« The banking crisis with interbank market freeze »

Auteurs

Jin Cheng, Meixing Dai, Frédéric Dufourt

Document de Travail n° 2015 – 20

Octobre 2015
The banking crisis with interbank market freeze

Jin Cheng, Meixing Dai and Frederic Dufourt

October 14, 2015

\footnote{Jin Cheng: Université d’Auvergne (CERDI), UMR CNRS 6587. E-mail: michellechengjin@hotmail.com. Jin Cheng thanks the Region of Auvergne for funding during the period when this paper is written. Meixing Dai: Université de Strasbourg (BETA), CNRS. E-mail: dai@unistra.fr. Frederic Dufourt: Aix-Marseille Université (Aix-Marseille School of Economics), CNRS-GREQAM & EHESS and Institut Universitaire de France. E-mail: frederic.dufourt@univ-amu.fr.}
Abstract

This paper studies banking crises characterized by interbank market freeze, fire sale and contagion in a model with collateralized interbank loans. We examine the role of interbank market in spreading and amplifying crises by distinguishing three sources of liquidity risks, i.e., panic-induced run, foreign sovereign debt crisis and gambling behavior. Our results underline that ex-post insufficient bank capital and/or liquidity reserves could lead the interbank market to malfunction. However, implementing more restrictive regulations to reinforce banks’ resilience to shocks could hamper their role as financial intermediary and may have perverse effects on their gambling behaviors. Therefore, the crisis management by a government even without monetary sovereignty is crucial in ensuring the well-functioning of the interbank market and hence the stability of the banking system, and it is effective as long as the scale of bailout is credible in the sense of not compromising the government’s solvency.

Keywords: Banking crisis, interbank market, market discipline, capital ratio, multiple equilibria, bank run, gambling asset, asymmetric information, and sovereign debt crisis.

JEL Classification Numbers: E43, G01, G11, G12, G18, G02, G28.
1 Introduction

The 2008 global financial crisis and the 2010-12 euro-zone crisis have revealed the fragility of the banking system strongly dependent on interbank funding and the inadequacy of regulatory mechanisms, based on Basel II, largely relied on market discipline. The failure of interbank market discipline to prevent excessively risky investment and to send any early warning signals, along with the freeze of the interbank market facing adverse shocks, represent some main characteristics of recent financial meltdowns. Despite great concerns among economists and policymakers worldwide aroused by the disastrous financial and economic impacts of these deficiencies, the literature on financial crisis does not pay due attention, especially from a theoretical perspective, either to the operating ‘principle’ of the interbank market based on market discipline or to the ‘rationale’ of its freeze during a banking crisis.

This paper aims at analyzing the market discipline in terms of capital requirement and liquidity reserves imposed by interbank lenders and its effect on banking contagion during financial crises. We examine the role of market discipline in ensuring the stability of the banking sector and the extent to which a government without monetary sovereignty can rescue illiquid banks affected by the malfunctioning of the interbank market, and discuss some proposals for the reform of the banking regulation. In our model, a financial crisis could be associated with banks’ ex-post inappropriate capital and/or reserve ratios, although these ratios are ex-ante in accordance with the interbank market participation constraint.

Our model allows examining how the functioning of the interbank market affects the financial position of commercial banks during normal and crisis times. We investigate three sources of liquidity risks with different implications for the role of the interbank market in spreading and amplifying the crisis in the banking system: (i) a confidence crisis triggered by the self-fulfilling bank-run; (ii) a sudden depreciation of banks’ ex-ante safe assets; (iii) a liquidity shock resulting from asymmetric information about the balance sheet of borrowing banks following the appearance of gambling assets. In particular, our model allows to explain why the interbank market freeze, by amplifying the needs of fiscal bailouts, became an important factor contributing to drawing several euro-periphery countries into a twin banking and sovereign debt crisis given the absence of alternative solutions other than massive fiscal bailouts.

In our framework, commercial banks pool resources by issuing deposits and ordinary shares, and then invest them in liquid government bonds and illiquid long-term projects. Initially
identical banks will be divided into two types following an idiosyncratic shock that affects a fraction of banks in the intermediate stage. Banks redistribute the idiosyncratic risk of illiquid projects through collateralized-lending in an interbank market that imposes resources constraints on borrowing banks. The presence of this market has considerable impacts on banks’ balance sheet in both normal and crisis times. Due to that, it substantially affects the effectiveness of prudential regulation and the credibility of governmental crisis responses.

Our framework allows to elucidate the operating ‘principle’ of the interbank market thanks to the integration of the interbank-market discipline. The latter is defined as market-based resource constraints on banks’ balance sheet such as minimal capital and liquidity reserve ratios, which are observable by lenders with full transparency with the effects of limiting the risk-taking in the absence of asymmetric information and warranting the interbank lending in future dates. Although these constraints are not imposed mandatorily, banks voluntarily abide by them so as to access the interbank market to efficiently deal with potential liquidity shortage.

The inclusion of market discipline in a banking crisis model also allows to understand the “rationale” behind the interbank market freeze in crisis times and to better anticipate crisis contagion between banks, between the banking system and the government budget, and across countries. By allowing an insightful understanding about how the interbank market functions and affects the generation and propagation mechanism of financial crises, our framework allows to make some policy proposals relevant for future financial reforms.

The main results of this paper are summed up as follows: (1) Despite non-mandatory, the interbank market discipline provides banks with efficient incentives to respect minimal capital and liquidity ratios. (2) In normal times, a banking system with a well-functioning interbank market exhibits a greater stability and improves social welfare compared to a banking system with autarkic banks. (3) The lending decision of interbank lenders can act as a self-fulfilling prophecy in triggering or not a bank run by depositors. (4) When the scale of banking crisis is large relative to the government budget, and when the role of the central bank as lender of last resort is absent, the interbank market freeze can immediately lead to a mutually reinforcing twin banking and sovereign debt crisis, even if no fiscal bailout is implemented. A foreign sovereign debt crisis can induce similar consequences by creating an aggregate liquidity shortage in domestic interbank market. (5) The market discipline can have perverse effects in the presence of asymmetric information and gambling assets in the sense that the interbank market functions normally only when borrowers have an exceedingly illiquid balance sheet. (6) When
confronting an aggregate liquidity shock, the interbank market has the effect of generating a
more important decrease in the size of eligible collateral assets. (7) The risk of bank failure and
the malfunctioning of the interbank market can be reduced but cannot be completely eliminated
by prudential regulation except at the risk of depriving banks’ role as financial intermediaries.

The remainder of the paper is organized as follows. Section 2 reviews the literature. Section
3 describes the basic model with an interbank market. Section 4 examines the functioning of
this market in the presence of liquidity risks due to a bank run. Section 5 studies the response of
the interbank market to the depreciation of ex-ante risk-free sovereign bonds. Section 6 shows
the implications of gambling asset associated with asymmetric information for the functioning of
the interbank market. Section 7 discusses prudential regulation and the crisis management via
a fiscal bailout taking account of the government’s budget position. The last section concludes.

2 Related literature

We depart from traditional banking crisis model of Diamond and Dybvig (1983) by paying
attention to the interactions between banks with liquidity surplus and those enduring liquidity
shortage in the interbank market, and to the role of market discipline in limiting risk-taking.

This paper contributes to the strand of theoretical literature on banking that analyzed mar-
et discipline. In a single-bank economy, uninsured and liquid deposits keep a bank’s portfolio
choice in line with the preferences of depositors while the threat of a bank run by informed
depositors discourages the bank’s owners from investing in too risky projects or committing
fraud (Calomiris and Kahn 1991, Flannery 1994, Dwyer and Samartín 2009). Furthermore,
liquid deposits could even have disciplinary effects in models excluding asymmetric information
(Qi 1998, Diamond and Rajan 2001a,b, 2005). However, this literature does not deal with
the interaction between bank runs and contagion. In contrast, Hasman, Samartín and Bommel
(2013) analyze market discipline in a many-bank economy where contagion and bank runs in-
teract with market discipline exercised by informed depositors that withdraw their deposits to
punish banks. To their difference, we put accent on market discipline exercised by interbank
lenders in response to idiosyncratic shocks.

Our paper is related to a number of studies on banking crisis that emphasize the inefficiency

---

1 Market discipline could be weakened by explicit deposit insurance and regulatory discipline (Billett, Garnikel
discipline is important since according to Hasan, Siddique and Sun (2014), market-implied capital is more sensitive
to risk factors and seems to be a potential leading indicator of capital adequacy.
of the interbank market. Rochet and Tirole (1996), Aghion, Bolton and Dewatripont (2000), and Allen, Garletti and Gale (2009) underline that the interbank market can not only contribute to spreading shocks stemming from outside the local financial system, but can also be the original culprits in widespread crises. Diamond and Rajan (2005) show that due to the feedback interactions through the interbank market, a liquidity mismatch can induce insolvency, which in turn will aggravate the liquidity shortage and lead to a bank run. Freixas and Jorge (2008), and Heider, Hoevova and Holthausen (2015) highlight the effects of banks' asset risk on the functioning of the interbank market and the inefficiency resulting from asymmetric information. The major difference in our paper is that we investigate the operating principle of the interbank market by providing an explicit role to interbank-market discipline.

Allen and Gale (2004), Diamond and Rajan (2011) and Hasman, Samartín and Bommel (2013), consider shocks stemming from the liability side of banks' balance sheet as the major culprit of liquidity shortage in the interbank market. Our paper accounts for shocks arising from both the asset and the liability side of the balance sheet. Considering that banks are submitted to both risks of a self-fulfilling bank run and non-performing assets, our model is close to the spirit of Tirole (2011).

Through its emphasis on the role of the bank capital in improving the banking stability, our paper is reminiscent of some earlier works such as Rochet and Tirole (1996), Aghion, Bolton and Dewatripont (2000), Allen and Gale (2000), and Brusco and Castiglionesi (2007). One important difference with these works is that banks in our setting endogenously choose a capital level that realizes their optimal resource allocation and respects interbank market discipline. In Kharroubi and Edouard (2009), due to banks' incentives to hoard liquidity, it can result in a liquidity provision in the interbank market lower than the social optimum. Our model shows that during normal times the interbank market permits banks to hoard an optimal liquidity provision that is less than in an autarkic banking system. Nevertheless, the issue of insufficiency of liquidity provision can arise in crisis times since banks can see their financial position greatly worsened by the malfunctioning of the interbank market.

Of great relevance to our paper is the literature on "risk shifting" or "gambling for resurrection" initiated by the seminal work of Kareken and Wallace (1978).² In the spirit of Agliardi and Andergassen (2009) and Boyd and Kakenes (2014), our model deals with the issue of gambling asset that is attractive for liquidity-deficit banks. Our results elucidate that, under asymmetric

---

²See Gorton and Winton (2003) for a literature review.
information, the suspicion of gambling behaviors can lead to an interbank market freeze while the complicity of liquidity-surplus banks encourages the gambling behavior of borrowing banks in some circumstance. Moreover, we show that a pre-committed penalty tax, as an efficient regulatory instrument, should be imposed on banks’ profits from gambling investments. Such a tax plays the same role as the penalty on looting considered by Boyd and Hakenes (2014) that reduces both the looting and the gambling behaviors of banks.

Finally, by examining the government crisis management similar to that carried out during the euro-zone crisis, this paper is closely linked to models of sovereign debt crisis built by Bolton and Jeanne (2011), Acharya, Drechsler and Schnabl (2014), and Cheng, Dai and Dufourt (2014), considering that assistance from the ECB was usually deferred and insufficient. To their difference, this paper focuses on the effectiveness of fiscal bailout in dealing with the interbank market freeze and the resulting banking crisis.

3 Basic framework

3.1 Environment

We consider a small open economy populated by a large number of ex-ante identical domestic residents of mass 1. Each period is divided into three stages, i.e., the planning stage \( t_0 \), the intermediate stage (short-term) \( t_1 \) and the final stage (long-term) \( t_2 \), respectively.

Domestic residents are of two types: impatient or patient. An impatient agent derives utility only from consuming at \( t_1 \), whereas a patient agent derives utility from consuming at \( t_2 \). Each domestic resident is endowed with an amount \( e > 0 \) of tradable good at \( t_0 \). Yet, information about agent types is private and is revealed only at \( t_1 \). Thus, domestic residents are uncertain about their type at \( t_0 \), but know the probability \( \lambda \) of being impatient, which is identical for all agents. Denoting by \( x \) the amount of good consumed at \( t_1 \) and by \( y \) the amount of good consumed at \( t_2 \), the expected utility of the representative domestic resident at \( t_0 \) is:

\[
\lambda U(x) + (1 - \lambda)U(y), \quad (1)
\]

where \( U(\cdot) \) is a CRRA instantaneous utility function defined by \( U(c) = c^{1-\sigma}/(1 - \sigma) \) with \( \sigma \in (0, 1) \) representing a positive relative risk aversion coefficient.

Two investment vehicles exist in the economy: liquid government bonds accessible to all
agents, and illiquid long-term production projects in which only commercial banks can invest.\footnote{We assume that such projects requires a special human capital uniquely possessed by commercial banks.} Domestic residents will entrust their endowments to banks if the return on deposit dominates that on their direct investment in government bonds.\footnote{Given the uncertainty about residents’ type and the inaccessibility of long-term projects, the return from residents’ direct investment equals to that from government bonds. This implies that commercial banks, by financing long-term assets, are able to issue deposits that ensure a higher utility for residents.}

**Government bonds**

Both commercial banks and domestic residents can purchase liquid domestic and foreign government bonds at $t_0$. Denote respectively by $r^d_{ij}$ and $r^f_{ij}$ the interest rate on domestic and foreign government bonds from $t_i$ to $t_j$. We assume that these bonds are *ex-ante* risk-free, so that the interest rates applied to them equal to corresponding risk-free international interest rates, i.e., \( r^* \) for the long term:

\[
 r^d_{01} = r^f_{01} = 0 \quad \text{and} \quad r^d_{02} = r^f_{02} = r^*_0. \tag{2}
\]

Conditions (2) imply that the interest rates from $t_1$ to $t_2$ are such that $r^d_{12} = r^f_{12} = r^*$.\footnote{Other parameters, e.g., $r_0^*$, are used to distinguish between the short and long term.}

**Long-term production technology**

A long-term constant-return-to-scale production technology is used to produce the unique good of the economy, which is freely traded in the world market and can be consumed or invested. Each producer needs to borrow at $t_0$ one unit of good from a bank. But only a fraction of projects can be financed by banks due to the scarcity of available long-term funding. Owning to the competition between entrepreneurs to obtain funds, banks take the entire output from the projects that they have financed at $t_0$. Banks are ex ante identical at $t_0$. However, with probability $\pi \ (< 1)$ an idiosyncratic shock will hit a bank at $t_1$ so that its long-term projects require a refinancing of $\phi \ll 1$ units of good (fresh liquidity) to be brought to completion at $t_2$ and its financial situation deteriorates at $t_1$. These projects are labeled as ‘non-performing’. Banks unaffected by the idiosyncratic shock are labeled as ‘type 1’ and those hit by it are called ‘type 2’. A unit of matured long-term asset yields $(1 - \tau)R$ units of good at $t_2$ after tax levied at the rate $\tau$, regardless of its position at $t_1$. But, if a reinvestment equal to $\phi$ is not in place at $t_1$, a non-performing project yields nothing at $t_2$, and hence a large liquidity shock occurs.
At $t_1$, long-term projects can be prematurely liquidated at a net fire-sale price of $(1 - \tau)r_l^p$ for a performing asset and of $(1 - \tau)r_l^{np}$ for a non-performing asset, such that

$$R > 1 > r_l^p > r_l^{np}. \quad (3)$$

Thus, the premature liquidation is costly for any bank. The assumption that $r_l^p > r_l^{np}$ is justified by the fact that the continuation of a non-performing project requires refunding at $t_1$.

Moreover, the expected net return from a long-term asset completing at $t_2$ dominates that from maturing two-period government bonds, such that

$$(1 - \tau)R - (1 + \pi\phi) > r_0^a. \quad (4)$$

Condition (4) implies that it is optimal for banks to borrow as much as they can at $t_0$ so as to invest as many resources as possible in long-term projects.

**Shareholders**

Following Allen and Carletti (2006) and Allen and Gale (2007), we consider a type of investors with risk-neutral preferences, called ordinary shareholders, who have an initial endowment $a$ at $t_0$ and no more endowment in future dates. They either consume or buy banks’ common shares at $t_0$. Being banks’ shareholders, they can claim dividends at each stage $d_t$, with $t = 0, 1, 2$, after the payment to bank creditors. Their utility function is

$$u(d_0, d_1, d_2) = R(1 - \tau)d_0 + d_1 + d_2.$$ 

Shareholders obtain a utility of $R(1 - \tau)a$ if they consume all the endowments at $t_0$, and they are indifferent between consuming at $t_1$ and $t_2$. Therefore, they will buy the common share if its rate of return is no less than $R(1 - \tau)$ so as to compensate for the consumption renounced at $t_0$. Let $K(\leq a)$ be the bank capital. The incentive constraint for holding bank capital is

$$d_1 + d_2 \geq (1 - \tau)RK.$$ 

This constraint implies that banks should hold enough lucrative long-term assets to ensure the dividend payment. Given long-term projects maturing at $t_2$, dividends in normal times are
distributed only at $t_2$. The above incentive constraint for shareholders can be rewritten as:

$$d_2 \geq (1 - \tau)RK. \quad (5)$$

We assume for simplicity that, at the aggregate level, domestic investors’ endowments are always large enough to meet banks’ capital needs, i.e., $a > K$.

### 3.2 Commercial banks and interbank market discipline

In a perfectly competitive deposit market, commercial banks must promise the highest possible payment when issuing demand deposits. In the absence of an interbank market, an individual bank has no incentive to issue common shares (capital). In effect, the net return to shareholders $(1 - \tau)(R - 1)$ is higher than that from long-term investment $(1 - \tau)R - (1 + \pi\phi)$, implying a subsidy from depositors to shareholders. Thus, a higher capital level will result in a lower remuneration to depositors and make a bank less competitive in the deposit market.

Denote by $A$ a bank’s holding of long-term assets. Each bank in autarky should keep liquidity reserves no less than $\phi A$ to ensure potential refinancing when hit by an idiosyncratic shock. From the social planner’s viewpoint, this resource allocation is sub-optimal since each bank has only a probability $\pi$ to be hit by this shock. At the aggregate level, the excessive hoarding of liquidity, accounting for $(1 - \pi)\phi A$, leads to a lower output. As a result, banks in a competitive deposit market have interest in forming an interbank market in which they can deal with idiosyncratic shocks through the redistribution of the reserves held within the banking system. We assume for simplicity that all domestic banks do not borrow in the international financial market to finance neither long-term projects nor the reinvestment.\(^5\)

To be able to borrow, borrowing banks must reassure lending banks facing counter-party risks with a solid balance sheet. Confronted to similar idiosyncratic shocks, all banks have \textit{ex-ante} incentives to strengthen their balance sheet in order to obtain interbank loans when necessary. In this sense, the existence of the interbank market imposes a market discipline.

\textbf{Definition 1 Market discipline in the interbank market.} Market discipline refers to non-mandatory resource constraints imposed by the interbank market on banks’ balance sheet in order to reduce the riskiness of interbank lending in future dates. It is built on two pillars: the liquidity requirement and the capital requirement (as shown below).

\(^5\)Assuming that banks can borrow in the international financial market will slightly complicate the algebraic analysis but will not change our fundamental findings.
Participating in the interbank market, banks can reduce their liquidity reserves, kept in form of government bonds and necessary for refinancing, from $\phi A$ to $\pi \phi A$. We can then define

$$\beta \equiv \pi \phi$$

(6)

as the minimum liquidity ratio imposed by the market discipline. The latter, while ensuring the interbank lending for refinancing troubled projects, will not imposes a minimum liquidity ratio greater than $\beta$, as a regulator may do.

As each bank hoards a reserve $\pi \phi A$ and ‘type 1’ (or lending) banks account for a proportion $1 - \pi$ of all banks, the total liquidity surplus (or supply) in the interbank market at $t_1$ is $(1 - \pi)\pi \phi A$. As ‘type 2’ (or borrowing) banks stand for a proportion $\pi$ of all banks, their total liquidity shortage (or demand) at $t_1$ is $\pi(1 - \pi)\phi A$. As a result, all banks will keep their liquidity reserve ratio equal to $\beta$ at the equilibrium.

We assume that if a ‘type 2’ bank fails after receiving the loan, the ‘type 1’ bank takes over the collateral but can only collect $(1 - \tau)(R - \delta)$ units of good per collateral at $t_2$, with $\delta (< R)$ denoting its effort cost of supervising a long-term project that it starts monitoring from $t_1$. The after-tax gross return from the seized collateral is lower than that from government bonds, i.e.,

$$(1 - \tau)(R - \delta) < 1 + r_{12}^*,$$

(7)

so that taking over the collateral is unprofitable for ‘type 1’ banks. To avoid the loss caused by counter-party risks, ‘type 1’ banks will not lend when they expect the failure of ‘type 2’ banks.

Consequently, unlike the mutual insurance, the well-functioning of the interbank market requires borrowers to hold enough collateral to warrant their ability to repayment. Since the deposit payments are fixed and non-negotiable, the assets financed by deposits ($A - K$) cannot serve as collateral. Thereby, the quantity of pledgeable assets is given by the investment financed by bank capital, and thus equals to $K$. Provided that a maturing project yields $(1 - \tau)R$ at $t_2$, the size of an interbank loan per unit of collateral at $t_1$ is at most equal to $\frac{(1 - \tau)R}{1 + r_{12}^*}$, i.e., the present value of collateral at $t_1$ with the discount rate $r_{12}^*$ corresponding to the net risk-free interest rate from date $t_1$ to $t_2$. Denoted by $L$ the scale of interbank loan, we have:

$$L \leq \frac{(1 - \tau)R}{1 + r_{12}^*}K.$$

(8)

*It is easy to show that $\pi \phi$ is also the liquidity reserve ratio required by a central planner.*
Taking account of its own liquidity reserves $\pi \phi A$, a ‘type 2’ bank needs to borrow at least $(1 - \pi) \phi A$ from the interbank market to fulfill the full scale continuation of long term projects $\phi A$. Substituting $L$ by $(1 - \pi) \phi A$, we rewrite condition (8) as

$$
(1 - \pi) \phi A \leq \frac{(1 - \tau) R}{(1 + r_{12}^*)} K, 
$$

i.e., the liquidity demand of a ‘type 2’ bank (the left-hand side (LHS) of (9)) should be entirely backed by the value of its pledgeable collateral (the right-hand side (RHS) of (9)) at $t_1$. Clearly, uncertain about their type, each bank must raise a capital $K$ to satisfy condition (9). Denote by $k_i = \frac{K_{\text{min}}}{A}$ the minimal capital ratio imposed by the market discipline with $K_{\text{min}}$ representing the minimal level of capital that a bank must have to participate in the interbank market. Using (9) and $r^* = r_{12}^*$, we obtain

$$
k_i = \frac{(1 + r^*)(1 - \pi) \phi}{(1 - \tau) R}. \tag{10}
$$

It results that $k_i$ is decreasing with the risk of assets $\pi$ and is hence pro-cyclical. In contrast, the liquidity ratio $\beta$ is countercyclical according to (6).

The expected net return from a unit of long-term project and a unit of capital are $(1 - \tau) R - 1 - \pi \phi$ and $(1 - \tau) R - 1$, respectively. Thus on average, a bank needs to subsidize shareholders with an amount of $\pi \phi$, which comes to reduce the return for deposits, implying that the cost of issuing bank capital increases with $\pi$. On the other hand, the opportunity cost of holding liquidity reserves instead of long-term assets equals to $[(1 - \tau) R - 1 - \pi \phi] - (1 + r_{12}^*)$, which decreases with $\pi$. As a result, when $\pi$ rises, it is optimal for banks to hold less capital but more liquidity to implement refinancing, and vice versa.

If banks respect the market discipline (minimal capital and liquidity ratios), the interbank market functions well in normal times. Using the definition of $\beta$ and $k_i$ given by (6) and (10) respectively, the expected rate of return to banks’ shareholders is:

$$
E[d_2] = \frac{\text{Gain if the bank is of ‘type 2’}}{(1 - \tau) R (K - k_i A)} + \frac{\text{Gain if the bank is of ‘type 1’}}{(1 - \pi) [(1 - \tau) R K + (1 + r_{12}^*) \beta A]}
$$

$$
= (1 - \tau) R K. \tag{11}
$$

Given that $E[d_2]$ satisfies constraint (5), banks respecting the market discipline can honestly promise the dividend payment and thus have no difficulty in raising capital. Otherwise, the interbank market will not provide enough funds for full scale reinvestment, implying a premi-
ture liquidation and hence a potential loss to banks’ shareholders since \( r_l < \frac{R}{1+r_1} \). Thereby, banks will voluntarily respect the market discipline, although it is not a mandatory government regulation. The following proposition encapsulates the above results:

**Proposition 1** Despite not mandatory, the interbank market discipline provides banks with efficient incentives to respect its minimal capital and liquidity reserve ratios, with the first ratio being pro-cyclical and the second being countercyclical.

We note that inverting the capital ratio of banks leads to the leverage ratio, which is also pro-cyclical in the sense that it is low when the risk is low (small \( \pi \)) and *vice versa*.

### 3.3 Optimization problem of commercial banks

In the benchmark case, there is no shock affecting the aggregate liquidity, and domestic and foreign government bonds are assumed to be risk-free. The latter assumption implies that the composition of banks’ bond portfolio is undetermined. Here, we simply assume that domestic banks choose to allocate a fraction \( \gamma \in (0, 1) \) and \( 1 - \gamma \) of their total bond purchases \( B \) to the purchase of domestic and foreign government bonds, respectively.

Commercial banks compete for deposits by offering the highest possible return on it, then invest the largest possible amount in long-term projects. Their investment decisions are restricted by two kinds of constraints. First, they must obviously synchronize their available liquidity with the withdrawal requests of depositors at each stage under any circumstances. Second, to ensure full scale reinvestment when hit by liquidity shocks, banks must comply with the market discipline. Besides, given the inefficiency of premature liquidation, banks will certainly avoid its occurrence when designing the investment plan, such that the ex-ante expected scale of liquidation is \( l = 0 \). The constraints faced by the representative bank at \( t_0 \) are:

\[
K \geq k' A, \quad (12)
\]

\[
\frac{B}{1+r^*} + A \leq \kappa + K, \quad (13)
\]

\[
\lambda x + \beta A \leq \frac{B}{1+r^*}, \quad (14)
\]

\[
(1 - \lambda)y + (1 - \tau)RK \leq (1 + r^*)(\frac{B}{1+r^*} - \lambda x - \beta A) + (1 - \tau)RA, \quad (15)
\]

Condition (12) is the capital requirement constraint. Condition (13) is the bank’s resource constraint at \( t_0 \) and specifies that its total investment including \( A \) units of long-term assets and
$B$ units of government bonds cannot exceed its available resources $e + K$. Condition (14) is the bank’s solvency constraint at $t_1$. It indicates that the bank’s available liquidity from selling government bonds, $\frac{B}{1 + r_2}$, must be adequate relative to the scale of the withdrawal by impatient depositors, $\lambda x$, and the expected amount of required refinancing, $\beta A$. When (14) is satisfied, there will be no concern over the solvency of the bank in normal circumstances. Otherwise, the bank is insolvent and thus needs to liquidate all its long-term assets. Finally, (15) is the bank’s solvency condition at $t_2$. The liquidity available to the bank should be sufficient to clear all the claims by patient depositors, $(1 - \lambda)y$, and shareholders, $(1 - \tau)RK$.

### 3.4 The optimal resource allocation

Banks act to maximize the social welfare by making the optimal allocation of resources. This allocation, obtained as the good Nash equilibrium of the demand deposit system described above, characterizes the economy in “normal times”. The equilibrium values of variables at the social optimum are denoted by a tilde over the variables. Condition (4) implies that resource constraints (13) and (14) must bind to maximize the social welfare. In a perfectly competitive deposit market, banks realize no profit after the payment to depositors and shareholders so that constraint (15) is also binding.

#### 3.4.1 Optimal allocation with the interbank market

The binding constraint (14) implies that at the optimum, the liquidity reserves of banks should just meet the minimal liquidity ratio $\beta$. Clearly, given the return from safe bonds is dominated by that from long-term projects, banks have no interest to hold extra bonds relative to the amount required by the interbank borrowing.

To minimize the costs linked to the refinancing of long-term projects, each bank must constitute a capital that allows it to participate in the interbank market at $t_1$. The high opportunity cost of capital and the perfectly competitive deposit market incite all banks to choose a level of capital with the capital ratio verifying condition (10) so that:

$$\tilde{K} = k^i \tilde{A},$$

(16)

In fact, keeping a capital ratio lower than $k^i$ makes banks unable to borrow enough funds from the interbank market and implies hence a risk of costly premature liquidation. On the contrary,
holding a capital ratio higher than \(k^i\) implies a lower return to deposits and makes a bank uncompetitive in the deposit market if other banks keep a ratio equal to \(k^i\).

Banks maximize the expected utility of depositors:

\[
\max E[\lambda U(x) + (1 - \lambda)U(y)],
\]

subject to constraints (12)-(15). Given that the latter are binding at the optimum, the optimal consumption allocation must satisfy the following social transformation curve:

\[
\Phi \lambda x + (1 - \lambda)y = \Phi e
\]

where the composite coefficient \(\Phi \equiv \Phi(k^i, \beta) \equiv \frac{(1-k^i)(1-\gamma)R}{(1-k^i+\beta)} > 1\) stands for the expected gross return on deposits withdrawn at \(t_2\). The LHS and the RHS of (18) are respectively the expected value of deposits and the total wealth of residents if they all withdraw and consume at \(t_2\). Here, \(\Phi\) also represents the marginal rate of substitution between the consumption at \(t_1\) and that at \(t_2\). It means that if impatient residents renounce to consume \(x\) at \(t_1\), they obtain \(\Phi x\) at \(t_2\).

Domestic residents will entrust their resources to a bank if the return from deposits is no smaller than that from government bonds. Thus, the incentive constraint \(\Phi \geq 1 + r^*\) should be satisfied. We assume that \(\Phi \equiv \Phi(k^i, \beta)\) is sufficiently high so that (19) always holds.

Using (18) and the CRRA utility function, we easily obtain the first-order condition:

\[
\frac{\tilde{x}}{\tilde{y}} = (\frac{\tilde{x}}{\tilde{y}})^{-\sigma} = \Phi.
\]

For patient residents to have no interest in mis-presenting their type at \(t_1\), the truth-telling condition \((1 + r^*)x < y\) must be verified. Given (19), it is straightforward to show that the relation between \(\tilde{x}\) and \(\tilde{y}\) defined by (20) satisfies this truth-telling condition so that patient residents will honestly report their type and withdraw only at \(t_2\) in normal times. Thereby, banks are able to design efficient deposit contracts for each type of resident.
Combining (18) with (20) yields the best consumption plan:

\begin{align*}
\tilde{x} &= \frac{\theta}{\lambda} e, \\
\tilde{y} &= \frac{1 - \theta}{1 - \lambda} \Phi e, \\
\end{align*}

(21) (22)

where the composite coefficient \( \theta \equiv \left( 1 + \frac{1 - \lambda}{\lambda} \Phi \frac{1 - \sigma}{\sigma} \right)^{-1} \), being within the unit interval, is important for determining the revenue distribution between impatient and patient residents. It decreases with \( \Phi \), meaning that the higher is \( \Phi \), the smaller is the proportion of the pay-off to impatient residents. It increases with \( \sigma \), indicating that the higher is the degree of risk aversion, the lower the residents’ willingness to substitute consumption over time.

Given (12)–(15) and (21)-(22), the bank’s optimal investment plan is:

\begin{align*}
\tilde{A} &= \frac{1 - \theta}{1 - k^i + \beta} e, \\
\tilde{B} &= \frac{(1 + r^*)[\beta + \theta(1 - k^i)]}{1 - k^i + \beta} e. \\
\end{align*}

(23) (24)

According to (23)–(24), the optimal investment in risky long-term assets is negatively related to \( \theta \), and the inverse is true for the optimal holding of government bonds.

### 3.4.2 Optimal allocation with autarkic banks

To highlight the role of interbank lending, we now examine the optimal allocation of an autarkic banking system by removing the interbank market. Commercial banks in this autarkic banking system will not raise any capital, i.e., \( K = 0 \), so as to maximize the return to depositors. The maximization problem of a representative autarkic bank is subject to following constraints:

\begin{align*}
\frac{B^a}{1 + r^*} + A^a &\leq e, \\
\lambda x^a + \phi A^a &\leq \frac{B^a}{1 + r^*}, \\
(1 - \lambda)y^a &= (1 + r^*) \left( \frac{B^a}{1 + r^*} - \lambda x^a - \phi A^a \right) + (1 - \tau) R^a A^a, \\
\end{align*}

(25a) (25b) (25c)

where the superscript ‘\( a \)’ stands for an ‘autarkic’ banking system. The absence of the interbank lending implies that banks must keep a liquidity ratio no less than \( \beta^a \equiv \phi \ (> \beta) \). The new
The social transformation curve is:

$$
\Phi^0 \lambda x^0 + (1 - \lambda) y^0 \equiv \Phi^0 e,
$$

where $$\Phi^0 \equiv (1 - \tau)R(1+\phi).$$ Using the definition of $$\beta$$ and $$k^i$$, it is straightforward to show that $$\Phi^0 < \Phi,$$ $$\forall \pi < 1$$, implying that the presence of the interbank market improves social welfare.

The optimal resource allocation in an economy with an autarkic banking system is:

$$
\tilde{x}^a = \frac{\theta^a}{\lambda} e, \quad \tilde{y}^a = \frac{1 - \theta^a}{1 - \lambda} \Phi^0 e, \quad \tilde{A}^a = \frac{1 - \theta^a}{1 + \phi} e \quad \text{and} \quad \tilde{B}^a = \frac{(1 + r^*)(\beta^a + \theta^a)}{1 + \phi} e,
$$

where $$\theta^a = \left[1 + \frac{1}{\lambda}(\Phi^0)^{\frac{1}{\sigma}}\right]^{-1}.$$ The fact that $$\Phi^0 < \Phi$$ yields immediately $$\theta^a > \theta$$ and:

$$
\tilde{x}^a < \tilde{y}^a, \quad \tilde{B}^a < \tilde{B}^0 A^a, \quad \tilde{y}^a > \tilde{y}^0 a \quad \text{and} \quad \tilde{A} > \tilde{A}^a.
$$

Comparing the equilibria with and without the interbank market shows that, in normal times, the latter affects the banking system in three aspects: (i) The market discipline imposed for liquidity funding incites banks to voluntarily keep a minimal level of capital that constitutes a buffer to absorb the liquidity risk due to mismatch in maturity, while such buffer is absent in the autarkic banking system. (ii) Due to the interbank market, the proportion of short-term liability ($$x$$) in banks’ balance sheet decreases and the remuneration for long-term liability ($$y$$) increases, i.e., $$\frac{\tilde{x}}{\tilde{y}} < \frac{\tilde{x}^a}{\tilde{y}^a},$$ enhancing hence the utility of patient residents, i.e. $$\tilde{y} > \tilde{y}^a.$$ (iii) The funding capacity of the banking system is reinforced by the interbank market. The fact that $$\frac{\tilde{B}}{\tilde{A}} < \frac{\tilde{B}^0}{\tilde{A}^a}$$ and $$\tilde{A} > \tilde{A}^a$$ shows that the interbank market encourages banks to fund more long-term projects in both relative and absolute terms, bolstering thus the output level and social welfare in normal times. It is to notice that (i) and (ii) reveal that the well-functioning of the interbank market permits to generate a more stable banking system than an autarkic one.

**Proposition 2** A banking system with a well-functioning interbank market exhibits a greater stability, increases the output and improves social welfare compared to an autarkic banking system, given that the interbank market discipline encourages banks to issue common shares, reduces their reliance on short-term funding, and allows them to smoothly cope with idiosyncratic liquidity shocks and to allocate more resources to long-term projects.

The advantages of the interbank market incite banks to not stay in autarky. However, such
an innovation carries new policy and regulatory issues for the government.

4 Confidence crises in the interbank market

A well-known result in the literature on banking crisis is that a run on a bank with mismatch between liabilities (deposits) and assets (illiquid investments) can occur, if its depositors lose confidence in its repayment ability (Diamond and Dybvig 1983).

In this subsection, we show that the root behind a bank run can be the malfunctioning of the interbank market, such that a self-fulfilling loss of confidence of lending banks rather than depositors can be the original culprit that triggers the crisis. There is no concern over the safety of government bonds and we characterize the run equilibrium as a rare event, corresponding to an inefficient situation where the first-best allocation considered above, is no longer feasible due to premature withdrawals.

At $t_1$, ‘type 2’ banks, hit by the idiosyncratic shock, need to borrow from the interbank market for full scale reinvestment. Burdened by non-performing assets, they are more vulnerable than ‘type 1’ banks with performing projects yielding a relatively high liquidation value. Their solvency depends on the expectations of both depositors and ‘type 1’ banks whereas the latter are subject only to the risk of premature withdrawal. According to the type of banks affected by a bank run, there are different implications for the functioning of the interbank market.

4.1 The bank run on ‘type 1’ banks

A self-fulfilling run on ‘type 1’ banks implies a run on ‘type 2’ banks and hence a systemic banking crisis. It occurs if ‘type 1’ banks’ liquidity needs exceed its liquidity inflows at $t_1$, i.e.,

$$z_p(r^*) \equiv \tilde{x} - (1 - \tau)r^*_1 \bar{A} - \frac{\bar{B}}{1 + r^*} > 0. \quad (29)$$

Condition (29) illustrates the situation where a ‘type 1’ bank fails to honor deposit withdrawals $\tilde{x}$ in the event of a run even after depleting all liquidity reserves $\frac{\bar{B}}{1 + r^*}$ and restructuring all long-term assets $\bar{A}$. They cannot obtain loans due to the disappearance of interbank lenders and cannot refinance (by assumption) in the international financial market.

Using (21)–(24) and the definition of $\theta$, we express (29) in terms of structural parameters:

$$r^*_1 < \frac{r^+_1}{1 - \tau} \quad (30)$$
where \( r_1^+ \equiv (1 - k^i + \beta)\Phi^{\sigma - 1} - \beta \) is a measure of illiquidity, with a lower \( r_1^+ \) meaning less illiquidity.\(^7\) The term \( r_1^+ \frac{1}{1-\tau} \) defines the threshold of the fire-sale price that the performing asset must attain to eliminate the run equilibrium for ‘type 1’ banks. It is straightforward to show that \( \frac{\partial r_1^+}{\partial \sigma} > 0 \), \( \frac{\partial r_1^+}{\partial \beta} < 0 \) and \( \frac{\partial r_1^+}{\partial k^i} < 0 \), implying that banks are more vulnerable to a run if depositors have a greater risk aversion and when the interbank market discipline in terms of capital requirement and liquidity reserve ratios is less severe. Given \( r_1^+ \), the ‘type 1’ bank is solvent if the fire-sale price of immature assets \( r_p^l \) is high enough to fill the liquidity gap.

The failure of ‘type 1’ banks leads to the depletion of interbank market liquidity. Unable to refinance, ‘type 2’ banks definitely cannot honor the deposit payments at \( t_2 \), triggering thus a bank run associated with an interbank market freeze that has a ‘knock-on’ effect by spreading the crisis from one bank to another and thus induces a systemic collapse.

**Proposition 3** *Existence of a run on ‘type 1’ banks.* The risk of run on ‘type 1’ banks, i.e., banks not hit by the idiosyncratic shock at the intermediate stage, and the risk of interbank market failure increase with the degree of risk aversion of depositors but decreases with the fire-sale prices of their immature long-term assets and the severity of interbank market discipline in terms of capital requirement and liquidity reserve ratios.

From now on, we are more interested by less harsh situations (in which the interbank market is resilient during a self-fulfilling crisis) than a systemic confidence crisis. Henceforth, we assume that ‘type 1’ banks are immune from the run equilibrium such that

\[
z_p(r^*) \leq 0 \quad \text{or} \quad r_i^p \geq \frac{r_1^+}{1-\tau}.
\]

This reflects the fact that a bank with a well-managed balance sheet (i.e., with promisingly profitable assets and no need for external funding) is generally resilient to liquidity shocks.

### 4.2 The bank run on ‘type 2’ banks

For ‘type 2’ banks, the condition for the existence of a confidence crisis depends on the expectations of their depositors and ‘type 1’ banks.

\(^7\)Provided that \( 0 < \sigma < 1 \), the illiquidity measure is always smaller than 1, i.e., \( r_1^+ < 1 \).
4.2.1 Liquidity situation with interbank lending

The interbank loan is granted during a run on ‘type 2’ banks if their illiquidity index is such that

\[ z_{np}(r^*) = \tilde{x} - \frac{\tilde{B}}{1 + r^*} - (1 - \tau)r_{l1}^{np}\tilde{A}(1 - k^i) - \frac{(1 - \tau)R}{1 + r^*}k^i\tilde{A} \]

\[ = z_p(r^*) + (1 - \tau)(r_{l1}^p - r_{l1}^{np})\tilde{A} - (1 - \tau) \left( \frac{R}{1 + r^*} - r_{l1}^{np} \right) k^i\tilde{A} < 0. \quad (31) \]

As a non-performing asset is less valuable than a performing one, i.e., \( r_{l1}^p - r_{l1}^{np} > 0 \), (31) implies that a ‘type 2’ bank is more vulnerable than a ‘type 1’ bank during a bank run, i.e., \( z_{np}(r^*) > z_p(r^*) \). The term \((1 - \tau)\left( \frac{R}{1 + r^*} - r_{l1}^{np} \right) k^i\tilde{A} > 0 \) in (31) is the additional liquidity brought by interbank loans per unit of long-term asset relative to that delivered by its restructuring and \( k^i\tilde{A} \) is the quantity of collateral.

In terms of structural parameters, condition (31) is equivalent to

\[ r_{l1}^{np} > \frac{r_2^+}{1 - \tau} \quad (32) \]

where \( r_2^+ = [\Phi_{np}^{-1}(1 - k^i + \beta) - \phi](1 - k^i)^{-1} = r_1^+ + \frac{k^i r_1^+ - \phi(1 - \pi)}{1 - k^i} \). The fact that \( r_2^+ < 1 \) and \( k^i < \phi(1 - \pi) \) indicate that \( r_2^+ < r_1^+ \) so that ‘type 2’ banks become less illiquid when the interbank market functions well. When (31) or its equivalent (32) does not hold, the interbank loan is definitively suspended when a run on ‘type 2’ banks occurs.

4.2.2 Liquidity situation without interbank lending

Following a freeze in interbank lending, a run on ‘type 2’ banks is possible if

\[ z_{np}^1(r^*) = \tilde{x} - (1 - \tau)r_{l1}^{np}\tilde{A} - \frac{\tilde{B}}{1 + r^*} \]

\[ = z_p(r^*) + (1 - \tau)\tilde{A}(r_{l1}^p - r_{l1}^{np}) > 0. \quad (33) \]

In terms of structural parameters, (33) can be expressed as:

\[ r_{l1}^{np} < \frac{r_1^+}{1 - \tau}. \quad (34) \]

Condition (33) (or (34)) is more restrictive than (31) (or (32)) as regard to ‘type 2’ banks liquidity position in a bank run. From (31), we obtain immediately \( z_{np}^0(r^*) < z_{np}^1(r^*) \) so that the
liquidity condition of a ‘type 2’ bank deteriorates when the interbank market is frozen. When (33) (or (34)) does not hold, a ‘type 2’ bank survives a bank run without the interbank loan. In such a case, the ‘type 1’ bank has no concern over the counter-party risk so that it will always grant the loans even during a run on the ‘type 2’ bank.

We summarize above results in the following proposition:

**Proposition 4** Existence of a run equilibrium on ‘type 2’ banks. A run equilibrium for a ‘type 2’ bank hit by the idiosyncratic shock at $t_1$ exists if its illiquidity index is such that:

$$z^j_{np}(r^*) \equiv \tilde{x} - (1 - \tau)r^{np}_l \tilde{A} - \frac{\tilde{B}}{1 + r^*} - (1 - j)\{(1 - \tau)R - (1 - \tau)r^{np}_l\kappa_i \tilde{A}\} > 0,$$

where $j = 0, 1$ stands for ‘no freeze’ and ‘freeze’in interbank lending, respectively. Furthermore, the ‘type 2’ bank facing a run:

- (4a) goes broke even though the interbank loan is granted if $z^0_{np}(r^*) > 0$.
- (4b) survives even without interbank lending if $z^1_{np}(r^*) \leq 0$.

Proposition 4 underlines the important role of structural parameters, especially, the liquidation value of immature assets $r^{np}_l$, for the existence of a run equilibrium and for the functioning of the interbank market. More precisely, a sufficiently low $r^{np}_l$ means a rather illiquid and vulnerable balance sheet such as in case 4a of Proposition 4, where the counter-party risk is inevitable so that the interbank market is entirely frozen. In contrast, a high fire-sale price $r^{np}_l$ can ensure a relatively healthy balance sheet in the crisis time as in case 4b, in which the interbank loan is always granted due to the elimination of counter-party risk.

Combining case 4a and 4b of Proposition 4 immediately leads to the following corollary:

**Corollary 1** Multiplicity of equilibria. If $\frac{r^1_{np}}{1 - \tau} < r^{np}_l < \frac{r^1_{np}}{1 - \tau}$ (or equivalently, the liquidity measure is such that $z^0_{np}(r^*) > 0$ but $z^1_{np}(r^*) \leq 0$), there are a run and a non-run equilibrium, with the lending decision of ‘type 1’ banks – the lender in the interbank market – acts as a self-fulfilling prophecy. The run equilibrium for ‘type 2’ banks is entirely ruled out if the interbank loan is granted, and materializes if the loan is suspended.

Corollary 1 reveals the significant role of the interbank market in triggering or not a run equilibrium by showing that a bank run can be induced by the ‘sudden stop’ of interbank loans, instead of the pure abrupt loss of confidence among depositors. The interbank market freeze
sends a bad signal to the depositors of liquidity-deficit banks and urges them to fulfill the expectations of a run immediately, although they may originally have wished to withhold their claims until \( t_2 \). However, the failure of these banks can be avoided if the loan is granted. The decision of lending or not made by interbank lenders acts as a self-fulfilling prophecy such that it will appear \( ex-post \) as perfectly justified by the situation of liquidity-deficit banks, since the situation that materializes is actually determined by the lending decision.

Proposition 4 and Corollary 1 show the vulnerability of liquidity-deficit banks that are dependent on the interbank market to meet their liquidity need. The self-fulfilling run and the resulting bankruptcy can occur in crisis times when their balance sheet is somewhat soft (i.e., \( r_{l}^{h} \frac{\epsilon}{1-\tau} < r_{l}^{np} < r_{l}^{h} \frac{\epsilon}{1-\tau} \)) or considerably weak (i.e., \( r_{l}^{np} < r_{l}^{h} \frac{\epsilon}{1-\tau} \)). Although controversial, we believe that Corollary 1 may characterize the functioning of the interbank markets of euro-periphery countries since the onset of the global financial crisis. In these countries, banks’ reliance on market funding surged along with the rapid process of financial liberalization, leading the market discipline to gain an increasing importance relative to the regulation (Adrian and Shin 2008). All goes well except that, at the dawn of eurozone crisis, liquidity-surplus banks preferred to ‘hoard’ liquidity to avoid counter-party risk rather than lending it through the interbank market to liquidity-deficit banks. This kind of loss of confidence would have induced a self-fulfilling run on ‘type 2’ banks and hence materialized their failure if national governments had not intervened.

Consistent with the view that market discipline is essential for ensuring the stability of the banking system (Brunnermeier and Pedersen 2009, and Geanakoplos 2010), our above analysis shows that it imposes efficient resource constraints on banks and helps improve the output level in normal times. Nevertheless, compared to the banking regulation that aims at avoiding bank failure in a crisis, the market discipline is far away from warranting banks’ resilience to unexpected shocks in crisis times.

5 Banking and sovereign debt crises with contagion

Sovereign bonds were deemed to be ‘risk-free’ by most financial operators. Such a perception was invalidated by the events in the recent eurozone crisis. In a monetary union without a banking union, a banking crisis could lead to a sovereign debt crisis and vice versa. Due to the high degree of financial integration, such twin crises in a member state could also have
destructive impacts on other member states and causes their interbank market to malfunction.

5.1 Twin domestic banking and sovereign debt crisis

In the absence of a well-designed government bailout, the failure of liquidity-deficit banks caused by the interbank market freeze can develop into a twin systemic banking and sovereign debt crisis if the government does not have monetary sovereignty.

Consider the case described in Corollary 1 in which, given that \( r^+_{12} < r^+_{12} < r^+_{12} \), the run equilibrium for ‘type 2’ bank materializes as ‘type 1’ banks suspend interbank loans. Such self-fulfilling confidence crisis involves a risk of contagion that can affect the government’s solvency, a risk particularly prominent in a monetary union where a ‘diabolical’ loop exists between the deteriorated balance sheet of banks and the stressed budget of the government. For simplicity of exposition, we assume in this subsection that the portfolio of domestic banks is only composed of domestic government bonds, i.e. \( \gamma = 1 \), and these bonds are only held by domestic banks.

5.1.1 Impacts of the banking crisis on the government budget

At \( t_1 \) and \( t_2 \), the government collects taxes \( T \) on production projects to finance government expenditures \( G \). It can also issue an amount \( B_{12} \) of “short-term” government bonds at \( t_1 \) if it needs extra liquidity. These bonds also mature at \( t_2 \), but the discount rate \( r^d_{12} \) applied on them depends on market conditions at \( t_1 \). The government’s budget constraint is thus:

\[
D_2 = B_{02} + B_{12} + G - T
\]

The debt \( D_2 \) left at \( t_2 \) will be rolled over by issuing new long-term government bonds in international financial markets. We assume that there exists an exogenous ceiling \( g_f \) for the ratio of public debt over potential GDP, \( Y \), above which investors refuse to refinance the debt.\(^8\) Thus, refinancing will be done provided that the debt-to-GDP ratio does not exceed the ceiling, i.e.,

\[
\frac{D_2}{Y} \leq g_f
\]  

\(^8\)Imposing an exogenous ceiling on national debt is a simplifying assumption, which characterizes well the sudden shot-up in the interest rate on the government bonds of some euro-periphery countries. For a discussion of endogenous debt ceiling in economies with potential debt repudiation when there is no possible backstop from the monetary authority, see Eaton and Gersovitz (1981). For discussions of ‘fiscal limits’ imposed on governments with or without central bank intervention, see Leeper (2013).
We assume that, in normal times, the level of output \( \bar{Y} = RA \) and taxation \( \bar{T} = \tau RA = \tau \bar{Y} \) corresponding to the optimal allocation allow the verification of (36) for a constant level of public debt, i.e., \( D_2 = D_0 = \bar{D} \), with the corresponding debt-to-GDP ratio given by \( \bar{g} = \frac{\bar{D}}{\bar{Y}} \).

Consider the case in which ‘type 1’ banks remain solvent and ‘type 2’ banks fail due to the interbank market freeze. The failure of ‘type 2’ banks means that all their long-term assets are liquidated prematurely at \( t_1 \). The resulting output and tax-revenue losses are \( \Delta Y(\pi, r_{1_{np}}^{np}) = \pi(R - r_{1_{np}}^{np})\bar{A} \) and \( \Delta T(\pi, r_{1_{np}}^{np}) = \tau\pi(R - r_{1_{np}}^{np})\bar{A} \), respectively. If the initial debt ratio is near to \( g_f \) and these losses are important, the resulting debt-to-GDP ratio could be such that:

\[
g_c(\pi, r_{1_{np}}^{np}) = \frac{\bar{B} + \Delta T(\pi, r_{1_{np}}^{np})}{\bar{Y} - \Delta Y(\pi, r_{1_{np}}^{np})} > g_f,
\]

where \( g_c \) denotes the debt-to-GDP ratio in the state where market-dependent borrowing banks are broke while the discount rate on government bonds still equals to \( r^* \). The fact that \( \frac{\partial g_c}{\partial r_{1_{np}}} > 0 \) and \( \frac{\partial g_c}{\partial g_{1_{np}}} < 0 \) implies that the pressure on sovereign debt is greater when the scale of banking crisis is larger and the degree of illiquidity is higher. Condition (37) shows that even without bailout costs, the debt-to-GDP ratio rises due to the failure of a fraction of banks. Therefore, a sovereign debt crisis can be triggered by a non-systemic banking crisis when the latter induces \( g_c(\pi, r_{1_{np}}^{np}) > g_f \), even in the absence of costly fiscal bailout.

### 5.1.2 “Diabolical loop” of banking and sovereign debt crises

For the moment, we focus on the contagion effect of a partial banking crisis on the domestic sovereign debt and banking system, and abstract from bailout programs that the government could implement to restore the normal functioning of the interbank market.

To capture the sensitivity of risk premium to changes in government budget position, we assume that the yield on government bonds includes a risk premium which is increasing in the debt-to-GDP ratio at \( t_2 \), denoted by \( g_2 \):

\[
r_{12}^d(g_2) = r^* + \rho(g_2),
\]

with \( \rho(\bar{g}) = 0 \) and \( \rho'(g_2) > 0 \) for \( g_2 > \bar{g} \), with the highest risk premium being given by \( \rho(g_f) \) which could be attained when \( g_2 > g_f \). This notion of a “debt-elastic interest rate” has become increasingly popular in the literature on international lending for its empirical relevance (Schmitt-Grohe and Uribe 2003, Garcia-Cicco et al. 2010, and Farhi, Gopinath and Itskhoki...
To the difference of the existing literature, we assume that the interest rate is sensitive to the future, instead of the current, debt-to-GDP ratio. In effect, this assumption is more realistic since, as argued Cheng, Dai and Dufourt (2014), what matters for investors is whether the government will be solvent at the maturity date of the bonds, and not at their issue date.

The institutional design of the monetary union matters for the debt elasticity of the interest rate. The sensitivity of the risk premium to a change in the debt-to-GDP ratio is likely to be less acute in standalone countries with monetary sovereignty or in a monetary union where the central bank backs sovereign debt markets as lender of last resort than in a monetary union in which the central bank’s statutes prevent it from doing so.

If the debt-to-GDP ratio rises about $\tilde{g}$ following a run on ‘type 2’ banks, the interest rate on domestic government bonds rises according to (38). Thus, the present value at $t_1$ of government bonds held by the entire domestic banking system decreases to

$$\frac{B}{1+r_{12}^d(g_c)}$$

where $r_{12}^d(g_c) = r^* + \rho(g_c) \equiv r_{12}^g$. The erosion of the market price of government bonds (from $1/(1+r^*)$ to $1/(1+r_{12}^g)$) in turn reduces the liquidity buffer of banks obtained from selling their bond holdings and thus deteriorate their balance sheet since the illiquidity index of ‘type 1’ banks jumps from $z_p$ to

$$z_p(r_{12}^g) \equiv z_p + \left(1 - \frac{1 + r^*}{1 + r_{12}^g}\right) \frac{\beta + \theta(1 - k^i)}{1 - k^i + \beta} e,$$

(39)

where $z_p(r_{12}^g)$ is the illiquidity index of ‘type 1’ banks when the market value of government debt erodes with the rise of its discount rate to $r_{12}^g$. According to (39), we have $z_p(r_{12}^g) > z_p$.

It is thus possible to have $z_p(r_{12}^g) > 0$ such that ‘type 1’ banks with initial liquidity-surplus turn to be insolvent as the market value of government bonds erodes. The breakdown of ‘type 1’ banks undermines further the government revenue so that the debt-to-GDP ratio shoots up, leading finally to the verification of (37) that materializes a domestic sovereign debt crisis and hence to a mutually reinforcing twin crisis in banking system and sovereign debt.

**Proposition 5** Twin banking and sovereign debt crises with contagion via the interbank market. A run on a fraction of banks leads to such a twin crisis if

- (5a) a non-systemic banking crisis affecting borrowing banks induces the debt-to-GDP ratio to rise about its normal level $\tilde{g}$, and hence the contagion from the banking system to sovereign debt market so that the discount rate on government bonds $r_{12}^g$ rises with $g_c$;

- (5b) and if the initially interbank lending banks becomes illiquid and fails following the
depreciation of sovereign debt due to higher $r_{12}^{gc}$, i.e., $z_p(r_{12}^{gc}) > 0$. The resulting systemic banking crisis further increases the public debt and $r_{12}^{gc}$ leading to $g_c > g_f$, materializing thus a mutually reinforcing twin banking and sovereign debt crisis.

Proposition 5 helps to understand why countries in the Euro-periphery, like Ireland or Spain where banks’ balance sheet is significantly weakened by the collapse of their own real estate market (since domestic banks are heavily engaged in long-term mortgage loans), have been the most exposed to a systemic banking crisis. During the crisis, the abrupt depreciation of these assets, as captured by a large decline in $r_{12}^{np}$, puts these banks under significant stress and induces the freeze of the interbank market. When the scale of illiquid crisis is sufficiently large, a banking crisis leads immediately to a sovereign debt crisis in a monetary union given that no monetary instrument exists either to stop the bank failure or to protect the solvency of the government, and finally results in mutually reinforcing twin banking and sovereign debt crises.

5.2 The impact of foreign sovereign crisis on the interbank market

We analyze how the interbank market in a relatively ‘healthy’ country of a monetary union functions, when confronting a foreign debt crisis. This analysis illustrates the situation in some eurozone countries like France or Germany. Their domestic banking systems suffered contagion from sovereign debt crisis originated in peripheral countries.

5.2.1 The effects of foreign sovereign crisis on banks’ balance sheet

Assume that, for some exogenous reasons (bad economic fundamentals and/or negative self-fulfilling expectations of investors), the foreign country is involved in a twin banking and sovereign debt crisis, so that the risk premium on its sovereign bonds jumps to $\rho^* > 0$ resulting in $r_{12}^f > r^*$ at $t_1$. The risk premium on foreign government bonds decreases the liquidity buffer that domestic banks can obtain by selling these bonds, thus weakening their balance sheet and reducing the liquidity available in the interbank market. Given the share of foreign bonds in domestic banks’ bonds portfolio $(1 - \gamma) \in (0, 1)$, the rise in $\rho^*$ leads immediately to an aggregate loss in liquidity reserves of domestic banks measured by

$$
h(\rho^*) = \frac{(1 - \gamma)\rho^*\hat{B}}{(1 + r^* + \rho^*)(1 + r^*)} \quad (40)$$
Disequilibrium results in the interbank market as such losses decrease the liquidity supply of lending banks and increase the liquidity demand of borrowing banks.

5.2.2 The functioning of the interbank market during the contagion

When foreign bonds depreciate following a rise in the risk premium on foreign bonds, ‘type 1’ banks will divert part of their liquidity reserves invested in government bonds, initially meant to be lent through the interbank market, to ensure the payments to impatient depositors. The aggregate liquidity supply in the domestic interbank market becomes:

\[ I^s = (1 - \pi)(\beta \tilde{A} - h(\rho^*)) \]  

(41)

while the liquidity demand before liquidation of any long-term projects is given by

\[ I^d = \pi \left[(1 - \pi)\phi \tilde{A} + h(\rho^*) \right] \]  

(42)

Comparing (41) and (42) yields immediately \( I^s < I^d \) and hence a credit crunch in the domestic interbank market as the risk premium on foreign bonds rises. The situation could deteriorate to a point so that \( I^s < 0 \), i.e., ‘type 1’ banks suffer also from liquidity deficits and are forced to carry out premature liquidation of long-term projects, implying a systemic liquidity crisis with an interbank market that is entirely frozen.

For \( I^s > 0 \), i.e., ‘type 1’ banks still have liquidity surplus to lend in the interbank market. However, the interbank market becomes very strained as the liquidity offer decreases by \((1 - \pi)h(\rho^*)\) while the liquidity demand increases by \(\pi h(\rho^*)\). Furthermore, the aggregate liquidity shortage \(h(\rho^*)\) determines that ‘type 2’ banks liquidate \(\frac{h(\rho^*)}{\phi}\) units of troubled long-term projects that cannot be refinanced. Without refinancing, these projects yield nothing at \(t_2\) and hence cannot serve as collateral. Given that depositors have the priority over wholesale creditors (i.e., lenders in the interbank market) and shareholders to recoup their funds, the quantity of pledgeable assets for interbank loans decreases from \(\pi k^i \tilde{A}\) to \(\pi k^i \tilde{A} - \frac{h(\rho^*)}{\phi}\) and the maximal amount of borrowing that can be backed by collateral is now reduced from \(\frac{(1 - \tau)R}{(1 + r^*)} \pi k^i \tilde{A}\) to

\[ \mu = \frac{(1 - \tau)R}{1 + r^*} \left[\pi k^i \tilde{A} - \frac{h(\rho^*)}{\phi}\right] \]  

(43)

Using (6) and (41)-(43), it is straightforward to show that \(I^d > I^s > \mu\). The initial liquidity
shortage in the interbank market has more severe detrimental consequences on borrowing banks’ pledgeable assets. More precisely, the shortfall of one unit of liquidity leads the quantity of eligible collateral to decrease by $\frac{1}{\phi}$ and the corresponding interbank loans to shrink by $(1-\tau)R \left(\frac{1-r}{(1+r)^2}\right)$. This is because non-refinanced projects $h(\rho^*) \neq 0$ yield nothing at $t_2$. Thus they will be liquidated at $t_1$ to obtain $(1-\tau)r_{np}^1h(\rho^*)$. The illiquidity index of ‘type 2’ banks is then:

$$z_{np}^0(\rho^*) = z_{np}^0(r^*) + \hat{h}(\rho^*) + (1-\tau)(\frac{R}{1+r^*} - r_{np}^1)\frac{h(\rho^*)}{\phi}. \quad (44)$$

The second and third terms represent losses due to the depreciation of foreign government bonds and the premature liquidation of long-term projects, respectively.

**Proposition 6** *Contagion from foreign sovereign debt crisis to domestic banks – the role of the interbank market.* A sufficiently severe foreign sovereign debt crisis could lead to the depletion of market liquidity and hence the collapse of the interbank market, by inducing

(6a) a direct loss in liquidity reserves for both lending and borrowing banks, inducing thus an aggregate liquidity shortage in the domestic financial system equal to $h(\rho^*) > 0$.

(6b) an indirect loss of liquidity $\Theta > 0$ for borrowing banks, reflecting the amplification effect of the contagion by the foreign crisis through the liquidity contraction in the interbank market, which yields a more severe decrease in the size of eligible collateral, and reduce further the loans that borrowing banks can obtain.

The existence of the interbank market with its associated market discipline implies that the contagion of foreign debt crisis could induce a domestic banking crisis, with potentially large negative consequences for the domestic government’s solvency.

**6 Gambling assets and the market discipline**

The interbank market freeze during recent financial crises is often explained by commentators as a result of gambling behaviors of some banks excessively engaged in risky assets, like sub-prime lending in the USA or real-estate lending in Spain and Ireland. However, gambling by banks might not explain such freeze except when asymmetric information, with its implications for the stability of the banking system and the role of market discipline, is taken into account.
6.1 Gambling asset and asymmetric information

We assume that once investors become banks’ shareholders, they establish a managing board to affect banks’ management in future dates. This board accountable to shareholders does not change banks’ best plan defined at \(t_0\) (section 3) since the economic environment remains unchanged. Once depositors have entrusted their resources to banks, the board can put pressure on the latter to deviate, at the arrival of gambling asset, from the optimal plan given asymmetric information. The very existence of this asset is perceived by banks only at \(t_1\) and it is unknown by depositors. Moreover, gambling by a bank is a private information. However, if banks gamble, the information will be revealed and obtained free of cost by any agents at \(t_2\).

The gambling asset yields \(\psi R > R\) and thus an excess return \((\psi - 1)R > 0\) at \(t_2\) with the probability \(\vartheta < 1\) for each unit of good invested at \(t_1\), and 0 otherwise. A gambling asset is characterized by a relatively low probability of success such that its expected return is lower than that from a long-term project, i.e.,

\[
\vartheta \psi R < R. \quad (45)
\]

Investing in such an asset is thus socially inefficient. Any risk-averse or risk-neutral investors will not fund the gambling asset with their own endowments.

6.2 The perverse effect of the market discipline

‘Type 2’ banks, subject to limited liability and under the pressure of their shareholders, will consider purchasing gambling assets as a rational choice, if

\[
\vartheta \left\{ (1-\tau)R \left[ (1-\tau)(1-k^i)^{\bar{A}r_{lp}^{np}} + (1-\pi)\phi\bar{A} \right] - (1-\tau)Rk^i\bar{A} - (1-\lambda)y \right\} > (K - k^i\bar{A})(1-\tau)R. \quad (46)
\]

The LHS of (46) represents the expected return for ‘type 2’ banks’ shareholders, if they divert the funds obtained from interbank loans and from prematurely liquidating a part \((1-k^i)\) of their long-term projects, i.e., \((1-\tau)(1-k^i)\bar{A}r_{lp}^{np}\), given that \(k^i\bar{A}\) is used as collateral to obtain interbank loans \((1-k^i)\bar{A}r_{lp}^{np}\). Thus ‘type 2’ banks can purchase gambling assets up to an amount equal to \((1-\tau)(1-k^i)\bar{A}r_{lp}^{np} + (1-\pi)\phi\bar{A}\) but has to reimburse \((1-\tau)Rk^i\bar{A}\) to the interbank lenders and pay \((1-\lambda)y\) to patient depositors. The RHS of condition (46) is the opportunity cost of investing in gambling assets since the interest rate on loans is such that the collateral
generates no dividend for shareholders in contrast to the projects financed by capital but not used as collateral, i.e., \((K - k^i\tilde{A})\). Given that banks at the optimum hold a capital equal to \(K = k^i\tilde{A}\), the RHS of (46) is zero. Using (22)-(23) to obtain \((1 - \lambda)y = (1 - \tau)R(1 - k^i)\tilde{A}\) and substituting the latter and \(K = K = k^i\tilde{A}\) into (46) yield:

\[
r^{np}_l > \frac{r^+_3}{1 - \tau},
\]

where \(r^+_3 \equiv \frac{1-(1-\tau)\phi}{(1-k^i)\psi} < 1\) is a measure of feasibility of gambling.\(^9\) When condition (47) is satisfied, ‘type 2’ banks will invest all their resources in the gambling asset since this is more profitable for their shareholders, even though it is socially inefficient. Given that \(\frac{\partial r^+_3}{\partial k^i} < 0\), condition (47) is more easily verified when the minimal capital ratio imposed by the interbank market is higher. This result is justified by the fact that refinancing through the interbank market is less costly than the premature liquidation, and a higher minimal capital ratio increases the liquidity that can be raised in the interbank market for investing in the gambling asset.

The information revealed by condition (47) is costless for all banks but not available to depositors at \(t_1\). Thus, the arrival of the gambling asset will not induce a run in the first place. Although ‘type 1’ banks cannot directly monitor ‘type 2’ banks’ gambling behavior but can check (47). If (47) does not hold, ‘type 1’ banks will grant interbank loans and the presence of gambling asset has no influence on the equilibrium, which remains the same as in section 3. Otherwise, there are two options left for ‘type 1’ banks. They may either refuse granting interbank loans to avoid the counter-party risk caused by the gambling or demand a higher return from the loans so as to compensate for the additional risk. Nevertheless, for risk neutral banks, the second option remains more appealing because it could yield higher expected dividends for their shareholders than the first. The interbank market under asymmetric information is thus inefficient because it functions ‘normally’ when ‘type 2’ banks have a less liquid balance sheet, i.e., \(r^{np}_l\) is too low to satisfy (47), and ‘abnormally’ in the contrary case.

When \(r^{np}_l > \frac{r^+_3}{1 - \tau}\), lending banks may gamble with ‘type 2’ banks by requiring a higher return for their loans equal to \(\frac{1 - \tau}{1 - \tau} Rk^i\tilde{A}\) (instead of \((1 - \tau)Rk^i\tilde{A}\)) to compensate for the risk due to gambling. ‘Type 2’ banks will carry out the premature liquidation of \((1 - k^i)\tilde{A}\) units of

\(^9\)Provided that \(k^i = \frac{(1+\tau)(1-\tau)\phi}{(1-\tau)R}\), we directly obtain \(k^i < (1 - \pi)\phi\), which ensures that \(r^+_3 < 1\).
long-term assets and invest the proceeds as well as the borrowed funds in the gambling asset if

$$\psi[(1 - \tau)(1 - k^i)\tilde{A}r_{i1}^{pp} + (1 - \pi)\phi\tilde{A}] - \frac{k^i\tilde{A}}{\phi} - \frac{(1 - \lambda)y}{(1 - \tau)R} > 0. \quad (48)$$

When (49) is verified, investing in gambling asset remains the better choice for ‘type 2’ banks despite the rise in the cost of interbank borrowing. The existence of asymmetric information determines that even if ‘type 2’ banks renounce to the gambling behavior, ‘type 1’ banks will not reduce the cost of borrowing. The risk of failure in the absence of interbank loans compels ‘type 2’ banks to accept unfavorable loan terms and gamble. ‘Type 2’ banks’ shareholders may then recoup their investment with a probability $\vartheta$ while their depositors potentially suffer a huge loss if the gambling fails. Condition (48) can be rewritten with structural parameters as

$$r_{i1}^{pp} > \frac{r_{4}^{+}}{1 - \tau}, \quad (49)$$

where $r_{4}^{+} = \frac{\vartheta + k^i(1 - \vartheta) - \vartheta\psi(1 - \pi)}{\psi(1 - \vartheta)}$ measures the feasibility of gambling with higher borrowing costs.

A bank run on ‘type 2’ banks happens at $t_1$ if the interbank market freezes. This is possible if ‘type 2’ banks cannot honor the repayment of the interbank loans borrowed at the risky rate $(1 - \tau)R$. Given that $r_{4}^{+} > r_{3}^{+}$, a situation exists in which (47) holds but not (49). In such a situation, a bank run on ‘type 2’ banks occurs at $t_1$ since depositors acquire the information that, due to a lack of refunding, their banks cannot ensure the payments at $t_2$. As a result, neither the refinancing of long-term projects nor the investment in the gambling asset will materialize.

Thus, asymmetric information increases the chance of failure of ‘type 2’ banks.

**Proposition 7** *Gambling, asymmetric information and the interbank market.* When banks are under their shareholders’ pressure, gambling assets and asymmetric information

(a) do not affect the banking system if $r_{i1}^{np} < \frac{r_{3}^{+}}{1 - \tau}$, i.e., a too illiquid balance sheet prevents ‘type 2’ banks from having gambling incentive;

(b) induce the interbank market to react to the potential gambling of ‘type 2’ banks according to the latter’s degree of illiquidity if $r_{i1}^{np} > \frac{r_{4}^{+}}{1 - \tau}$, such that:

(i) for $r_{i1}^{np} \in \left(\frac{r_{4}^{+}}{1 - \tau}, \frac{r_{3}^{+}}{1 - \tau}\right)$, the interbank market is completely frozen and a run on ‘type 2’ banks is triggered immediately, thwarting thus the plan of gambling.
(7b-ii) for \( r_{lp}^{np} > \frac{r^*_1}{1 - \tau} \), the interbank market functions with a rise in borrowing costs and the whole banking system adopts a gambling behavior.

Comparing (32) with (49), we find that in normal times, interbank loans are granted to banks with a relatively solid balance sheet (i.e., a higher \( r_{lp}^{np} \)), whereas, with the arrival of gambling asset, the interbank market ‘functions’ if borrowing banks have either an exceedingly illiquid or an exceptionally solid balance sheet as shown in cases (7a) and (7b-ii) of Proposition 7, with lending banks indirectly involving themselves in gambling in the latter case. Case (7b-i) of Proposition 7 describes a specific situation where the interbank lending is completely suspended regardless whether ‘type 2’ banks gamble or not. The fact that \( \frac{\partial r^*_1}{\partial k^i} < 0 \) and \( \frac{\partial r^*_4}{\partial k^i} > 0 \) indicates that the market discipline can have perverse effect on the stability of the banking system in the presence of gambling assets in the sense that both case (7a) and (7b-ii) are less likely to hold when \( k^i \) increases, contributing thus to bring a complete freeze of the interbank market despite its original purpose of ensuring the well-functioning of this market by enhancing banks’ balance sheet. Proposition 7 leads immediately to the following corollary:

**Corollary 2 The perverse effect of the interbank market discipline.** When banks are under the pressure of shareholders, the market discipline designed in normal times can have perverse effects in the presence of asymmetric information and gambling assets in the sense that, given that \( \frac{\partial r^*_1}{\partial k^i} < 0 \) and \( \frac{\partial r^*_4}{\partial k^i} > 0 \), a rise in the minimal capital ratio \( k^i \) enlarges the interval \( \left( r_{lp}^{np}, \frac{r^*_1}{1 - \tau} \right) \), thus increasing the scope of realization of a complete freeze in the interbank market.

7 Prudential regulation and crisis response

Previous results show that the market discipline, although works efficiently in normal times in stabilizing the banking system in normal times and in ensuring a higher output, can aggravate rather than mitigate banks’ illiquidity in crisis times. Therefore, the role of regulation and government intervention in protecting the financial system becomes crucial particularly when the monetary authority does not play the role of lender of last resort.

7.1 Prudential regulation

Given that banking crises are often linked with a high-leveraged and/or weak balance sheet, a prudential regulation more severe than what imposes the market discipline is *a priori* welcomed
in terms of financial stability when banks are confronted to confidence crisis and gambling asset associated with asymmetric information.

However, a higher liquidity reserve ratio implies that banks must hold more government bonds that may become risky. This could have important perverse effects on banks’ balance sheet and make the “diabolic loop” between sovereign debt crisis and banking crisis more probable. To avoid such twin crisis, banks should be better informed about the real riskiness of "safe assets". Meanwhile, in the presence of gambling asset and asymmetric information, a higher capital ratio can also have perverse effects by making the interbank market freeze more likely. Finally, an effective regulation must eliminate the risk of bank failures without hampering banks’ role as efficient financial intermediaries. The latter exigency implies upper bounds on capital requirement and liquidity reserve ratios.

7.1.1 Upper bounds on the intensity of prudential regulation

As shown in section 3, banks can efficiently pool resources from domestic residents and optimally allocate them in investment vehicles only when condition (19) is verified, i.e., \( \Phi \geq 1 + r^* \). Given \( \Phi = \frac{(1-k^i)(1-r^*)R}{(1-k^i+\beta)} \), we obtain immediately \( \frac{\partial \Phi}{\partial k^i} < 0 \) and \( \frac{\partial \Phi}{\partial \beta} < 0 \), implying that the gross return on deposits (thus residents’ welfare) decreases with \( k^i \) and \( \beta \). This effectively sets upper bounds \( \overline{k} \) and \( \overline{\beta} \) on the intensity of prudential regulation to keep residents' incentive in entrusting their funds to banks. These two upper bounds are interdependent so that the one cannot be determined without the other being fixed. Consider two extreme cases. For \( \beta = \pi \phi \), using (19) and the definition of \( \Phi \), we obtain the maximum value of capital ratio

\[
\overline{k} = \frac{(1 - \tau)R - (1 + \pi \phi)(1 + r^*)}{(1 - \tau)R - (1 + r^*)} \in (0, 1).
\]  

(50)

For \( k^i \) given by (10), (19) implies that the maximum value of liquidity ratio is

\[
\overline{\beta} = \frac{[(1 - \tau)R - (1 + r^*)][(1 - \tau)R - (1 + r^*)(1 - \pi)\phi]}{(1 + r^*)(1 - \tau)R} \in (0, 1).
\]  

(51)

7.1.2 Prudential regulation to rule out the confidence crisis

The existence of self-fulfilling run equilibrium can be entirely eliminated if (34) is reversed, i.e., \( r_{1}^* \leq (1 - \tau)r_{1}^{pp} \), meaning that the whole banking system is immune to bank run in a confidence

\( ^{10} \)This explain why banks only issue the threshold level of capital, defined by (16), in equilibrium .
crisis. Since $\frac{\partial r_i^+}{k^i} < 0$ and $\frac{\partial r_i^+}{\beta} < 0$, there exist $k^g(\beta)$ and $\beta^g(k^i)$ such that

$$r_i^+(k^g, \beta^g) \leq (1 - \tau)r_{ip}^p. \quad (52)$$

However, this regulation will be effective if it does not deprive banks’ role as efficient financial intermediaries, i.e., $k^g(\beta) \leq \bar{E}(\beta)$ and $\beta^g(k^i) \leq \bar{B}(k^i)$. The minimal regulatory capital (or liquidity) ratio ensuring the elimination of bank run cannot be expressed explicitly in terms of structural parameters given the non-linearity in (52). For $\sigma = 0.5$, $\tau = 0.1$, $\phi = 0.2$, $r_{ip}^p = 0.6$, $R = 2$ and various values of $\pi$ (from 0.1 to 0.7), numerical simulations show that, to eliminate the risk of confidence crisis, $k^g$ (or $\beta^g$) should be raised to a level so that $k^g > k$ for $\beta^g = \beta$ (or $\beta^g > \bar{B}$ for $k^g = k^i$). It is to notice that if $k^g$ and $\beta^g$ are simultaneously imposed above the levels implied by the market discipline, both upper bounds will be lower. Therefore, in our model, the government cannot set a practicable regulatory capital or liquidity ratio to completely rule out the risk of bank runs without undermining banks’ capability to pool resources.

### 7.1.3 Prudential Regulation to avoid gambling behavior

The perverse effects of the interbank market discipline, in the presence of asymmetric information and gambling asset, imply the necessity to increase shareholders’ cost of gambling through prudential regulation. Reinforcing capital regulation is appropriate since this directly affects the interests of banks’ shareholders. A ‘type 2’ bank will have no incentive to gamble if (46) is not verified. It is easy to see that condition (46) breaks down easily, if the government enacts a prudential regulation that requires banks to keep a capital ratio $k^g$ such that\footnote{Condition (53) is obtained by inverting (46) while using (22)-(23) and the definition of $\Phi$.}

$$k^g > k^i + \frac{\theta}{1 - \theta}[(1 - \tau)(1 - k^i)r_{ip}^p + \psi(1 - \pi)\phi - 1] \equiv k_{\min}^g. \quad (53)$$

Condition (53) indicates that a bank capital higher than the minimal level $k^i\bar{A}$ required for obtaining interbank lending increases the cost of premature liquidation (i.e., the term $(K - k^i\bar{A})(1 - \tau)R$ on the RHS of (46)), and thus incites ‘type 2’ banks to give up their gambling behavior. However, given the fact that $\psi$ is high and $\frac{\partial k^g}{\partial \psi} > 0$, $k_{\min}^g$ could exceed the ceiling $\bar{k}$ defined by (50).

To enhance the stability of the interbank market, the government should implement a capital regulation that requires banks to hold a level of capital higher than the minimal capital
imposed by the interbank market discipline. Nevertheless, as a regulation that is fully efficient in eliminating banking crises could imply a capital ratio that exceeds the capital ratio ceiling $\bar{k}$ and hence is too high to be compatible with banks as financial intermediaries, especially when the gambling is highly attractive (i.e., $\delta$ and $\psi$ are relatively high). As a result, the government’s ex-post crisis management could play a crucial role in stabilizing the banking system and in restoring the normal conditions of the interbank market during crisis times.

We summarize the results of this subsection in the following proposition:

**Proposition 8 Limited efficiency of prudential regulation.** The prudential regulation imposing more severe capital requirement and liquidity reserve ratios than the market discipline is efficient in fighting against banking crises and the malfunctioning of the interbank market by reinforcing banks’ resilience to diverse risks through improving banks’ balance sheet, i.e., $\beta^g > \beta$ and $k^g > k^i$ (or $k^g > k^g_{\text{min}}$ in the presence of gambling asset and asymmetric information). In general, it cannot completely preclude the risk of financial crisis because, by doing that, it could be so severe that these ratios exceed the upper bounds on the intensity of regulation and deprive banks’ role as efficient financial intermediary.

### 7.2 Crisis response

We first explore the effect of adding a government deposit guarantee in our benchmark economy, seen as a commitment by the government to raise any possible additional resources in financial markets to bail out banks with insufficient liquidity to avoid a severe economic crisis. This arrangement is particularly useful in the Euro zone, since according to the initial institutional design of the Eurozone, the central bank should not be involved in intervention of any form. We then study how to implement a penalty tax to prevent gambling behaviors.

#### 7.2.1 Bailouts during a pure confidence crisis

To resolve the interbank market crisis and the bank failure caused by the lack of confidence of the type in section 4, the government can conduct a bailout through injecting into illiquid banks at $t_1$ an amount of liquidity $\pi z_{\text{np}}^1$. Without the power of creating money, the government must issue new (short-term) bonds with a face value $B^g_d$ sold at a discount of the par value. We start

---

12 The run equilibrium described in proposition 4, can be eliminated and the normal functioning of the interbank market can be restored if the liquidity gap $z_{\text{np}}^1$ is filled. Provided that illiquid banks accounting for $\pi$ percent of total banks, the liquidity injection should be $\pi z_{\text{np}}^1$ at the optimum.
with the case in which the feasibility of the bailout package are not an ex ante concern so that the discount rate on these bonds is \( r_{\text{d}}^b = r^* \). Thus this bailout should satisfy

\[
G_{\text{d}}^b = \frac{B_{\text{d}}^b}{1 + r^*} = \frac{\pi z_{\text{np}}}{1 + r^*}
\]  

(54)

in equilibrium. The bank run and hence the interbank market crisis will be immediately eliminated and thus no premature liquidation will be required at intermediate stage, if

\[
g_2 \equiv \frac{\tilde{D} + \Delta T + (1 + r^*)G_{\text{d}}^b}{Y} < g_f
\]  

(55)

holds.\(^{13}\) Condition (55) describes the case in which a mere announcement of bailout can rectify the lack of confidence in domestic banks if the government is able to credibly afford the cost of liquidity injection \( G_{\text{d}}^b \) and the tax-revenue losses \( \Delta T \). Comparing (55) with (37), we find that \( g_2 \) can be smaller than \( g_c \) defined by (37) such that the implementation of bailout does not necessarily impair the government budget and can effectively protect banks against the loss of confidence of depositors and interbank market lenders.

However, in a country with soft fundamentals (the initial debt-to-GDP ratio is neither too high nor too low), condition (55) can be broken. Thus the bailout becomes infeasible. In addition, according to condition (38), the implementation of such a bailout will lead the discount rate on government bonds to jump to \( r_{12}(g_2) = r^* + \rho(g_f) > 0 \) when \( g_2 > g_f \). As a result, we turn to a situation similar to that in subsection 5.1 in which a twin banking and sovereign debt crisis emerges.

### 7.2.2 Bailouts during a crisis resulting from a foreign debt crisis

There are two pre- eminent differences between a confidence crisis and a crisis originating from depreciating assets. In terms of crisis origins, the first is due to the liquidity mismatch in banks’ balance sheet and the lack of confidence, while the second is caused by the insolvency of ‘type 2’ banks as a result of the aggregate liquidity shortfall in the interbank market. In terms of crisis management, the first could be ruled out with a mere announcement of a credible bailout, while the second must be dealt with by effectively injecting liquidity to fill the liquidity gap.

To cope with a crisis induced by the depreciation of foreign sovereign bonds, the government should inject sufficient liquidity to fill the aggregate liquidity gap \( h(\rho) \) to avoid the credit crunch

\(^{13}\)Here, \( \Delta T = \tau \pi (R - r_{\text{d}}^n)\bar{A} \), representing tax-revenue losses, has been defined in section 5.1.
and the resulting failure of ‘type 2’ banks. The total cost of the bailout package in this case is simply $G_f^b = h(\rho)$. As in the above, the government without monetary sovereignty should issue at $t_1$ an amount of short-term bonds equal to $\frac{B_f^b}{1 + r^*} = G_f^b$, where the discount rate on these bonds is $r_f^b = r^*$ as the government solvency is not an ex ante concern of market participants. Consequently, this bailout program is feasible under the condition that

$$g_2 = \frac{\tilde{D} + \Delta T + (1 + r^*)G_f^b}{Y} < g_f.$$  \hspace{1cm} (56)$$

The implication of (56) is similar to that of (55). For $g_2 < g_f$, this bailout is feasible and credible. Its implementation can not only fill the aggregate liquidity shortfall, but also restore the order in the interbank market, stopping thus the crisis contagion. If $g_2 > g_f$, the government is unable to rescue the banking sector contaminated by the foreign sovereign debt crisis. If the bailout is going to be implemented, the discount rate on government bonds jumps to $r_{12}(g_2) = r^* + \rho(g_f)$ according to (38), inducing the erosion of the value of domestic bonds and hence imperiling the solidity of banks’ balance sheet and the solvency of the government. Thus, a twin banking and sovereign debt crisis may materialize in the domestic economy as in Proposition 5.

**Proposition 9** The domestic government, without monetary sovereignty, can restore the well-functioning of the interbank market and the banking system if it has enough room to maneuver the bailout program in the sense that the debt-to-GDP ratio after implementing the bailout remains below the threshold level ensuring the government’s solvency, i.e., $g_2 < g_f$. Otherwise, such a fiscal bailout can aggravate rather than mitigating the interbank market freeze and the financial crisis, and can accelerate the contagion between banking sector and government budget, resulting finally in a mutually reinforcing twin banking and sovereign debt crisis.

Proposition 9 depicts well the financial straits faced by many euro-periphery countries during the recent crisis. In the absence of the central bank’s role as the lender of last resort, the member state governments’ capacity of restoring the normal functioning of the interbank market and bailing banks out is largely limited by their fiscal position. The linkage between the financial position of banks and the fiscal position of the government determines the existence of a “diabolical” loop between sovereign debt and banking crises. Therefore, the monetary instrument and the assistance from international institutions become the key to disentangle this “diabolical” loop. Such an impasse can be avoided, if the central bank purchases the debt of distressed governments so as to allow them to credibly rescue domestic banks.
7.2.3 Policy response to asymmetric information and gambling assets

The interbank market discipline can encourage instead of prevent the gambling behavior, while the prudential regulation can be too costly thus infeasible to rule out the consequences of the sudden arrival of a gambling asset at $t_1$. In addition, given that direct supervision over ‘type 2’ banks’ balance sheet will not be practicable for the government, the efficient preventive policy response to such a situation should consist of eliminating banks’ incentive to gamble.

The domestic government does not receive more information about the soundness of banks’ balance sheet than market participants, but has the capacity to verify at $t_2$ whether banks have gambled or not. To avoid the destabilizing effects caused by gambling, the government, as the banking regulator, can announce at $t_1$ that it will raise a penalty tax on gambling income at $t_2$. Let $\tau^p$ denote the penalty tax rate. This preventive policy is efficient if it can effectively destroy banks’ incentive to gamble such that

$$(1 - \tau - \tau^p)R[\psi(1 - \tau)(1 - k^i)\hat{A}r_{ji}^p + \frac{\psi(1 - \tau)Rk^i\hat{A}}{1 + r^*}] - [(1 - \tau)Rk^i\hat{A} - (1 - \lambda)y] < 0. \tag{57}$$

The LHS of (57) stands for ‘bad’ banks’ gain from gambling, which is lower than that in condition (46) due to the presence of a penalty tax. The RHS, identical to that in (46), represents ‘type 2’ banks’ revenue loss due to the abandonment of viable long-term projects. Arranging the terms of (57), we obtain that, to prevent gambling behavior, the penalty tax rate should verify

$$\tau^p > (1 - \tau)\left\{1 - \frac{1}{\psi(1 - \tau)(1 - k^i)r_{ji}^p + (1 - \pi)\phi}\right\} \equiv \tau^p_{\min}. \tag{58}$$

A penalty tax rate $\tau^p$ satisfying (58) eliminates the incentive of ‘type 2’ banks to gamble by making the expected return of gambling negative. Therefore, ‘type 1’ banks are not anymore uncertain about the potential gambling of ‘type 2’ banks and grant the interbank loans under normal conditions. As a result, funds will not be invested in gambling assets, and both the banking system and the interbank market keep their normal track.

**Proposition 10** The existence of a credible preventive policy, in the form of a sufficiently high penalty tax on gambling revenues, i.e., $\tau^p > \tau^p_{\min}$, can completely eliminate the gambling incentive, and hence efficiently protects the banking system and the interbank market from the destabilizing effects of gambling assets in the presence of asymmetric information.

The limit of this regulation is that when the gambling is successful and the financial market
functions well, policymakers and regulators may not be urged to punish the banks that take too much risk by investing in gambling assets. There is also much difficulty in distinguishing a good investment from a gambling asset that has been successful.

8 Conclusion

The model presented in this paper captures several features of recent banking crises characterized by the malfunctioning of the interbank market. It is shown that, while the latter improves the social welfare during normal times, it could be a factor of instability by disseminating the effects of various shocks to the entire banking system during crisis times. In particular, the interbank market could aggravate or induce a confidence crisis by making possible the self-reinforcing panic of both borrowing banks’ depositors and interbank-lending banks. It could increase banks’ risk-taking in the presence of gambling asset and even aggravate the negative effects of the contagion from a foreign sovereign debt crisis on the domestic banking system. In addition, facing an aggregate liquidity shortage, the malfunctioning of interbank market will induce a contraction in the size of eligible collateral.

The minimal capital/liquidity ratios required by the interbank market discipline to achieve optimal risk-sharing in the banking system during normal times could be too low for being efficient during crisis times. Implementing a more severe regulation by the government is hence essential for enhancing banks’ resilience during crises. However, a too restrictive regulation is impractical because it could hamper the role of banks as efficient financial intermediaries. Moreover, such regulation as well as the interbank market discipline may have perverse effects on the gambling behaviors of banks, and the government should introduce a penalty tax conditional on the ex-post discovery of such behaviors.

These measures should be complemented by the government’s crisis management in the form of banking bailout. The latter can play a key role in stabilizing the interbank market and hence the banking sector during a crisis that could originate from self-fulfilling bank runs, contagion from foreign sovereign debt crisis and gambling behaviors of banks if the government’s initial budgetary position is relatively strong. Otherwise, an engagement by a government without any monetary sovereignty in bailing out banks during an interbank market crisis could increase the national public debt to an unsustainable level and hence result in twin banking and sovereign debt crises, as it is the case in the Eurozone.
References


