« The signaling role of policy action »

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Abstract

This paper analyzes the conduct of the optimal monetary policy with imperfect information on the shocks hitting the economy where firms’ prices are strategic complements. Monetary policy entails a dual stabilizing role, as a policy response that influences directly the economy and as a vehicle for information that shapes firms’ beliefs. In the case where more information is welfare detrimental, the central bank faces a dilemma, for its monetary instrument aimed at stabilizing the economy may harmfully shape firms’ beliefs. Recognizing the signaling role of its instrument, the central bank finds it optimal to distort its policy response in order to mitigate the detrimental information that it may convey.

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1 Introduction

In the ongoing debate on the social value of public information, most of the literature considers information as being disclosed by means of an explicit and official statement made by an institution like a central bank.\(^1\) The contribution of this paper is to argue that taking a policy action also conveys information as an implicit communication. For instance, the implementation of monetary policy reveals to the market the economic assessment of the central bank. When it is perfectly observed, the policy action implemented by a policy maker discloses public information even in the absence of an explicit statement. So, the response of a policy maker entails a dual role, as an action that influences directly economic outcomes and as a vehicle for information that influences the beliefs of market participants. The main contribution of this paper is to highlight the signaling role of policy choices and to show how the design of the optimal policy should take into account the value of the information conveyed by the policy action. Although there is a growing literature on the desirability of central bank transparency, it has largely abstracted from the interaction between the choice to be transparent and the optimal design of monetary policy.\(^2\)

To illustrate the signaling role of the policy action, we consider the optimal conduct of monetary policy in an economy where monetary frictions are caused by imperfect information. In this environment, communication turns out to be an essential component in designing the optimal monetary policy pattern because it drives the degree of information imperfection and the real effects of policy choices. In particular, we distinguish two channels by which the central bank discloses information to the private sector. First, the central bank explicitly discloses information by making an announcement; this is the form of communication with which the literature usually deals. Second, the central bank implicitly discloses information about its assessment on the economy by implementing its monetary policy. The signaling role of monetary policy has been well documented by Romer and Romer (2000). Using US data, they show that "the Federal Reserve’s actions

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\(^2\)Some exceptions are Angeletos et al. (2006), Hellwig et al. (2006), or Walsh (2007) who analyze the signaling role of policy choices on market participants in different contexts.
signal its information” and that “commercial forecasters raise their expectations of inflation in response to contractionary Federal Reserve actions [...]” (Romer and Romer (2000, p. 430)). Considering the signaling role of the instrument set by the central bank points out the intertwined relation between the implementation of a policy instrument and the communication strategy of the central bank. On the one hand, the optimal monetary policy is a function of firms’ beliefs and thereby of the communication strategy of the central bank. On the other hand, since monetary policy conveys information to firms, it is a constitutive part of the communication strategy of the central bank. As a result, the central bank should choose its instrument by optimally balancing both its direct impact on the economy and the information it conveys to market participants. In particular, if the central bank wishes to withhold information from the markets, it should adjust the conduct of monetary policy in order not to release too much information through its policy action.

The economy that we consider is hit by two types of disturbances, namely a stochastic labor supply shock that induces parallel variations in both the efficient and the equilibrium level of output and a stochastic mark-up shock that induces variations in the equilibrium level of output but leaves the efficient level unaffected. Providing better information reduces frictions and helps economic agents reach the equilibrium level of output that would prevail in a frictionless economy. As Angeletos and Pavan (2007) emphasizes, providing more information is welfare improving to the extent that the equilibrium and the efficient level of output are symmetrically affected by shocks. So, withholding information about the mark-up shock is welfare improving because it prevents the economy from moving too closely to the frictionless equilibrium level of output and thereby from deviating too much from the efficient level.\(^3\) This creates a case for constructive ambiguity that can be exploited by the central bank when taking its policy decision.

\(^3\)With a Lucas-type Phillips curve, Cukierman (2001) shows that transparency with respect to mark-up shocks is detrimental to welfare because it impedes employment stabilization by the central bank. Under opacity the central bank is able to optimally trade between employment and inflation stabilization through inflation surprise. This mechanism is different from ours in so far as in our model transparency exacerbates firms’ reaction to mark-up shocks.
central bank does not disclose any explicit announcement about its economic assessment, firms cannot properly decipher the rationale behind the instrument. For instance, the central bank may implement an expansionary instrument in response either to a positive labor supply or a negative mark-up shock. The central bank can exploit this ambiguity by distorting its response to labor supply and mark-up shocks in order to optimally balance the direct stabilizing role of its instrument and its indirect stabilizing role through shaping firms’ beliefs.

The paper is structured as follows. Section 2 presents the economy. Section 3 derives the optimal monetary policy for benchmark cases with homogeneous information. This highlights the welfare effect of information with respect to labor supply and to mark-up shocks. Section 4 presents the optimal monetary policy under heterogeneous information as a function of the communication strategy of the central bank. Finally, section 5 concludes.

2 The economy

The economy is derived from a small scale general equilibrium model with flexible prices, populated by a representative household, a continuum of monopolistic competitive firms, and a central bank. Two types of stochastic shocks hit the economy, a labor supply shock and a mark-up shock. The nominal aggregate demand is determined by the central bank that maximizes the utility of the representative household. Apart from the informational structure, we base our analysis on the model developed by Adam (2007).4

2.1 Representative household

The representative household chooses its aggregate composite good \( Y \) and labor supply \( L \) in order to maximize its utility subject to its budget constraint,

\[
U(Y) - \nu V(L),
\]

4For convenience, we only present here the main components of the economy and refer to Adam (2007) for the detailed derivation of the model.
\[ s.t. \ W L + \Pi = P Y + T. \]

\( W \) denotes the competitive wage, \( \Pi \) the profits the household gets from firms, and \( T \) the nominal transfer from the central bank. The parameter \( \nu \) is a stochastic labor supply shock with \( E(\nu) = 1 \), that induces variations in the efficient level of output. \( Y \) is the composite good defined by the Dixit-Stiglitz aggregator

\[ Y = \left[ \int_{0}^{1} (Y_i)^{\frac{\theta - 1}{\theta}} di \right]^{\frac{\theta}{\theta - 1}}, \]

where \( \theta > 1 \) is the parameter of price elasticity of demand and where \( Y_i \) is the good produced by firm \( i \). \( \theta \) is stochastic with \( E(\theta) = \bar{\theta} \) and induces variations in the desired mark-up of firms and thereby in the equilibrium level of prices and of the output. \( P \) is the appropriate price index which solves \( P Y = \int_{0}^{1} P_i Y_i di \).

### 2.2 Firms

Each firm \( i \) produces a single differentiated good \( Y_i \) with one unit of labor \( L_i \) according to the simple production function

\[ L_i = Y_i. \]

The profit maximization problem of firm \( i \) is given by

\[ \max_{P_i} E[(1 + \tau) P_i Y_i(P_i) - W Y_i(P_i) | \Gamma_i], \]

where \( \tau \) is an output subsidy that offsets the efficiency detrimental effect of the mark-up and \( \Gamma_i \) is the information set of firm \( i \). Linearizing the first order condition of firm \( i \)'s problem around its steady state delivers

\[ p_i = E_i[p + \xi (y - y^*) + u], \tag{1} \]

where \( E_i \) is the expectation operator conditional on firm \( i \)'s information \( \Gamma_i \) and where small letters indicate percentage deviation from the steady state. The pricing rule (1)
states that firms set their price as a function of their expectations of the overall price level $p$, the real output gap $y - y^*$, and the mark-up shock $u$. The deviation of the efficient level of output $y^*$ from its steady state is determined by the stochastic labor supply shifter $\nu$. The parameter $\xi = -\frac{U''(\bar{Y})\bar{Y}}{U'(\bar{Y})} + \frac{V''(\bar{Y})\bar{Y}}{V'(\bar{Y})}$ determines the sensitivity of the optimal price to the output gap and is increasing in the risk aversion of the household. The optimal pricing also depends on the expected value of the mark-up shock $u$ given by $u = -\frac{1}{(\tilde{G} - \bar{G})}$, where $\tilde{G}$ is the price elasticity of demand at its steady state level. Firms find it optimal to increase their price when the price elasticity of demand $\theta$ falls below its steady state value $\bar{\theta}$.

Using the fact that the nominal aggregate demand $q$ can be expressed as $q = y + p$, we rewrite the pricing rule (1) as

$$p_i = E[(1 - \xi)p + \xi q - \xi y^* + u].$$

$\xi$ determines whether prices are strategic complements or substitutes. We realistically assume that prices are strategic complements, i.e. $0 < \xi \leq 1$. This means that each firm tends to rise its own price when it expects other firms to do so.

The labor supply shock and the mark-up shock are assumed to be normally and independently distributed with the following properties:

$$y^* \sim N(0, \sigma_{y^*}^2)$$

$$u \sim N(0, \sigma_u^2).$$

### 2.3 Central bank

The central bank seeks to maximize the expected utility of the representative household by adjusting its monetary instrument, the nominal demand $q$, conditional on its own information. Appendix A derives the approximation of the welfare of the representative household according to the informational structure of our economy. The monetary policy
problem implies that the central bank seeks to minimize the expected loss

\[
\mathbb{E}_{cb}(L) = \min_q \mathbb{E}_{cb}\left[(y - y^*)^2 + \frac{\bar{\theta}}{\xi}\left(\frac{1 - \alpha}{\alpha}p^2 + \alpha\gamma_3^2\sigma^2_{\rho}\right)\right],
\]

subject to the pricing equation of firms (2). Coefficients \(\alpha\) and \(\gamma_3\) depend upon the informational structure and the equilibrium condition of the economy as defined below. \(\sigma^2_{\rho}\) captures the dispersion of firms’ private information.

### 2.4 Informational structure

Monetary frictions arise in our economy because of information imperfections. The recent revival of interest in Phelps (1970) insight – according to which information imperfections play a crucial role in the monetary transmission mechanism – includes the work of Adam (2007), Hellwig (2002), Mankiw and Reis (2002), and Woodford (2003). These authors emphasize the realistic dynamics of models relying on information imperfections when firms’ prices are strategic complements.

#### 2.4.1 Information of the central bank

The central bank receives in private a signal on each the labor supply and the mark-up shock. Each signal deviates from the true value of the shock by an error term that is normally distributed:

\[
y^*_{cb} = y^* + \eta, \quad \text{with} \quad \eta \sim N(0, \sigma^2_{\eta})
\]

\[
u_{cb} = u + \mu, \quad \text{with} \quad \mu \sim N(0, \sigma^2_{\mu}),
\]

where \(\eta\) and \(\mu\) are independently distributed.

Since both fundamental shocks and error terms are independently normally distributed, the optimal instrument rule of the central bank that determines the nominal aggregate
demand $q$ is a linear combination of its signals and can be written as

$$ q = \zeta_1 (y^* + \eta) + \zeta_2 (u + \mu). $$

$\zeta_1$ and $\zeta_2$ describe how the central bank sets the nominal aggregate demand in response to its signal on both shocks.

The central bank may disclose information to firms through two different channels. While the monetary instrument conveys information about its economic assessment whenever it is observed, the central bank may also make an explicit announcement that renders the interpretation of the instrument unequivocal.

### 2.4.2 Information of firms

We introduce two sources of information imperfections with respect to firms.

On the one hand, following Mankiw and Reis (2002), we assume that information spreads slowly through the economy. According to this information stickiness assumption, only a fraction $\alpha$ of firms is informed about the contemporaneous economic development while the remaining fraction $1 - \alpha$ of firms does not receive any contemporaneous information update at all. As emphasized in section 3, this first source of frictions enable us to solve the problem of price level indeterminacy and to derive the optimal monetary policy.

On the other hand, the information received by the fraction $\alpha$ of informed firms is noisy and heterogeneous, what entails fundamental and strategic uncertainty. The information received by the informed $\alpha$-type firms is threefold.

First, each informed $\alpha$-type firm $i$ receives a private signal on the mark-up shock $u_i$ that may be interpreted as a private estimate. The private signal of each firm deviates from the true mark-up shock by an error term that is normally distributed:

$$ u_i = u + \rho_i, \quad \text{with} \quad \rho_i \sim N(0, \sigma^2_\rho), $$

where $\rho_i$ are identically and independently distributed across $\alpha$-type firms.
Second, the \( \alpha \)-type firms eventually observe the monetary instrument implemented by the central bank. The signal released by the central bank on its instrument can be generally expressed as

\[
q_i = q + \varphi_i, \quad \text{with} \quad \varphi_i \sim N(0, \sigma^2_{\varphi}).
\]

Whenever the central bank is transparent with respect to its monetary instrument \((\sigma^2_{\varphi} = 0)\), the nominal level of aggregate demand \(q\) is common knowledge among the informed \(\alpha\)-type firms. By contrast, whenever the central bank is opaque with that respect \((\sigma^2_{\varphi} \to \infty)\), firms cannot observe the instrument. By making its instrument public, the central bank gives an indication to firms about its own beliefs on the state of the economy. However, firms are unable to properly understand the central bank’s assessment of the economy: since the central bank responds to two shocks, the monetary instrument does not allow firms to decipher the rationale behind the implemented policy, unless the central bank discloses more information.

Third, whenever the central bank is transparent with respect to its monetary instrument, the \( \alpha \)-type firms eventually observe an additional public signal that completely eliminates the informational asymmetry between the central bank and the \( \alpha \)-type firms. We assume that a fully transparent central bank directly discloses its signal on the efficient level of output \( y^*_\text{cb} \), so that the \( \alpha \)-type firms are able to properly interpret the rationale for the monetary instrument.\(^5\) The signal released by the central bank on its economic assessment can be generally expressed as

\[
y^*_{\text{cb},i} = y^*_\text{cb} + \phi_i, \quad \text{with} \quad \phi_i \sim N(0, \sigma^2_{\phi}).
\]

The case of transparency with respect to its economic assessment is captured by \( \sigma^2_{\phi} = 0 \) and the case of opacity by \( \sigma^2_{\phi} \to \infty \).

\(^5\)One may think of different types of announcement that would reveal central bank’s signals to firms. In practice, the publication of inflation forecast and/or target appears to be the main form of announcement adopted by transparent central banks.
2.5 Timing of events

The sequence of events is as follows. First, the communication strategy of the central bank is determined and is common knowledge among firms. Second, the nature draws the labor supply shock $y^*$ and the mark-up shock $u$. The central bank observes both shocks with an error term and sets its monetary instrument $q$. According to its communication strategy, it may reveal its instrument to the public and may make an explicit announcement $y_{cb}^*$. Based on their private signal on the mark-up shock $u_i$ and – when available – on the monetary instrument $q_i$ and on the announcement of the central bank $y_{cb,i}^*$, firms then simultaneously determine their price. Finally, the household demands products for consumption and production takes place. The central bank plays the role of a Stackelberg leader and will exploit its first mover advantage to shape firms’ expectations.

3 Homogeneous information

In this section, benchmark information settings are discussed to illustrate the mechanism of the model. First, it is shown that in a frictionless economy where both the central bank and firms have perfect information, the optimal monetary policy is indeterminate. Second, information stickiness is introduced for resolving the indeterminacy problem and for illustrating the welfare effect of information about mark-up shocks.

3.1 Perfect information

The case of perfect information is captured in our economy when there is no information stickiness ($\alpha = 1$) and when the error terms are zero: $\sigma^2_\eta = 0$, $\sigma^2_\mu = 0$, and $\sigma^2_\rho = 0$. With perfect information there is no price dispersion across firms because all firms set the same

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6In our setup the choice for the communication strategy occurs before the central bank observes its signals on the shocks. We abstract here from the discussion on whether it is optimal for the central bank to rely on its signals to choose its communication strategy and how firms’ would accordingly adjust their beliefs.
price and the monetary policy problem becomes

\[
E(L) = \min_q E \left[ (y - y^*)^2 \right]
\]  
(4)

s.t. \quad p = q - y^* + \frac{1}{\xi} u 
(5)

\quad q = y + p. 
(6)

Plugging (6) into (5) shows that an output gap (and a loss) appears whenever there is a mark-up shock:

\[ y - y^* = -\frac{1}{\xi} u. \]

Because the price level \( p \) does not enter into the loss function (4), the optimal monetary policy is indeterminate:

\[ q = y^* - \frac{1}{\xi} u + p. \]

The nominal aggregate demand \( q \) can be chosen arbitrarily by the central bank and the price level \( p \) will accept the corresponding value. In an frictionless economy, monetary policy does not have any role to play. The unconditional expected loss is given by

\[ E(L) = \frac{1}{\xi^2} \sigma_u^2 \]  
(7)

and is independent from the policy implemented by the central bank.

### 3.2 Perfect sticky-information

In order to solve the indeterminacy problem of monetary policy in the frictionless economy, we introduce sticky information as described in section 2.4.2. In this setup, only a fraction \( 0 < \alpha < 1 \) of firms is assumed to get an information update, the other \( 1 - \alpha \) firms remaining completely uninformed. Yet, the error terms of the central bank and of the \( \alpha \)-type firms remain zero: \( \sigma_u^2 = 0, \sigma_{\mu}^2 = 0, \) and \( \sigma_{\rho}^2 = 0. \) With perfect sticky-information the monetary
policy problem of the central bank becomes

\[ \mathbb{E}(L) = \min_q \mathbb{E} \left[ (y - y^*)^2 + \frac{\bar{\theta} (1 - \alpha)}{\xi} p^2 \right] \tag{8} \]

s.t. \[ p = \frac{\alpha}{1 - \alpha + \alpha \xi} (\xi q - \xi y^* + u) \]

\[ q = y + p. \]

Solving the problem (8) delivers the optimal monetary policy

\[ q = y^* - \frac{\alpha (\bar{\theta} - 1)}{1 - \alpha + \alpha \bar{\theta} \xi} u \]

and yields a price level and an output gap given by

\[ p = \frac{\alpha}{1 - \alpha + \alpha \bar{\theta} \xi} u \quad \text{and} \quad y - y^* = \frac{1 - \alpha}{1 - \alpha + \alpha \xi} (q - y^*) - \frac{\alpha}{1 - \alpha + \alpha \xi} u = \frac{-\alpha \bar{\theta}}{1 - \alpha + \alpha \bar{\theta} \xi} u \]

what implies an unconditional expected loss equal to

\[ \mathbb{E}(L) = \frac{\alpha \bar{\theta}}{\xi (1 - \alpha + \alpha \bar{\theta} \xi)} \sigma_u^2. \]

The optimal monetary policy indicates that labor supply shocks are perfectly accommodated what simultaneously closes the output gap and eliminates price deviations. By contrast, mark-up shocks cannot be neutralized by the central bank.

Increasing the share \( \alpha \) of informed firms strengthens the aggregated reaction to mark-up shocks. As a result, when the share of informed \( \alpha \)-type firms increases, the response of the central bank to mark-up shocks becomes stronger \( \left( \frac{\partial q}{\partial \alpha} < 0 \right) \), the price level and the output gap deviations increase what entails a larger unconditional expected loss. So, improving information among firms is welfare detrimental. While information about mark-up shocks is privately desirable according to the optimal pricing rule of firms (2), it is socially undesirable because of the efficiency wedges it creates.\(^7\)

\(^7\)At the limit, when \( \alpha \) converges to 1, the output gap and the unconditional expected loss are identical as under perfect information. However, the price level and the monetary policy are determinate at the limit.

11
The loss associated with mark-up distortions increases with the price elasticity of demand $\bar{\theta}$. So, the central bank responds more aggressively to mark-up shocks when the price elasticity of demand increases ($\frac{\partial q}{\partial \bar{\theta}} < 0$) and perfectly stabilizes the price level for infinite price elasticity.

4 Heterogeneous information

We now consider the more realistic case where firms have heterogeneous information. First, we describe the general equilibrium of the economy and then derive the optimal monetary policy according to three communication strategies of the central bank.

4.1 Equilibrium

This section solves the perfect Bayesian equilibrium and derives the optimal behavior of firms according to their information set on the monetary instrument and on the central bank assessment of the economy. The information set of $\alpha$-type firms is composed of a private signal on the mark-up shock $u_i$, a signal on the nominal aggregate demand $q_i$, and a signal on the central bank assessment of the labor supply shock $y_{cb,i}$.

For setting its optimal price according to (2), each $\alpha$-type firm solves the inference problem $\mathbb{E}[q, y^*, y_{cb}^*, u|q_i, y_{cb,i}^*, u_i]$ that is defined as

$$
\mathbb{E}
\begin{pmatrix}
q \\
y^* \\
y_{cb}^* \\
u
\end{pmatrix} | q_i, y_{cb,i}^*, u_i = \Omega
\begin{pmatrix}
q_i \\
y_{cb,i}^*
\end{pmatrix},
$$

(9)

with $\Omega = V_{uo} V_{oo}^{-1}$, where $V_{uo}$ is the covariance matrix of the expected variables and the signals and $V_{oo}$ is the covariance matrix of the signals themselves.

Following Morris and Shin (2002), we assume that $\alpha$-type firms set their price according

*Note that the signals on the nominal aggregate demand $q_i$ and on the central bank assessment of the labor supply shock $y_{cb,i}^*$ may be completely uninformative in the case of opacity.
to the following linear pricing rule:

\[ p_i = \gamma_1 q_i + \gamma_2 y^*_{cb,i} + \gamma_3 u_i. \]

As derived in Appendix B, the equilibrium response of \( \alpha \)-type firms to their signals is given by the system of simultaneous equations:

\[
\begin{align*}
\gamma_1 &= \frac{\alpha (1 - \xi)(\gamma_2 \Omega_{31} + \gamma_3 \Omega_{41}) + \xi(\Omega_{11} - \Omega_{21}) + \Omega_{41}}{1 - \alpha(1 - \xi)\Omega_{11}} \\
\gamma_2 &= \frac{\alpha (1 - \xi)(\gamma_1 \Omega_{12} + \gamma_3 \Omega_{42}) + \xi(\Omega_{12} - \Omega_{22}) + \Omega_{42}}{1 - \alpha(1 - \xi)\Omega_{32}} \\
\gamma_3 &= \frac{\alpha (1 - \xi)(\gamma_1 \Omega_{13} + \gamma_2 \Omega_{33}) + \xi(\Omega_{13} - \Omega_{23}) + \Omega_{43}}{1 - \alpha(1 - \xi)\Omega_{43}}.
\end{align*}
\]

The central bank chooses its monetary instrument to minimize the expected loss (3) subject to (10) given the precision of its information.

We derive in the next sections the optimal monetary policy for three central bank communication strategies. First, we consider the case of transparency where there is no information asymmetry between the central bank and informed \( \alpha \)-type firms. Second, we derive the optimal monetary policy for the case of opacity where the central bank does not disclose any information with respect to both its monetary instrument and its economic assessment. And third, the case of intermediate transparency depicts the more interesting situation where the central bank is transparent with respect to its monetary instrument but does not disclose any additional information about its economic assessment.

While the current section presents the equilibrium for any degree of information stickiness \( \alpha \), we concentrate in the remainder of the paper upon the limit case where the share of informed firms \( \alpha \) goes to one. This allows us to solve the indeterminacy problem that occurs in the absence of stickiness while focusing on the heterogeneous nature of information as frictions.\(^9\)

\(^9\)The sole presence of heterogeneous information leaves monetary policy indeterminate because the implied price dispersion does not depend upon the price level.
4.2 Optimal monetary policy under transparency

Under transparency, the informed \(\alpha\)-type firms perfectly observe the monetary instrument \(q\) and the central bank assessment of the labor supply shock \(y^*_{cb}\). Since there are two shocks affecting the economy, the combination of both observations removes the informational asymmetry between the central bank and the \(\alpha\)-type firms. In our setup, transparency is modeled with perfect firms’ signals on the monetary instrument \(q\) and on the central bank announcement \(y^*_{cb}\), i.e. with \(\sigma^2_\varphi = \sigma^2_\rho = 0\).

Solving the monetary policy problem under transparency and taking the limit when \(\alpha\) converges to one delivers the optimal coefficients of monetary policy:

\[
\begin{align*}
\zeta_1, T &= \frac{\sigma^2_\varphi}{\sigma^2_\varphi + \sigma^2_y} \\
\zeta_2, T &= -\frac{\bar{\theta} - 1}{\theta \xi} \frac{\sigma^2_u}{\sigma^2_u + \sigma^2_\mu}.
\end{align*}
\]

As \(\zeta_1, T\) indicates, the central bank fully accommodates variations in the efficient level of output according to the precision of its signal. The strength of central bank’s response to mark-up shocks \(\zeta_2, T\) increases with the precision of its information \(\sigma^2_\mu\), with the price elasticity of demand \(\bar{\theta}\), and with the degree of strategic complementarities \(1 - \xi\). The response of the central bank under transparency is reminiscent of its response in the case of perfect sticky-information derived in section 3.2.\(^{11}\)

Implementing the optimal monetary policy under transparency yields an unconditional expected loss given by

\[
\mathbb{E}_{cb}(L_T) = \frac{1}{\xi^2} \sigma^2_u + \frac{\sigma^2_y}{\sigma^2_y + \sigma^2_\eta} + \frac{\theta \xi \sigma^2_u \sigma^2_\mu}{\xi^2 (\sigma^2_u + \sigma^2_\mu) + \xi \sigma^2_u \sigma^2_\mu} \frac{\xi^2 (\sigma^2_u + \sigma^2_\mu)}{\xi^2 (\sigma^2_\mu + \sigma^2_u)} \\
&= \frac{\sigma^2_y}{\sigma^2_y + \sigma^2_\eta} + \frac{-2\theta \xi \sigma^2_u \sigma^2_\mu}{\xi^2 (\sigma^2_u + \sigma^2_\mu) + \xi \sigma^2_u \sigma^2_\mu} \frac{\xi^2 (\sigma^2_u + \sigma^2_\mu)}{\xi^2 (\sigma^2_\mu + \sigma^2_u)} \\
&\quad + \frac{\theta \xi \sigma^2_u \sigma^2_\mu}{\xi^2 (\sigma^2_u + \sigma^2_\mu) + \xi \sigma^2_u \sigma^2_\mu} \frac{\xi^2 (\sigma^2_u + \sigma^2_\mu)}{\xi^2 (\sigma^2_\mu + \sigma^2_u)} + \frac{\xi^2 (\sigma^2_u + \sigma^2_\mu)}{\xi^2 (\sigma^2_\mu + \sigma^2_u)} \frac{\xi^2 (\sigma^2_u + \sigma^2_\mu)}{\xi^2 (\sigma^2_\mu + \sigma^2_u)}.
\]

\(^{10}\)We consider the case of a credible central bank and abstract from the discussion on whether it would be optimal for the central bank to cheat the private sector by disclosing a falsified economic assessment. \(^{11}\)Appendix C presents the optimal monetary policy under transparency for general values of \(\alpha\).
The loss associated with the output gap is captured by $A_T$, $B_T$, and $C_T$ while the loss associated with the price dispersion is captured by $D_T$. $A_T$ stands for the loss under perfect information as derived in section 3.1. $B_T$ is the incremental loss associated with the output gap that arises when the central bank is unable to perfectly accommodate the labor supply shock because of its imperfect information. $C_T$ captures the mitigation of the loss associated with output gap that arises with the uncertainty surrounding the mark-up shock. $C_T$ gets smaller with the inaccuracy of firms’ private information $\sigma^2_\rho$, with the inaccuracy of central bank information $\sigma^2_\mu$, and with the degree of strategic complementarities $1 - \xi$. When firms’ private information and central bank information is totally noisy, $C_T$ perfectly offsets $A_T$, i.e. $\lim_{\sigma^2_\rho,\sigma^2_\mu \to \infty} C_T = -\sigma^2_\rho / \xi^2$.

The loss associated with the price dispersion $D_T$ increases with the inaccuracy of central bank information $\sigma^2_\mu$, with the degree of strategic complementarities $1 - \xi$, and with the price elasticity of demand $\bar{\theta}$. The precision of firms’ private information has an ambiguous effect on the price dispersion: $\frac{\partial D_T}{\partial \sigma^2_\rho} > 0 \Leftrightarrow \sigma^2_\rho < \frac{\xi \sigma^2_\rho \sigma^2_\mu}{\sigma^2_\rho + \sigma^2_\mu}$. When firms private information is highly precise, raising its inaccuracy increases the price dispersion because firms tend to assign a large weight to it. By contrast, when the inaccuracy of firms private information is sufficiently high, more inaccuracy reduces the price dispersion because firms respond less to their private information.

Overall, the uncertainty surrounding the mark-up shock is welfare improving when $\bar{\theta} < 2 + \left(\frac{\sigma^2_\rho + \sigma^2_\mu}{\xi \sigma^2_\rho \sigma^2_\mu}\right)^2$, i.e. when $C_T + D_T < 0$. The error terms on the mark-up shock are welfare improving when the loss associated with price dispersion is not too large, that is when the price elasticity of demand $\bar{\theta}$ is low.

4.3 Optimal monetary policy under opacity

Under opacity, the informed $\alpha$-type firms observe neither the monetary instrument $q$ nor the central bank assessment of the labor supply shock $y^*_{cb}$. They only get their private signal on the mark-up shock $u_i$. The case of opacity is modeled in our setup with infinite noise on firms’ signal on the monetary instrument and on the central bank assessment,
\[ \begin{align*}
\zeta_{1,O} &= \frac{\sigma_y^2}{\sigma_y^2 + \sigma_\theta^2} \\
\zeta_{2,O} &= -\frac{(\theta - 1)\sigma_\rho^2\sigma_u^4}{\sigma_\mu^2(\sigma_\rho^2 + \xi\sigma_u^2)^2 + \sigma_\rho^2\sigma_\mu^2(\xi\sigma_u^2 + \sigma_\rho^2)}.
\end{align*} \]

\( \zeta_{1,O} \) indicates that the central bank tries to fully accommodate variation in the efficient level of output according to the precision of its signal. The central bank’s response to mark-up shocks \( \zeta_{2,O} \) is always contractionary and becomes stronger with the precision of its information \( \sigma_\rho^2 \), with the price elasticity of demand \( \tilde{\theta} \), and with the degree of strategic complementarities \( 1 - \xi \). The effect of the precision of firms’ private information \( \sigma_\rho^2 \) is not monotone on the response to mark-up shocks: \( \frac{\partial \zeta_{2,O}}{\partial \sigma_\rho^2} > 0 \iff \sigma_\rho^2 > \frac{\xi^2\sigma_\mu^2\sigma_u^2}{\sigma_\rho^2 + \sigma_\mu^2} \). The central bank implements a strong response to mark-up shocks when the price dispersion is relatively high, that is to say when the inaccuracy of firms’ private information is intermediate. When firms’ private information is either perfectly accurate \( (\sigma_\rho^2 = 0) \) or perfectly noisy \( (\sigma_\rho^2 \to \infty) \) there is no price dispersion and the central bank does not respond to mark-up shocks.

Implementing the optimal monetary policy under opacity yields an unconditional expected loss given by

\[ E_{\Delta}(L_O) = \frac{1}{\xi^2} \left( \frac{\sigma_u^2}{A_O} + \frac{\sigma_y^2\sigma_\mu^2}{B_O} + \frac{\xi(\tilde{\theta} - 2)\sigma_\rho^2\sigma_\mu^2 - \sigma_\rho^2(\sigma_u^2 + \sigma_\mu^2)}{\xi^2(\sigma_\rho^2(\xi\sigma_u^2 + \sigma_\rho^2)^2 + (\xi\sigma_u^2 + \sigma_\rho^2)\sigma_\mu^2\sigma_u^2)^2} \right) \]

As in the case of transparency, \( A_O \) stands for the loss under perfect information and \( B_O \) for the incremental loss associated with the output gap that arises when the central bank is unable to perfectly accommodate the labor supply shock because of its imperfect information. When firms’ private information and central bank’s information is totally

\[ ^{12} \text{Appendix D presents the optimal monetary policy under opacity for general values of } \alpha. \]
noisy, \( C_O \) perfectly offsets \( A_O \), i.e. \( \lim_{\sigma_u^2, \sigma_\mu^2 \to \infty} C_O = -\sigma_u^2/\xi^2 \). Under opacity, the combined welfare effect of the uncertainty surrounding the mark-up shock on the output gap and on the price dispersion given by \( C_O + D_O \) is welfare improving when \( \tilde{\theta} < 2 + \frac{(\sigma_u^2 + \sigma_\mu^2)\sigma_\rho^2}{\xi^2 \sigma_u^2 \sigma_\mu^2} \).

**Unconditional expected loss under transparency vs. opacity** It can be easily shown that the unconditional expected loss under opacity is always smaller than or equal to the loss under transparency: \( C_O + D_O \leq C_T + D_T \). In the particular case where

\[
\tilde{\theta} = 2 + \frac{(\sigma_u^2 + \sigma_\mu^2)\sigma_\rho^2}{\xi^2 \sigma_u^2 \sigma_\mu^2},
\]

losses are equivalent in both disclosure regimes. This arises when the positive welfare effect of uncertainty surrounding the mark-up shock on the output gap \( C \) perfectly offsets its negative welfare effect on price dispersion \( D \). When \( \tilde{\theta} < 2 + \frac{(\sigma_u^2 + \sigma_\mu^2)\sigma_\rho^2}{\xi^2 \sigma_u^2 \sigma_\mu^2} \), the loss associated with the output gap under opacity is smaller than under transparency \((C_O < C_T)\) while the loss associated with price dispersion under opacity is larger than under transparency \((D_O > D_T)\). The opposite holds when \( \tilde{\theta} > 2 + \frac{(\sigma_u^2 + \sigma_\mu^2)\sigma_\rho^2}{\xi^2 \sigma_u^2 \sigma_\mu^2} \).

### 4.4 Optimal monetary policy under intermediate transparency

We now turn to the more interesting case of intermediate transparency where the monetary instrument is perfectly observed by the \( \alpha \)-type firms while the central bank does not make any explicit announcement about its economic assessment. Intermediate transparency is modeled in our setup with a perfect signal on the monetary instrument \( q \), but with an infinitely noisy signal for firms on the central bank assessment \( y^*_{cb} \), i.e. \( \sigma_\phi^2 = 0 \) and \( \sigma_\phi^2 \to \infty \).

By implementing its monetary instrument, the central bank implicitly discloses information to firms about its economic assessment. However, in the absence of an additional announcement, firms cannot unambiguously decipher the rationale behind the implemented instrument. For instance, the central bank may implement an expansionary instrument either because of a positive labor supply shock or because of a negative mark-
up shock. The central bank finds it optimal to adjust its response to labor supply and mark-up shocks in order to shape the information conveyed by its instrument and thereby the beliefs of firms with respect to mark-up shocks.

Figure 1 illustrates the optimal conduct of monetary policy as $\alpha$ goes to one for the three communication strategies considered in this section (transparency, opacity, and intermediate transparency) as a function of price elasticity of demand $\bar{\theta}$ (with $\xi = 0.15$, $\sigma^2_{y^*} = \sigma^2_{x} = \sigma^2_{\rho} = 1$, and $\sigma^2_{\eta} = \sigma^2_{\mu} = 0.2$). The first graph shows the optimal response of the central bank to labor supply shocks, the second graph its optimal response to mark-up shocks, the third graph the loss effect of uncertainty surrounding the mark-up shock $C + D$, as defined in sections 4.2 and 4.3, and the fourth graph the loss effect of uncertainty associated with the output gap $C$ and with price dispersion $D$ separately.

For the extreme cases of transparency and opacity, the optimal responses to labor supply shocks $\zeta_{1,T}$ and $\zeta_{1,O}$ are identical. As the central bank can manipulate in neither of these two extreme cases the beliefs of firms, it accommodates the labor supply shocks given the accuracy of its signal. However, under intermediate transparency, the central bank strongly distorts its response to the labor supply shock in order to optimally shape firms’ expectations and to trade off the output gap against price dispersion.

As previously seen, when $\bar{\theta} < 2 + \frac{\sigma^2_{y^*} + \sigma^2_{\rho}}{\xi \sigma^2_{x} \sigma^2_{\rho}}$, the uncertainty of firms with respect to the mark-up shock has a positive welfare effect associated with the output gap $C$ that is larger than its negative effect associated with price dispersion $D$ under both transparency and opacity regimes, i.e. $C_T + D_T < 0$ and $C_O + D_O < 0$. The same condition holds for intermediate transparency.

The central bank finds it optimal to strengthen under intermediate transparency its response to the labor supply shock when the price elasticity of demand is low in order to reduce the output gap at the expense of a larger price dispersion. The opposite holds when the price elasticity of demand is large: since firms’ uncertainty with respect to the mark-up shock has a negative welfare effect associated with price dispersion that is larger than its positive effect associated with the output gap, the central bank weakens its response to
the labor supply shock for mitigating price dispersion.

The table below shows the unconditional expected loss under transparency \( \mathbb{E}(L_T) \), under intermediate transparency \( \mathbb{E}(L_{IT}) \), and under opacity \( \mathbb{E}(L_O) \) with \( \sigma^2_{\mu^*} = \sigma^2_u = 1, \sigma^2_\eta = \sigma^2_\mu = 0.2 \), and for various parameter values of \( \sigma^2_\rho \), \( \xi \), and \( \bar{\theta} \). The loss under intermediate transparency is always larger than under opacity but smaller than under transparency. The advantage of opacity (or of intermediate transparency) is particularly large when the equivalence condition (11) is strongly violated. The table shows that opacity is particularly welfare improving for low price elasticity of demand when the degree of strategic complementarities \( 1 - \xi \) and the noise of firms’ private signal \( \sigma^2_\rho \) are large.

The fourth line of the table illustrates the unconditional expected loss under intermediate transparency when the central bank ignores the signaling role of its monetary instrument and implements – instead of the policy that is optimal under intermediate transparency – the policy that would be optimal in the case of transparency (\( \zeta_{1,T} \) and \( \zeta_{2,T} \)). The fifth line of the table shows the unconditional expected loss when the central bank implements the optimal policy under opacity (\( \zeta_{1,O} \) and \( \zeta_{2,O} \)) although its monetary instrument is common knowledge among the \( \alpha \)-type firms. This exercise shows that ignoring the signaling role of the policy action can result in large incremental losses when parameters combination strongly departs from the equivalence condition (11).

<table>
<thead>
<tr>
<th>( \sigma^2_\rho = 0.2 )</th>
<th>( \sigma^2_\rho = 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \xi = 0.15 )</td>
<td>( \xi = 0.5 )</td>
</tr>
<tr>
<td>( \bar{\theta} = 2 )</td>
<td>( \bar{\theta} = 60 )</td>
</tr>
<tr>
<td>( \mathbb{E}(L_T) )</td>
<td>38.76</td>
</tr>
<tr>
<td>( \mathbb{E}(L_{IT}) )</td>
<td>29.57</td>
</tr>
<tr>
<td>( \mathbb{E}(L_O) )</td>
<td>27.48</td>
</tr>
<tr>
<td>4. ( \mathbb{E}(L_{IT}) ) w. T policy</td>
<td>37.42</td>
</tr>
<tr>
<td>5. ( \mathbb{E}(L_{IT}) ) w. O policy</td>
<td>34.86</td>
</tr>
</tbody>
</table>
5 Conclusion

This paper analyzes the optimal conduct of monetary policy with imperfect information on the shocks affecting the economy where firms’ prices are strategic complements. It emphasizes that central bank communication intertwines with the design of the optimal monetary policy pattern: the choice of implementing a monetary instrument and the communication strategy of the central bank are two sides of the same coin. As discussed in the extreme cases of transparency and opacity, the optimal monetary policy is a function of firms’ beliefs and thereby of the communication strategy of the central bank. Moreover, the realistic case of intermediate transparency also shows that firms’ beliefs can be shaped by monetary policy whenever its interpretation is ambiguous.

The main contribution of this paper is to highlight that the monetary instrument entails a dual stabilizing role, as a policy response that influences directly the economy and as a vehicle for information that shapes firms’ beliefs. In the case where more information is welfare detrimental, the central bank faces a dilemma because its monetary instrument aimed at stabilizing the economy may harmfully shape firms’ beliefs. Recognizing the signaling role of its instrument, the central bank finds it optimal to distort its policy response in order to mitigate the detrimental information that it may convey.

References


Cukierman, A. (2001). Accountability, credibility, transparency and stabilization policy


A Utility of the representative household

As shown in Adam (2007), maximizing the second order approximation of the welfare of the representative household in our economy is equivalent to maximizing

\[-(y - y^*)^2 - \frac{\hat{\theta}}{\xi} \int_0^1 (p_i - p)^2 \, di. \tag{12}\]

The welfare of the representative household decreases with the output gap and the price dispersion across firms. In our informational setup as described in section 2.4 and assuming the absence of price dispersion previously to the occurrence of the current shocks, the price dispersion across firms can be rewritten as

\[
\int_0^1 (p_i - p)^2 \, di = \int_0^\alpha (p_i - p)^2 \, di + \int_\alpha^1 (p_i - p)^2 \, di
\]

\[
= \alpha \left[ \frac{(1 - \alpha)^2}{\alpha^2} p^2 + \frac{\gamma_3^2 \sigma^2_p}{\sigma_p^2} \right] + (1 - \alpha) p^2
\]

\[
= \frac{1 - \alpha}{\alpha} p^2 + \alpha \gamma_3^2 \sigma^2_p,
\]

where \(\gamma_3\) is the weight a firm assigns to its private signal on mark-up shocks in its pricing rule as defined in (10). Plugging the derivation of the price dispersion according to our informational setup into the general welfare representation (12) delivers the central bank’s objective (3) in the text.
B Linear pricing rule

This appendix solves the rational expectations equilibrium for the pricing rule of the informed α-type firms. Following Morris and Shin (2002), we assume that α-type firms set their price according to the following linear combination of their signals:

\[ p_i = \gamma_1 q_i + \gamma_2 y_{cb,i}^* + \gamma_3 u_i. \]

The optimal weights \( \gamma_1, \gamma_2, \) and \( \gamma_3 \) depend on firms’ expectations about the pricing behaviour of other firms. The conditional estimate of the average price is given by

\[ E_i(p) = \alpha \gamma_1 E_i(q) + \alpha \gamma_2 E_i(y_{cb}^*) + \alpha \gamma_3 E_i(u). \]

Plugging \( E_i(p) \) in the pricing rule (2) and replacing the expectations of firm \( i \) about \( q, y^*, y_{cb}^* \), and \( u \) as defined in (9) yields

\[
\begin{align*}
    p_i &= (1 - \xi)E_i(p) + \xi E_i(q) - \xi E_i(y^*) + E_i(u) \\
    &= (1 - \xi)[\alpha \gamma_1 E_i(q) + \alpha \gamma_2 E_i(y_{cb}^*) + \alpha \gamma_3 E_i(u)] + \xi E_i(q) - \xi E_i(y^*) + E_i(u) \\
    &= [\alpha(1 - \xi)(\gamma_1 \Omega_{11} + \gamma_2 \Omega_{31} + \gamma_3 \Omega_{41}) + \xi \Omega_{11} - \xi \Omega_{21} + \Omega_{41}]q_i \\
    &\quad +[\alpha(1 - \xi)(\gamma_1 \Omega_{12} + \gamma_2 \Omega_{32} + \gamma_3 \Omega_{42}) + \xi \Omega_{12} - \xi \Omega_{22} + \Omega_{42}]y_{cb,i}^* \\
    &\quad +[\alpha(1 - \xi)(\gamma_1 \Omega_{13} + \gamma_2 \Omega_{33} + \gamma_3 \Omega_{43}) + \xi \Omega_{13} - \xi \Omega_{23} + \Omega_{43}]u_i.
\end{align*}
\]

Identifying the coefficients, we get the equilibrium responses of α-type firms to their signals (10) in the text.

C Optimal monetary policy under transparency

This appendix presents the optimal monetary policy under transparency as described in section 4.2 for general values of \( \alpha \), the share of informed firms. The monetary policy
problem under transparency consists in minimizing the unconditional expected loss (3) subject to the equilibrium pricing rule defined in (10) when the monetary instrument \( q \) and the central bank’s assessment \( y_{cb}^* \) are perfectly observed by \( \alpha \)-type firms \((\sigma_\varphi^2 = \sigma_\phi^2 = 0)\).

The corresponding coefficients of monetary policy satisfy:

\[
\begin{align*}
\zeta_{1,T} &= \frac{\sigma_{y^*}^2}{\sigma_{y^*}^2 + \sigma_\eta^2} \\
\zeta_{2,T} &= -\frac{\alpha(\bar{\theta} - 1)}{1 - \alpha + \alpha\bar{\theta}\xi} \frac{\sigma_u^2}{\sigma_u^2 + \sigma_\mu^2}.
\end{align*}
\]

Taking the limit of \( \zeta_{1,T} \) and \( \zeta_{2,T} \) when \( \alpha \) goes to one delivers the optimal monetary policy presented in section 4.2. The equilibrium pricing rule coefficients (10) become under transparency:

\[
\begin{align*}
\gamma_{1,T} &= \frac{\alpha\xi((\bar{\theta} - 1)\sigma_\rho^2\sigma_\mu^2 - \sigma_\rho^2(\sigma_u^2 + \sigma_\rho^2))}{\alpha(\bar{\theta} - 1)(((1 - \alpha + \alpha\xi)\sigma_u^2 + \sigma_\rho^2)\sigma_\mu^2 + (\sigma_\rho^2 - \sigma_u^2)\sigma_\mu^2)}, \\
\gamma_{2,T} &= \frac{(\alpha\xi(1 - \bar{\theta})\sigma_u^2\sigma_\mu^2 + (\sigma_u^2 + \sigma_\rho^2)\sigma_\mu^2)}{\alpha(\bar{\theta} - 1)(((1 - \alpha + \alpha\xi)\sigma_u^2 + \sigma_\rho^2)\sigma_\mu^2 + (\sigma_\rho^2 - \sigma_u^2)\sigma_\mu^2)}, \\
\gamma_{3,T} &= \frac{\sigma_u^2\sigma_\rho^2}{\sigma_u^2\sigma_\rho^2 + (\sigma_\rho^2 - \sigma_u^2)\sigma_\mu^2 + (\sigma_\rho^2 + (1 - \alpha)\sigma_u^2)(\sigma_\mu^2 + \sigma_u^2\sigma_\rho^2)}.
\end{align*}
\]

### D Optimal monetary policy under opacity

This appendix presents the optimal monetary policy under opacity as described in section 4.3 for general values of \( \alpha \), the share of informed firms. The monetary policy problem under opacity consists in minimizing the unconditional expected loss (3) subject to the equilibrium pricing rule defined in (10) when neither the monetary instrument \( q \) nor the central bank’s assessment \( y_{cb}^* \) are observable by \( \alpha \)-type firms \((\sigma_\varphi^2, \sigma_\phi^2 \to \infty)\).

The corresponding coefficients of monetary policy satisfy:

\[
\begin{align*}
\zeta_{1,O} &= \frac{\sigma_{y^*}^2}{\sigma_{y^*}^2 + \sigma_\eta^2} \\
\zeta_{2,O} &= \frac{\alpha(\bar{\theta} - 1)(\sigma_\rho^2 + (1 - \alpha)\sigma_u^2)\sigma_u^4}{(\sigma_\rho^2 + (1 - \alpha + \alpha\xi)\sigma_u^2)\sigma_\mu^2 + (\sigma_\rho^2 + (1 - \alpha)\sigma_u^2)(\sigma_\mu^2 + (\sigma_\rho^2 + (1 - \alpha + \alpha\bar{\theta}\xi)\sigma_u^2)\sigma_\mu^2).}
\end{align*}
\]
Taking the limit of $\zeta_{1,O}$ and $\zeta_{2,O}$ when $\alpha$ goes to one delivers the optimal monetary policy presented in section 4.3. The equilibrium pricing rule coefficient becomes under opacity:

$$
\gamma_{3,O} = \frac{(1 - \alpha \xi (\bar{\theta} - 1)(\sigma^2 + (1-\alpha)\sigma^2_\nu))\sigma_\nu^4}{(1 - \alpha + \alpha \xi)\sigma_\nu^2 + \sigma^2_\rho}.
$$

E Graphs

Figure 1: Optimal monetary policy and welfare effects for alternative communication strategies
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This list contains the Working Paper written after January 2010, 1rst. The complete list is available upon request.