2 m - f'(m) f(m)(1 + f(m))}{mf(m)(1 + f(m))^2} > 0. \]

This establishes that the function \( \Psi \) is strictly increasing in \( m \).

From (37) \( \Psi(m) = -\lambda (\delta + \pi) \), and since function \( \Psi \) is strictly increasing in \( m \) we can write \( m = \Psi^{-1}(-\lambda (\delta + \pi)) \) implying an inverse relationship between \( m \) and \( \pi \). This completes the proof.

As in section 3, we assume that the constant per capita government budget deficit \( d \) is financed by issuing high-powered money, according to (21):

\[ d = g - \tau = \frac{\dot{M}}{P} \cdot \frac{\dot{M}_t}{M_t} \cdot m_t. \]

Using the latter expression, the definition of real per-capita money balances and the value of \( \pi \) extracted from equation (37) we have, omitting time index \( t \),

\[ m = \left( \frac{\dot{M}}{M} - \pi \right) = d + \delta m + \frac{mf'(m)}{\lambda (1 + f(m))}. \quad (38) \]

Equation (38) provides a complete characterization of real per-capita money balances dynamics before \( \tau \) is reached when \( t < \tau \).
Let us denote $\varepsilon$ the elasticity of function $f$, that is $\varepsilon(m) = -\frac{mf''(m)}{f'(m)}$, and $\lim_{m \to \overline{m}} \varepsilon(m) = \overline{\varepsilon}$. The following proposition shows that the current optimizing framework is appropriate to highlight both monetary hyperinflations and speculative hyperinflations.

**Proposition 6:** In this transaction costs optimizing framework:
if $d < \overline{\varepsilon} \overline{T} - \delta \overline{m}$, then (i) monetary hyperinflation paths exist, and (ii) speculative hyperinflation paths exist.

**Proof:** Proving the proposition requires three steps: firstly showing that the curve $\overline{m}$ is always increasing in $m$, secondly the condition for $\lim_{m \to \overline{m}} \dot{m} < 0$, thirdly identifying and showing qualitative dynamic properties of autonomous differential equation (38).

First step:

$$\frac{\partial \dot{m}}{\partial m} = \delta + \frac{f''(m)}{\lambda(1 + f(m))} + \frac{m f''(m)}{\lambda(1 + f(m))} - \frac{m(f'(m))^2}{\lambda(1 + f(m))^2}$$

Remembering that $f$ is isoelastic ($\frac{\partial \varepsilon(m)}{\partial m} = 0$), implying that $f''(m) = \frac{(f'(m))^2}{f(m)} - \frac{f'(m)}{m}$, gives

$$\frac{\partial \dot{m}}{\partial m} = \delta + \frac{m(f'(m))^2}{\lambda(1 + f(m))^2} > 0.$$  

Second step:

$$\lim_{m \to \overline{m}} \dot{m} = d + \delta \overline{m} + \frac{\overline{m} f'(\overline{m})}{\lambda(1 + f(\overline{m}))}$$

Rearranging this can be rewritten in the following way

$$\lim_{m \to \overline{m}} \dot{m} = d + \delta \overline{m} + \frac{\overline{m} f'(\overline{m})}{\lambda(\overline{c} + \overline{f}(\overline{m}))} = d + \delta \overline{m} + \frac{\overline{T}}{\lambda(\overline{c} + \overline{f}(\overline{m}))}(-\overline{\varepsilon})$$

Using (35) the latter simplifies in $\lim_{m \to \overline{m}} \dot{m} = d + \delta \overline{m} - \overline{\varepsilon} \overline{T}$ which is negative if $d < \overline{\varepsilon} \overline{T} - \delta \overline{m}$.

Third step:

As $\lim_{m \to +\infty} \dot{m} = +\infty$ the curve representing $\dot{m}$ is increasing and crossing the horizontal axis only once at the unique steady state $m^* > \overline{m}$.

All paths originating at the right of $m^*$ are hyperdeflationary paths that can be ruled out because violating the transversality condition given by (34).

When the government runs a positive fiscal deficit, $d > 0$, all paths starting at the left of $m^*$ are hyperinflationary paths since the level of per-capita money balances decreases continuously as time goes by, and thus, according to proposition 5, the inflation rate explodes. Moreover, these hyperinflationary paths are monetary hyperinflations because along these paths the rate of growth of the money supply explodes as well. Rewriting government budget constraint (21) as

$$\frac{\dot{M}}{M} = \frac{d}{m}, \quad (21)'$$
we see that along these paths of continuously declining \( m \), given that \( d > 0 \), the growth rate of money supply increases continuously. This proves (i).

When the government budget is balanced, \( d = 0 \), the condition \( d < \bar{\varepsilon} - \delta \bar{m} \) still holds and the same kind of monetary dynamics appear as with the case \( d > 0 \). However, in this case, hyperinflation paths originating at the left of \( m^* \) are not monetary hyperinflations but speculative hyperinflations since, according to (21), the stock of money is constant. This completes the proof.

![Figure 3](image)

Figure 3 illustrates the dynamic properties of differential equation (38). Along a monetary hyperinflation path, once the economy reaches \( \bar{m} \) real money balances vanish and the economy switches to a barter trading system. Then, the government can no longer finance its deficit by money creation.

In the case \( d > \bar{\varepsilon} - \delta \bar{m} \), we have \( \lim_{m \to \bar{m}} m > 0 \) and, from there, an increasing curve describing \( \dot{m} \). A continuum of hyperdeflationary paths would occur at the right of \( \bar{m} \) and there wouldn’t be no steady state. These hyperdeflationary paths can be ruled out by violating the transversality condition (34). This implies that the model is valid only for the case \( d < \bar{\varepsilon} - \delta \bar{m} \).

Proposition 6 is important because it establishes that a perfect foresight optimizing monetary model is able to produce monetary hyperinflation and not only speculative hyperinflation. This result has been obtained without any ad-hoc assumption implying the inclusion of friction in the adjustment of some nominal variable. Therefore, it suggests that monetary hyperinflation paths are consistent with the optimizing behaviour of representative households in an optimizing monetary framework where money is desired for its role as a medium of exchange. The transaction costs modelling approach may give an alternative to the standard Cagan’s model failing with perfect foresight.

5 Conclusions
The Cagan ad-hoc model has been designed to explain monetary hyperinflation, hyperinflation driven by money growth acceleration in the context of an excessive government budget deficit. In this paper, we confirm the results of Buiter (1987) establishing that in the Cagan ad-hoc model with perfect foresight deficits can never generate monetary hyperinflations. On the contrary, hyperdeflationary paths may get under way when deficits are excessive (Buiter hyperdeflation paradox). Moreover, we show that no monetary hyperinflation can be generated in this framework. Therefore, the Cagan ad-hoc model with perfect foresight is seriously flawed for the investigation of monetary hyperinflation. However, we show that this framework can be saved for the analysis of hyperinflationary processes because it can generate speculative hyperinflations or hyperinflationary bubbles in the context of a balanced government budget where there is no money growth.

This paper resorts to first principles to clarify the failure of the Cagan ad-hoc model and to assess the consistency of hyperinflationary paths with the optimizing behaviour of representative agents. We consider two standard optimizing monetary frameworks to model the use of money in a context where money growth relies on the government need for seigniorage revenues: a money-in-the-utility model and a transactions costs based model. In the context of a money-in-the-utility model we show that the Cagan ad-hoc model with perfect foresight can be derived from first principles by specifying appropriate agents preferences. We obtain the same qualitative dynamic properties as in the Cagan ad-hoc model with perfect foresight. The optimizing behaviour of the representative agents allows ruling out all hyperdeflationary paths as non optimal paths. In this model we show that monetary hyperinflations can't be generated. Moreover, we show that the only hyperinflationary processes that can be generated are speculative hyperinflations consistent with the agents optimizing behaviour. Therefore, the money-in-the-utility framework strongly confirms that the Cagan model with perfect foresight cannot be used to investigate monetary hyperinflation and that it is only accurate to investigate speculative hyperinflation.

In the context of an optimizing model with transaction costs, we show that deficits can generate monetary hyperinflations in a perfect foresight framework and speculative hyperinflations remain a strong possibility in the context of a balanced government budget. This result is in sharp contrast to that of the money-in-the-utility framework in which Buiter results were founded and confirmed. It implies a demand for real cash balances strictly decreasing with respect to the inflation rate but different from the Cagan money demand. Hence, monetary hyperinflation paths are consistent with the optimizing behaviour of representative households in an optimizing monetary framework where money is desired for its role as a medium of exchange. The transaction costs modelling approach may give an alternative to the standard Cagan’s model failing with perfect foresight. This result emerges without any ad-hoc assumption implying the inclusion of friction in the adjustment of some nominal variable and suggests that monetary hyperinflation analysis with perfect foresight requires removing the Cagan money demand.

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