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Incentives for cost misrepresentation in supply chains

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This paper investigates sequential manufacturer–retailer price determination and channel performance under possible misrepresentation by one member of its privately known cost. To the standard double marginalization game, we add a preliminary stage where the manufacturer (alternately the retailer) announces its privately known constant marginal cost. We prove that the manufacturer has no incentive to misrepresent its cost, and we give respective sufficient conditions on the demand function for the retailer to overreport and to underreport costs. Depending on the shape of the demand function, opportunistic behavior by the retailer may lower or raise the manufacturer's profit and channel performance.

Key words channel cooperation, channels of distribution, cost misrepresentation, double marginalization, retailing and wholesaling

JEL classification C72, D82, L11, M31

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1 Introduction

It has long been recognized that informational asymmetries between manufacturers and retailers can drastically affect pricing behavior and channel performance. The nature of these asymmetries is continuously evolving with the advent of new information technologies (such as marketing decision support systems; see Little 1979), which alter the division of power between manufacturers and retailers. Desiraju and Moorthy (1997) study channel performance when the retailer is better informed than the manufacturer about demand conditions. They prove that channel profit is maximized when this information about demand is indirectly shared by the manufacturer through the observation of the market price and service. An opportunistic behavior on the part of the informed channel member decreases channel performance. Type-revealing channel contracts are one way to alleviate this effect (Chu 1992).

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In the present paper, we investigate the incentives to share or distort privately-held cost information by one channel member, as well as the resulting pricing behavior and channel profits. Consider the standard double marginalization game (Spengler 1950). An upstream manufacturer produces a good at a constant marginal cost, which is common knowledge. The manufacturer then sells the good through an independent retailer. Double marginalization occurs because the manufacturer and the retailer successively choose the wholesale price and the selling price. Although the manufacturer has no direct control over the marketing policies of the retailer, it does have some influence on the final selling price, because it sets the wholesale price to maximize its own profit, taking into account the retailer's reaction. These sequential decisions are traditionally formalized as a Stackelberg-type two-stage game: at stage 1, the manufacturer chooses the wholesale price; and at stage 2, given this wholesale price, the retailer chooses the final selling price. Here, we expand this game by adding a preliminary stage at which one channel member reports, and thus can misrepresent its constant marginal cost. We assume that the other player believes the announced cost is the true cost. This could be justified by the fact that the other player does not possess any additional relevant information about the unknown cost.[1] Therefore, we focus exclusively on the incentives faced by one channel member to manipulate in its favor the other's naïve belief (given by the announced value) about its cost.

We define our setup for two scenarios, scenario M where the manufacturer has private information about its marginal manufacturing cost and scenario R where the retailer has private information about its marginal retail cost. For each scenario, we first give generalizations of existing results in the literature related to the reaction of the retail price to the wholesale price and the reaction of the wholesale price to an increase in the marginal manufacturing cost or in the marginal retail cost. The added generality is manifested in the absence of customary restrictive assumptions on the shape of the demand function (i.e. some form of concavity of the profit function). Then, we study the incentive of the player holding private information to lie about its marginal cost.

For scenario M, the manufacturer's incentive to misrepresent its marginal cost crucially depends on how the selling price reacts to the wholesale price and how the wholesale price reacts to the reported manufacturing cost. Our generalizations of existing results establish that these effects are unambiguous and independent of the shape of the demand function: as the reported manufacturing cost increases, the wholesale price increases and so does the selling price. This allows us to prove our first important result: when holding private information, the manufacturer would always choose to reveal its true cost. Effectively, if the manufacturer overreports costs, it increases its markup, but the retail price also increases and sales decrease, and the latter is the dominant effect. In turn, if it underreports costs, its markup decreases. This negative effect on its profit overwhelms the positive effect of a lower selling price and higher sales. To conclude, when the manufacturer has private

[1] In other words, we are implicitly assuming that the second player is rather naïve in that it takes for granted the announcement, although it ought to be aware, given the common knowledge structure of the model, that incentives for misrepresentation might be present. More precisely, we are not modeling the strategic interaction here as a Bayesian game where the cost of one player is private information, with the other player acting only on the basis of a prior belief about this cost. Nonetheless, although the analysis presented here is not in the spirit of Bayesian games, the results constitute a useful building block for such games.

information, pricing decisions and channel performance are identical to the standard double marginalization game with complete information.

For scenario *R*, the retailer's incentive to overreport or to underreport costs depends on how the selling price reacts to the sum of the wholesale price and the reported retail cost and how the wholesale price reacts to the reported retail cost. As in the previous case, the first effect is unambiguous. Nevertheless, the sign of the second effect crucially depends now on the shape of the demand function. The intuition is easy to grasp. When the retail cost increases, the manufacturer knows that the retailer will increase the selling price. If the demand function is such that this does not severely reduce demand, the manufacturer will choose to increase the wholesale price. This obtains for a restricted class of demand functions that includes the class of isoelastic (or hyperbolic) demands that is frequently used to study channel performance. In this case, to generate a lower wholesale price, the retailer chooses to underreport costs. Therefore, we can prove our second main result: for a small class of demand functions, the retailer underreports costs. The more intuitive result that the retailer overreports costs obtains for a larger class of demand functions that includes the standard class of linear demands, following the converse intuition. A precise global condition involving the log-concavity of the demand function and of the negative of its derivative captures which of the two cases (if any) will prevail. This is illustrated with three specific groups of demand functions. As a special case of one of these examples, the exponential demand is shown to yield truthful cost reporting, thus forming the boundary between the two classes of demand described above.

As to the comparison between the two scenarios, we point out that the sequential nature of the double marginalization game implies that only the follower, namely, the retailer, can benefit from manipulating private cost information. Our third main result deals with the implications on prices and profits. We prove that for the intuitive case, the upward misrepresentation of the retail marginal cost in scenario *R* increases the selling price and decreases the wholesale price, increasing simultaneously the retailer's profit and decreasing the manufacturer's profit, relative to scenario *M*. Inversely, for the counterintuitive case, the downward misrepresentation of the retail marginal cost in scenario *R* will reduce the selling price and the wholesale price, relative to scenario *M*. In this case, the opportunistic behavior of the retailer that emerges, when it has private information, does not only increase its own profit but also the manufacturer's profit and, therefore, channel profit. Hence, for a robust class of demand functions, including isoelastic demands, it makes sense for the manufacturer to be naïve and to accept the cost announced by the retailer and the related price contract specified in scenario *R*. In addition, the result, taken for granted in the literature, that the opportunistic behavior of the retailer decreases the manufacturer's profit and channel performance is only valid for the identified class of demand functions, which includes the class of linear demands.

The counterintuitive conclusions are in line with some well-known results on the effects of cost increases on firm profits in static oligopoly settings. Indeed, with identical firms, whether equilibrium per-firm profit decreases with the firms' common unit cost depends on the shape of the demand function. For a class of convex demand functions that includes the class of isoelastic demands, the counterintuitive conclusion that equilibrium per-firm profit increases with the firms' common unit cost is valid. In addition, this seemingly counterintuitive result is robust to changes in modes of competition and product differentiation. See Seade (1985), Kimmel (1992) and Fevrier and Linnemer (2004), among others, for standard Cournot competition and Anderson, De Palma, and Kreider (2001) for Bertrand competition with differentiated products.

The present paper is related to a number of published papers that find that the introduction of information flows in a supply chain increases channel performance. Information flows ensure a better perception by a channel member of his or her environment (e.g. Cachon and Fisher 2000). In addition, information allows the channel member to internalize the spillovers induced by his or her own decisions in the distribution channel. This internalization is a prerequisite to solving the well-known double marginalization problem uncovered by Spengler (1950). It can be achieved through a coordination mechanism, such as the quantity discount schedule discussed by Jeuland and Shugan (1983).

The rest of the paper is organized as follows. Section 2 characterizes the scenario where the manufacturer alone has private information on own marginal cost. Section 3 considers the alternative scenario where the private information on own cost is held by the retailer. Section 4 establishes a comparison between the two scenarios and its implications on prices and profits. Section 5 contains a brief conclusion. Section 6 provides two specific illustrative examples of demand functions that admit closed-form solutions. Section 7 presents a self-contained and simple summary of the supermodularity notions used, as well as the proofs of all the results of this paper.

2 The manufacturer's cost as private information

In this section, we consider the scenario M where the private information on own marginal cost is held by the manufacturer. Hence, the sequential decisions are formalized as a three-stage game. At stage 0, the manufacturer announces the marginal cost $c_1 + \beta$, where $\beta \in [-c_1, +\infty)$ and c_1 is its true marginal cost, anticipating the wholesale price set in the next period and the retailer's reaction to the wholesale price. At stage 1, given its announcement $c_1 + \beta$, the manufacturer chooses p_1 anticipating the retailer's reaction to p_1. Finally, at stage 2, the retailer chooses p_2 given p_1 and implicitly $c_1 + \beta$.

The (subgame perfect) equilibrium of this three-stage game can be computed through backward induction.

At stage 2, the retailer chooses the selling price $p_2 \in [p_1 + c_2, +\infty)$ given the announced manufacturing marginal cost $c_1 + \beta$ and the wholesale price $p_1(c_1 + \beta)$ in order to maximize profit:

$$\pi_2(p_2, p_1) = (p_2 - p_1 - c_2)D(p_2),$$

with β denoting the cost distortion. The following assumption is maintained throughout.

Assumption 1 *The demand function is C^1 at all $p_2 \geq 0$ such that $D(p_2) > 0$, and* $\lim_{p_2 \to +\infty} p_2 D(p_2) = 0$.

Under Assumption 1, the solution $p_2(p_1)$ is always interior,[2] that is, $p_2(p_1)$ is always in $(p_1 + c_2, +\infty)$, and satisfies

$$(p_2 - p_1 - c_2)D'(p_2) + D(p_2) = 0. \tag{1}$$

We establish now that, whatever the shape of the demand function, the manufacturer knows that the retailer will increase the market price when the wholesale price increases.

Lemma 1 *Under Assumption 1, every selection of $p_2(p_1)$ is strictly increasing in p_1.*

All proofs are in the Appendix. Note that the proof of this lemma follows the line of arguments given by Amir, Maret, and Troge (2004), who use lattice-theoretic techniques developed by Topkis (1978, 1998) in a study of monopoly pricing.[3]

At stage 1, the manufacturer chooses the wholesale price p_1 to maximize its announced profit; that is,

$$\max_{p_1 \in [c_1+\beta, +\infty)} \pi_1^A(p_1, c_1 + \beta) = (p_1 - c_1 - \beta)D(p_2(p_1)).$$

To simplify the analysis, we introduce another assumption on the demand function.

Assumption 2 *The demand function is C^3, $D'(p_2) < 0$ and $D(p_2)D''(p_2) - 2D'^2(p_2) < 0$ at all $p_2 \geq 0$ such that $D(p_2) > 0$.*

Under Assumptions 1 and 2, the solution $p_1^M(c_1 + \beta)$ is always interior,[4] that is, $p_1^M(c_1 + \beta) \subset (c_1 + \beta, +\infty)$, and it fulfills:[5]

$$D\left[p_2\left(p_1^M\right)\right] + \left(p_1^M - c_1 - \beta\right) p_2'\left(p_1^M\right) D'\left(p_2\left(p_1^M\right)\right) = 0. \tag{2}$$

Our second general result is that, whatever the shape of the demand function, the wholesale price is strictly increasing in the manufacturer's announced marginal cost.

Lemma 2 *Under Assumption 1, every selection of $p_1^M(c_1 + \beta)$ is strictly increasing in $(c_1 + \beta) \in [0, +\infty)$.*

At stage 0, the manufacturer chooses to misrepresent its marginal cost by the amount $\beta \in [-c_1, +\infty)$ in order to maximize its (true) profit. The price rule the manufacturer takes into account is, of course, the announced price rule $p_1^M(c_1 + \beta)$. Otherwise, its behavior

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2 From Assumption 1, it follows that:

$$\begin{aligned}\lim_{p_2\to+\infty} \pi_2(p_2, p_1) &= \lim_{p_2\to+\infty} p_2 D(p_2) - (p_1 + c_2) \lim_{p_2\to+\infty} D(p_2)\\ &= -(p_1 + c_2) \lim_{p_2\to+\infty} D(p_2) = 0.\end{aligned}$$

In addition, $\pi_2(p_1 + c_2, p_1) = 0$, and there exists $\overline{p_2} \in (p_1 + c_2, +\infty)$ such that $\pi_2(\overline{p_2}, p_1) > 0$. Hence, for all p_1, every selection of the price argmax is interior.

3 Appendix I summarizes the relevant notions from this theory for the present paper.

4 This follows from the fact that by definition of $p_2(p_1)$, $\lim_{p_1\to+\infty} p_2(p_1) = +\infty$.

5 Under the assumption that $2(D'(p_2))^2 - D(p_2)D''(p_2) \neq 0$ and $D'(p_2) \neq 0$ for all (p_1, p_2) that fulfills (1), the implicit function theorem ensures that $p_2(p_1)$ is a C^1 function.

would be incoherent, and the retailer would know the manufacturing cost was incorrectly reported:

$$\max_{\beta\in[-c_1,+\infty)} \pi_1(\beta) = \left[p_1^M(c_1+\beta) - c_1\right] D\left(p_2\left(p_1^M(c_1+\beta)\right)\right).$$

The marginal profit of the manufacturer with respect to β can then be written:[6]

$$\frac{d\pi_1}{d\beta}(\beta) = p_1^{M\prime} D + \left(p_1^M - c_1\right) p_2' p_1^{M\prime} D'.$$

From (2), we deduce that:

$$\left(p_1^M - c_1\right) p_2' D' = \beta\, p_2' D' - D.$$

Therefore,

$$\frac{d\pi_1}{d\beta}(\beta) = \beta\, p_1^{M\prime} p_2' D'.$$

Hence, the manufacturer's incentive to misrepresent its marginal cost crucially depends on how the wholesale price reacts to the reported manufacturing cost (i.e. on the sign of $p_1^{M\prime}$) and how the selling price reacts to the wholesale price (i.e. on the sign of p_2'). These effects are unambiguous and independent of the shape of the demand function: as the reported manufacturing cost increases, the wholesale price increases and so does the selling price whatever the shape of the demand function. Therefore, under Assumption 2 we deduce that $\pi_1(\beta)$ is strictly decreasing in β for all $\beta > 0$, and $\pi_1(\beta)$ is strictly increasing in β for all $\beta < 0$. Denote by β^M the optimal level of β. Therefore, we can establish the following proposition.

Proposition 1 *Under Assumptions 1 and 2, at the subgame perfect equilibrium of the three-stage game with private information held by the manufacturer, the latter chooses to reveal its true manufacturing cost; that is,* $\beta^M = 0$.

The manufacturer always has an incentive for truth-telling. When it overreports costs, it increases its mark-up, but the retail price also increases and sales decrease, and the latter is the dominant effect. When the manufacturer underreports costs, its markup decreases. This negative effect on its profit overwhelms the positive effect of a lower selling price and higher sales.

3 Retailer's cost as private information

In this section, we consider the alternative scenario where the private information on own marginal cost is held by the retailer, and not by the manufacturer. Hence, the sequential decisions are now formalized by the following three-stage game. At stage 0, the retailer

[6] Under the additional assumption that $2p_2'(p_1)D'(p_2) + (p_1 - c_1 - \beta)[p_2'^2 D''(p_2) + p_2''(p_1)D'(p_2)] \neq 0$ for all p_1 that fulfills (2), the implicit function theorem ensures that $p_1(c_1+\beta)$ is a C^1 function.

announces the marginal cost $c_2 + \delta$, where $\delta \in [-c_2, +\infty)$ and c_2 is its true marginal cost, anticipating the manufacturer's reaction to this announcement and its own reaction to the wholesale price. At stage 1, the manufacturer chooses p_1, taking for granted $c_2 + \delta$ and knowing the retailer's reaction to p_1. The manufacturer's influence on the market price is now biased in the sense that the retailer's reaction expected by the manufacturer for any given wholesale price is conditional on the announced retailer's marginal cost; that is, $c_2 + \delta$. Finally, at stage 2, the retailer chooses p_2 given $c_2 + \delta$ and p_1.

We ask the same question the other way around; namely, whether the retailer now has any incentive to misrepresent its marginal cost, under the assumption that the manufacturer will accept the announcement at face value. We shall establish that the answer is positive.

The (subgame perfect) equilibrium of this three-stage game can be computed through backward induction, with computations at stage 2 defining the retailer's reaction function as expected by the manufacturer.

At stage 2, the manufacturer expects that the retailer will choose a selling price $p_2 \in [p_1 + c_2 + \delta, +\infty)$ in order to maximize its (announced) profit:

$$\pi_2^A(p_2, p_1 + c_2 + \delta) = (p_2 - p_1 - c_2 - \delta)D(p_2),$$

given p_1.

Under Assumption 1, the solution $p_2(p_1 + c_2 + \delta)$ is always interior,[7] that is, $p_2(p_1 + c_2 + \delta) \subset (p_1 + c_2 + \delta, +\infty)$, and it fulfills:

$$(p_2 - p_1 - c_2 - \delta)D'(p_2) + D(p_2) = 0. \tag{3}$$

From Lemma 1, we know that whatever the shape of the demand function, the manufacturer knows that the retailer will increase the market price when the sum of the wholesale price and the reported retail cost increases; that is, every selection of $p_2(p_1 + c_2 + \delta)$ is strictly increasing in $(p_1 + c_2 + \delta)$.

At stage 1, the manufacturer chooses the wholesale price p_1 to maximize profit; that is,

$$\max_{p_1 \in [c_1, +\infty)} \pi_1(p_1, c_2 + \delta) = (p_1 - c_1)D(p_2(p_1 + c_2 + \delta)).$$

Under Assumption 1, the solution $p_1^R(c_2 + \delta)$ is always interior,[8] that is, $p_1^R(c_2 + \delta) \subset (c_1, +\infty)$, and it fulfills:

$$D(p_2) + \left(p_1^R - c_1\right) p_2' \left(p_1^R + c_2 + \delta\right) D'(p_2) = 0. \tag{4}$$

Following the same line of argument as in Lemma 2, one can easily check that whatever the shape of the demand function, the sum of the wholesale price and the reported retail cost is strictly increasing in the reported retail cost.

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[7] From Assumption 1, it follows that there exists $\overline{p_2} \in (p_1 + c_2 + \delta, +\infty)$ such that $\pi_2^A(\overline{p_2}, p_1 + c_2 + \delta) > 0$ and $\lim_{p_2 \to +\infty} \pi_2^A(p_2, p_1 + c_2 + \delta) \leq 0$ for all $p_1 + c_2 + \delta$. Because $\pi_2(p_1 + c_2 + \delta, p_1 + c_2 + \delta) = 0$, for all $p_1 + c_2 + \delta \geq 0$ every selection of the price argmax is interior.

[8] Note that although the function $p_2(\cdot)$ is identical in both scenarios, the function $p_1^R(\cdot)$ differs from the function $p_1^M(\cdot)$ of the previous scenario.

Lemma 3 *Under Assumption 1, every selection of* $p_1^R(c_2+\delta)+c_2+\delta$ *is strictly increasing in* $(c_2+\delta)\in[0,+\infty)$.

At stage 0, the retailer chooses to misrepresent its marginal cost by the amount $\delta \in [-c_2,+\infty)$ in order to maximize its (true) profit. The price rule that the retailer takes into account is, of course, the announced price rule, $p_2^\delta = p_2(p_1^R(c_2+\delta)+c_2+\delta)$. Otherwise, the retailer's behavior would be incoherent, and the manufacturer would know that the retail cost was incorrectly reported:

$$\max_{\delta\in[-c_2,+\infty)} \pi_2(\delta) = \left[p_2^\delta - p_1^R(c_2+\delta) - c_2\right] D\left(p_2^\delta\right).$$

Under Assumption 2, the optimal selling price $p_2(p_1^R+c_2+\delta)$ is a single-valued function of $(p_1^R+c_2+\delta)$ that is twice-continuously differentiable and the optimal wholesale price $p_1^R(c_2+\delta)$ is a continuously differentiable function of $(c_2+\delta)$.[9] The marginal profit of the retailer with respect to δ can then be written:

$$\begin{aligned}\frac{d\pi_2}{d\delta}(\delta) &= \left[p_2'\left(p_1^{R\prime}+1\right) - p_1^{R\prime}\right] D(p_2) + \left(p_2 - p_1^R - c_2\right)\left(p_1^{R\prime}+1\right) p_2' D'(p_2)\\ &= p_2'\left(p_1^{R\prime}+1\right)\left[D(p_2)+\left(p_2-p_1^R-c_2\right)D'(p_2)\right] - p_1^{R\prime}D(p_2).\end{aligned}$$

From (3), we deduce that:

$$\frac{d\pi_2}{d\delta}(\delta) = \delta\, p_2'\left(p_1^{R\prime}+1\right) D'(p_2) - p_1^{R\prime} D(p_2). \tag{5}$$

Hence, the retailer's incentive to overreport or to underreport costs depends on how the selling price reacts to the sum of the wholesale price and the reported retail cost (i.e. on the sign of p_2') and how the wholesale price reacts to the reported retail cost (i.e. on the sign of $p_1^{R\prime}$). As in the previous scenario, the first effect is unambiguous. Nevertheless, the sign of the second effect crucially depends now on the shape of the demand function. Denote by δ^R the optimal level of δ.

Now, we shall establish: (i) that under a broad set of demand specifications, the retailer has an incentive to overreport costs, but (ii) that under a rather restricted although robust set of demand specifications, the retailer has an incentive to underreport costs. Whereas the latter conclusion is clearly counterintuitive at first sight, it becomes less so when contrasted with well-known results on the effects of exogenous unit cost increases on equilibrium profits in Cournot oligopoly, as we shall argue below.[10]

The key determinant of whether the retailer will find it profitable to overreport or to underreport costs is the sign of:

$$\Delta \equiv D^2\left(D'D''' - D''^2\right) - 2\left(DD'' - D'^2\right)^2.$$

[9] To be more rigorous, the latter property required an additional assumption introduced in Proposition 2 (i.e. $\Delta \neq 0$ for all p_2, where the definition of Δ is given below).

[10] See Seade (1985), Kimmel (1992) and Fevrier and Linnemer (2004), among others.

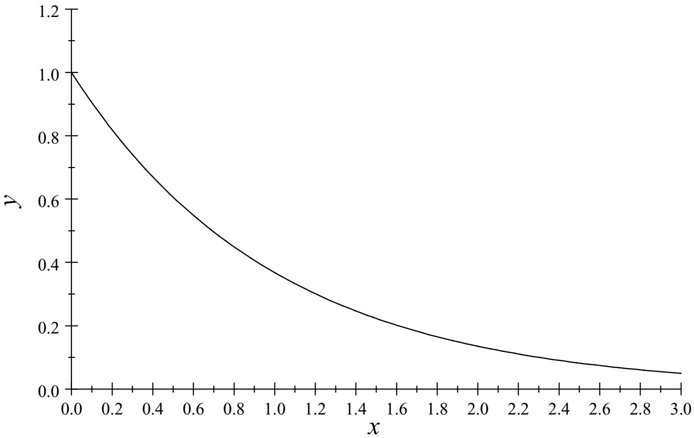

Figure 1 Exponential demand function.

Proposition 2 *Under Assumptions 1 and 2, at the subgame-perfect equilibrium of the three-stage game with private information held by the retailer:*[11]

(i) if the demand function is such that $\Delta < 0$ for all p_2, then the retailer has an incentive to announce a higher marginal cost, that is, $\delta^R \geq 0$; and
(ii) if the demand function is such that $\Delta > 0$ for allp_2, then the retailer has an incentive to announce a lower marginal cost, that is, $\delta^R \leq 0$.

As the conditions $\Delta < 0$ and $\Delta > 0$ are rather uncommon, we now provide some insight into the nature of the underlying restrictions on demand, supplemented by illustrative examples using well-known demand specifications. Following this, we also provide a general economic interpretation of Proposition 2.

The separating case of $\Delta = 0$ holds for the exponential demand function $D(p_2) = e^{-p_2}$, which makes the two terms in Δ both equal to 0.[12] This function is plotted in Figure 1.

The logarithm of this function is linear; therefore, in this case, D (which is equal to $-D'$) is simultaneously log-concave and log-convex.

The condition $\Delta < 0$, which yields the intuitive conclusions, holds for demand functions that are log-concave (i.e. $DD'' - D'^2 < 0$) or strongly log-convex (i.e. $DD'' - D'^2$ is very >0) and have $-D'$ being log-concave (i.e. $D'D''' - D''^2 < 0$).[13] This is the case, for example, for the class of demand functions $D(p_2) = e^{-p_2^{\alpha}}$, with $\alpha > 1$, where the price-elasticity of

[11] Note that this proposition does not require that the demand function be strictly decreasing (but only for any p_2 that fulfills (3) and (4)), and it only requires that $D''D - 2D'^2 \neq 0$ for any p_2 that fulfills (3). In addition, whenever $p_2(p_1 + c_2 + \delta)$ is C^1, under Assumption 1, Lemma 1 implies that $D''D - 2D'^2 \leq 0$ for all p_2 where the equality holds only for isolated prices in $\mathbb{R}_+$ (see Appendix II).

[12] For more on this demand function, see Amir (1996a) and Amir and Lambson (2000).

[13] Note that because log-concavity is only defined for positive functions, it makes sense to talk about $-D'$ being log-concave, but not about D' being log-concave.

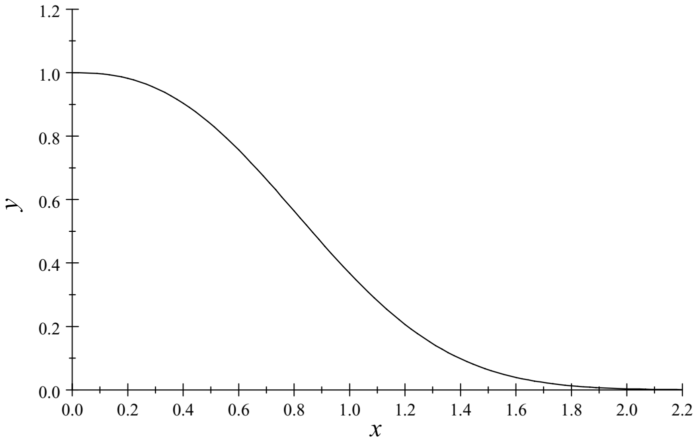

Figure 2 The demand function $D(p_2) = e^{-p_2^{2.5}}$.

the price-elasticity of demand, α, is now larger than 1. For $\alpha = 2.5$, the demand function is plotted in Figure 2. The logarithm of this function is concave yielding to a demand function that is intially concave and then convex.

Another class of demand functions that satisfies the intuitive case is the one most frequently used in industrial organization, namely, the class of linear demands $D(p_2) = \max\{a - bp_2, 0\}$, where $a, b > 0$ are constant with $b(c_1 + c_2) < a$ and the logarithm is concave.

The condition $\Delta > 0$, which yields the counterintuitive conclusions, holds for demand functions that are log-convex (i.e. $DD'' - D'^2 > 0$) and have $-D'$ being log-convex (i.e. $D'D''' - D''^2 > 0$). This is the case, for example, for the class of demand functions $D(p_2) = e^{-p_2^{\alpha}}$, with $\frac{1}{2} \leq \alpha < 1$, where the price-elasticity of the price-elasticity of demand, α, is now lower than 1. For $\alpha = \frac{1}{2}$, the demand function is plotted in Figure 3. The logarithm of this function is convex.

Another well-known example of a class of demands satisfying the counterintuitive case is the class of isoelastic demands, given by $D(p_2) = a/p_2^b$, with $a > 0$, $b > 1$ and b being the elasticity of demand. For $a = 1$ and $b = 2$, the demand function is plotted in Figure 4. The logarithm of this function is convex.

For a better understanding of Proposition 2, we shall explain how the manufacturer and the retailer change their decision at each stage of the game depending on the shape of the demand function (i.e. on the sign of Δ).

Note first that the decision of the retailer in the last stage of the game is independent of the shape of the demand function. More precisely, when the reported retail cost increases, whatever the shape of the demand function (Lemma 1), the retailer sets a higher selling price at stage 2.

At stage 1, given this higher selling price in the next period, the manufacturer changes the wholesale price such that the sum of the wholesale price and the reported retail cost increases (Lemma 2). Nevertheless, whether the manufacturer chooses to increase the

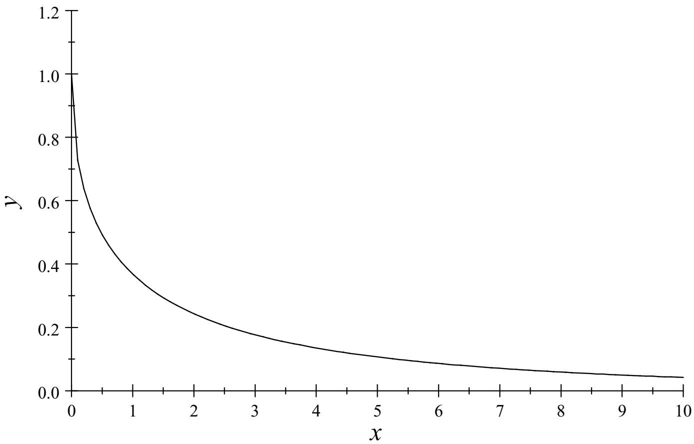

Figure 3 Demand function $D(p_2) = e^{-\sqrt{p_2}}$.

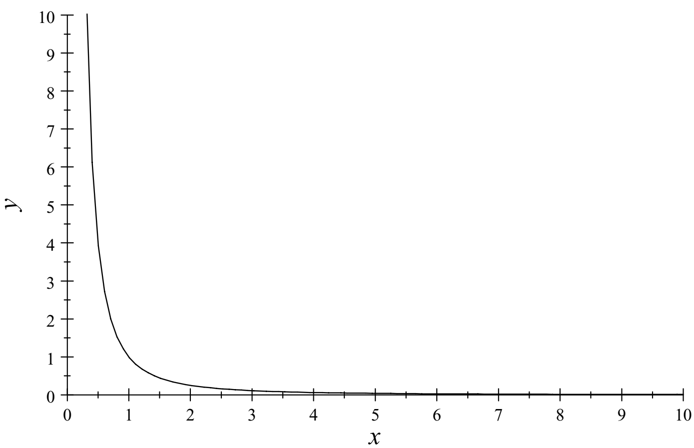

Figure 4 Demand function $D(p_2) = 1/p_2^2$.

wholesale price crucially depends on the shape of the demand function. If the demand function is strongly reduced by the higher selling price (i.e. when $\Delta < 0$), the manufacturer finds it advisable to decrease the wholesale price to stimulate market demand. Inversely, if demand is not severely lowered by the increase in the selling price (i.e. when $\Delta > 0$), the manufacturer increases the wholesale price.

To understand how the retailer revises its choice at stage 0, we shall study the impact of the two future price changes on the retailer's profit at this stage. The forecast increase in the selling price generates two effects, a negative effect due to the fall in demand and a positive effect due to the higher retailer's markup. The global selling price effect can be written $\delta p_2'(p_1^{R\prime} + 1) D'(p_2)$, which is negative when the retailer overreports costs (i.e. when $\delta > 0$) and is positive when the retailer underreports costs (i.e. when $\delta < 0$). In turn,

Table 1 Comparison of scenarios M and R

	δ^R	p_1	p_2	π_1	π_2
$\Delta < 0$	$\delta^R \leq 0$	$p_1^R \leq p_1^M$	$p_2^M \leq p_2^R$	$\pi_1^R \leq \pi_1^M$	$\pi_2^M \leq \pi_2^R$
$\Delta > 0$	$\delta^R \geq 0$	$p_1^R \leq p_1^M$	$p_2^R \leq p_2^M$	$\pi_1^M \leq \pi_1^R$	$\pi_2^M \leq \pi_2^R$

the forecast change of the wholesale price generates an additional mark-up effect. Formally, this effect can be written $-p_1^{R\prime} D(p_2)$. The sign of this effect crucially depends on whether the wholesale price increases in the next stage. If $\Delta < 0$, the wholesale price decreases in the next stage; therefore, the retailer's markup increases and the wholesale price effect on the retailer's profit is positive. If $\Delta > 0$, the wholesale price increases in the next stage; therefore, the retailer's markup decreases and the wholesale price effect on the retailer's profit is negative.

To conclude, if $\Delta < 0$, when the retailer underreports costs, the selling price effect and the wholesale price effect on the retailer's profit are both positive. Hence, the retailer always has an incentive to increase its reported marginal cost; therefore, it finally chooses $\delta^R \geq 0$. If $\Delta > 0$, when the retailer overreports costs, the selling price effect and the wholesale price effect on the retailer's profit are both negative. Hence, the retailer always has an incentive to decrease its reported marginal cost; therefore, it finally chooses $\delta^R \leq 0$. The equilibrium choices are summarized in Table 1. They are compared with the equilibrium values of scenario M, which match those of the standard double marginalization game with complete information.

4 Comparison of the two scenarios

We shall now compare the two scenarios. The manufacturer, when holding private information about cost, always chooses to reveal the true cost, while the retailer, when holding such information, generally chooses to lie about the cost. This comes from the sequential nature of the double marginalization game and the leader position of the manufacturer. At an intuitive level, this result is not surprising because the retailer reacts *directly* to p_1 only, and not to c_1, so that any advantage of reporting a suitable c_1 for the manufacturer can also be achieved by choosing the corresponding p_1 in the standard double marginalization game. Hence, untruthful reporting of c_1 is tantamount to choosing a price distinct from the leader's optimal price in the subgame-perfect equilibrium, something that is clearly ruled out by this equilibrium concept. The sequential nature of the game thus implies that only the follower, here the retailer, can benefit from manipulating private cost information.

Let us now compare prices and profits for the two scenarios. In scenario M, because the manufacturer chooses to reveal its true cost, the equilibrium values are those of the standard double marginalization game. In turn, the latter game matches the last two stages of scenario R when the cost misrepresentation is set to 0. Hence, the equilibrium prices in scenario M are such that:

$$\begin{cases} p_1^M = p_1^R(c_2) \\ p_2^M = p_2\left(p_1^R(c_2) + c_2\right) \end{cases},$$

whereas in scenario R the equilibrium price values also depend on the cost misrepresentation chosen by the retailer; that is,

$$\begin{cases} p_1^R = p_1^R\left(c_2 + \delta^R\right) \\ p_2^R = p_2\left(p_1^R\left(c_2 + \delta^R\right) + c_2 + \delta^R\right) \end{cases}.$$

As to the market price, we already pointed out that when the reported retail cost increases, whatever the shape of the demand function (Lemma 1), the retailer increases the selling price at stage 2. As a consequence, for the class of demand functions where the retailer chooses to overreport its cost ($\Delta < 0$), the selling price is higher when the retailer has private information; that is, $p_2^M \leq p_2^R$. For the class of demand functions where the retailer chooses to underreport its cost ($\Delta > 0$), we obtain the counterintuitive result that the selling price is lower when the retailer has private information; that is, $p_2^R \leq p_2^M$.

The consequences for the wholesale price are as follows. As noted previously, whether the wholesale price increases with the reported retail cost ($c_2 + \delta^R$) and, therefore, with the selling price crucially depends on the shape of the demand function. If the demand is strongly reduced by a higher selling price (i.e. when $\Delta < 0$), the manufacturer finds it advisable to decrease the wholesale price in order to stimulate market demand. In this case, when the retailer has private information and increases the selling price, the manufacturer decreases the wholesale price; that is, $p_1^R \leq p_1^M$. Inversely, if demand is not severely lowered by the increase in the selling price (i.e. when $\Delta > 0$), the manufacturer increases the wholesale price with the selling price. In this case, when the retailer has private information and decreases the selling price, because the beneficial impact on market demand is not strong enough, the manufacturer also chooses to decrease the wholesale price; that is, $p_1^R \leq p_1^M$. To conclude, despite the incentives going in opposite directions for the two classes of demand functions, irrespective of the shape of the demand function, the manufacturer always chooses a larger wholesale price when it has private information about its cost.

Let us now compare the retailer's profit. Obviously, in scenario R, where the retailer has private information and the opportunity to lie about its cost, its profit cannot be lower than in scenario M, where it does not have this opportunity, given that the equilibrium values match those of the standard double marginalization game.

Finally, we establish in the proposition below that the manufacturer's profit cannot increase with the retailer's announcement δ. The intuition is as follows. An increase in the reported retail cost generates two effects on the manufacturer's profit. The first effect is a negative market price effect: market demand and, consequently, the manufacturer's profit decrease with the selling price. The second effect is a mark-up effect that is negative when the wholesale price decreases ($\Delta < 0$) and positive when the wholesale price increases ($\Delta > 0$). It is proved in Proposition 3 that the first effect is stronger. Therefore, whatever the shape of the demand function, the manufacturer's profit does not increase with the reported retail cost. Consequently, for the class of demand functions defined by $\Delta < 0$, we obtain the intuitive result that the manufacturer's profit is larger when it has private information $\pi_1^R \leq \pi_1^M$. Nevertheless, for the class of demand function defined by $\Delta > 0$, we obtain the counterintuitive result that, when the retailer has private information, its opportunistic behavior does not only increase its own profit but also the manufacturer's profit, or $\pi_1^M \leq \pi_1^R$.

The comparison between prices and profits of the two scenarios is summarized in Proposition 3.

Proposition 3 *Under Assumptions 1 and 2:*

(i) if $\Delta < 0$ *for all* p_2*, one has*

$$\begin{cases} p_1^R \leq p_1^M \text{ and } p_2^M \leq p_2^R \\ \pi_1^R \leq \pi_1^M \text{ and } \pi_2^M \leq \pi_2^R \end{cases}; \text{ and}$$

(ii) if $\Delta > 0$ *for all* p_2*, one has*

$$\begin{cases} p_1^R \leq p_1^M \text{ and } p_2^R \leq p_2^M \\ \pi_1^M \leq \pi_1^R \text{ and } \pi_2^M \leq \pi_2^R \end{cases}.$$

In other words, whereas an upward misrepresentation of the retail marginal cost increases the market price and decreases the wholesale price, increasing simultaneously the retailer's profit and decreasing the manufacturer's profit, a downward misrepresentation will decrease the selling price and the wholesale price. In this case, the opportunistic behavior of the retailer that emerges in scenario *R* does not only increase its own profit but also the manufacturer's profit. Hence, for a robust class of demand functions including isoelastic demands, it makes sense for the manufacturer to be naïve and to accept the cost announced by the retailer and the related price contract specified in scenario *R*.

The results of Propositions 2 and 3 are summarized in Table 1.

We now return to the interpretation of the counterintuitive conclusions for the case $\Delta > 0$. The conclusions are not all that surprising when viewed in light of some well-known results on the effects of cost increases on firm profits in a Cournot setting. Indeed, with identical firms, equilibrium per-firm profit increases with the firms' common unit cost when inverse demand satisfies $P'(Q) + QP''(Q) > 0$ at the equilibrium output, or when the elasticity of $P'(Q)$, $QP''(Q)/P'(Q)$ is less than -1.[14] This condition requires that the inverse demand be strongly convex, with the prototypical example being the class of isoelastic demands that fulfills the condition $\Delta > 0 : P(Q) = a^{1/b}/Q^{1/b}$, where $a > 0$, $1 < b < 2$, with *b* being the elasticity of demand; see Kimmel (1992) for details.[15]

For Bertrand competition with differentiated products, Anderson, de Palma, and Kreider (2001) establish that per-firm profit increases with unit cost for a class of demand

[14] It is worthwhile stressing that this conclusion is robust, even though the assumption on inverse demand is rather restrictive. See Seade (1985) for the first exposition of this result and Fevrier and Linnemer (2004) for an application of more general forms of cost increases.

[15] Therefore, although neither of the two conditions at hand ($\Delta > 0$ and $P'(Q) + QP''(Q) > 0$) implies the other, they appear to share some common properties through their inclusion of this commonly used class of very convex demands.

functions satisfying an elasticity condition reflecting a strong notion of convexity. Their finding is quite closely analogous to the conclusion derived by earlier authors for Cournot competition, confirming once more the robust nature of this seemingly counterintuitive result, even to changes in modes of competition and product differentiation.

5 Two illustrative examples

We now provide two separate examples as illustrations of Proposition 2 and Proposition 3. For each of these examples, closed-form solutions for all the variables of interest allow for an easy verification of the conclusions of Proposition 2 and Proposition 3, the details of which are left to the reader. These examples focus on the conditions $\Delta = 0$ and $\Delta > 0$. As pointed out previously, a class of demand functions that fulfills the condition $\Delta < 0$ is the most frequently used in industrial organization, namely the class of linear demands for which the equilibrium values are easily computed.

Example 1 Consider the class of demand functions $D(p_2) = e^{-p_2^{\alpha}}$ with $\alpha > 0$. The price-elasticity of demand is given by $\varepsilon(p_2) = -\frac{p_2 D'}{D} = \alpha p_2^{\alpha}$. Furthermore, the price-elasticity of the price-elasticity of demand is given by $\widehat{\varepsilon}(p_2) \stackrel{\Delta}{=} \frac{p_2 \varepsilon'(p_2)}{\varepsilon(p_2)} = \alpha$ for all p_2, as is easily verified. Call this the second-order elasticity of demand.

One can easily verify that $D(\cdot)$ fulfills Assumptions 1 and 2 for all $\alpha > 0$. In addition, standard computations yield:

$$\Delta = -\frac{e^{-4p_2^{\alpha}}\alpha^2}{p_2^4}(\alpha - 1)\left[\alpha p_2^{3\alpha} + (2\alpha - 1)p_2^{2\alpha}\right].$$

For $\frac{1}{2} \le \alpha < 1$ (the second-order elasticity of demand is low), D and $-D'$ are log-convex, and one can easily verify that $\Delta > 0$ for all p_2. It follows that $\delta^R \le 0$ from Proposition 2. (Note that we have no closed-form solutions in this case.)

For $\alpha = 1$, a standard calculation yields $\Delta = 0$ for all p_2 and

$$\begin{cases} p_2(p_1 + c_2 + \delta) = p_1 + c_2 + \delta + 1 \text{ and } p_1^R(c_2 + \delta^R) = 1 + c_1 \\ \delta^R p_2' \left[p_1^{R\prime} + 1\right] D'(p_2) = 0 \end{cases}.$$

The solution of this system gives the equilibrium values of scenario R that match the equilibrium values of scenario M, because for this specific demand function the retailer, like the manufacturer, has an incentive for truth-telling.

$$\begin{cases} \beta^M = \delta^R = 0 \\ p_2^M = 2 + c_1 + c_2 = p_2^R \\ p_1^M = 1 + c_1 = p_1^R \\ \pi_1^M = \pi_2^M = e^{-(2+c_1+c_2)} = \pi_1^R = \pi_2^R \end{cases}.$$

For $\alpha > 1$ (the second-order elasticity of demand is high), D and $-D'$ are log-concave and one can easily check that $\Delta < 0$. Therefore, $\delta^R \ge 0$ from Proposition 2.

Example 2 Consider the class of iso-price-elastic demand models, which is frequently used to study channel performance:

$$D(p_2) = a/p_2^b \text{ where } a > 0, b > 1.$$

Here, the parameter b is the price-elasticity of demand. The higher b is, the more sensitive demand is to a change in price. We focus on price-elastic commodities by assuming $b > 1$. (This restriction follows from Assumption 1. Indeed, if $b < 1$, the optimal price for the retailer's optimization problem would go to infinity. Any iso-price-elastic demand with $b > 1$ fulfills Assumptions 1 and 2.)

A standard computation yields $\Delta = p_2^{-4b-4} a^4 b^2 (b^2 + 2b - 1)$.

Because $\Delta > 0$ for all p_2, we conclude from Proposition 2 that $\delta^R \leq 0$.

To verify this conclusion, a standard computation yields:

$$\begin{cases} p_2\,(p_1 + c_2 + \delta) = \dfrac{b(p_1 + c_2 + \delta)}{b-1} \\ p_1^R\,(c_2 + \delta) = \dfrac{bc_1 + c_2 + \delta}{b-1} \\ \dfrac{\partial \pi_2}{\partial \delta}(\delta) = -p_1^{R\prime} D(p_2) + p_2'\left(p_1^{R\prime} + 1\right)\delta D'(p_2) = -\dfrac{1}{b-1} a p_2^{-(b+1)} \left(p_2 + \dfrac{b^3}{b-1}\delta\right) \end{cases}.$$

Solving this, we obtain the equilibrium values:

$$\begin{cases} \delta^R = -\dfrac{c_1 + c_2}{b^2 - b + 1} < 0 = \beta^M \\ p_1^R = \dfrac{(b^2+1)c_1 + bc_2}{b^2 - b + 1} < \dfrac{bc_1 + c_2}{b-1} = p_1^M \\ p_2^R = \dfrac{b^3(c_1 + c_2)}{(b^2 - b + 1)(b-1)} < \dfrac{b^2}{(b-1)^2}(c_1 + c_2) = p_2^M \\ \pi_1^M = \dfrac{a\,(b-1)^{2b-1}}{b^{2b}\,(c_1 + c_2)^{b-1}} < \dfrac{a\left(b^2 - b + 1\right)^{b-1}(b-1)^b}{b^{3b-1}\,(c_1 + c_2)^{b-1}} = \pi_1^R \\ \pi_2^M = \dfrac{a\,(b-1)^{2b-2}}{b^{2b-1}\,(c_1 + c_2)^{b-1}} < \dfrac{a\left(b^2 - b + 1\right)^{b}(b-1)^{b-1}}{b^{3b}\,(c_1 + c_2)^{b-1}} = \pi_2^R \end{cases}.$$

6 Conclusion

This paper has characterized the respective incentives faced by the manufacturer and the retailer to communicate private information on own cost to the other party. Although the manufacturer always has an incentive for truth-telling, the retailer will overreport or underreport own cost, depending on the shape of the demand function. Hence, when studying the impact of information sharing policies, focusing on specific demand functions (often adopted in the marketing literature) might yield insights that are not robust.

For one broad class of demand functions, including linear demand, an intuitive result, taken for granted in the published literature, is confirmed. If allowed to misrepresent its marginal cost, the retailer would exaggerate own cost and, thereby, increase the market price, decrease the manufacturer's profit and the channel performance,[16] relative to the alternative scenario where the manufacturer has private information. In other words, the well-known social benefit of vertical integration is reinforced with the introduction of an informational asymmetry in the decentralized channel.

For another robust but somewhat restricted class of demand functions, including isoelastic demands, a more counterintuitive result is obtained. When holding private information, the retailer underreports its cost and, therefore, sets a lower selling price (relative to the alternative scenario where private information is held by the retailer). In contrast to the previous case, this opportunistic behavior of the retailer increases the manufacturer's profit and channel performance. In other words, the well-known social benefit of vertical integration is mitigated by the introduction of the retail cost misrepresentation in the decentralized channel. Indeed, it is to the manufacturer's advantage to be naïve and to accept the cost announced by the retailer at face value. This may be viewed as an interesting new form of collusion over information transmission, which benefits not only the two firms but consumers as well (through the lower selling price), in contrast to classical forms of collusion.

Appendix

Here, we first present a concise summary of the relevant supermodularity notions and results invoked in this paper, and then provide the proofs of our results.

Appendix I: Mathematical preliminaries

Consider a parametrized family of optimization problems, where $S \subset R$ is a parameter set, $A_s \subset A \subset R$ (for some action set A) is the set of feasible actions when the parameter is s, and $F : S \times A \to R$ is the objective function:

$$a^*(s) = \arg\max\{F(s, a) : a \in A_s\}. \tag{6}$$

The aim is to derive sufficient conditions on the objective and constraint set that yield monotone optimal argmax.[17]

A function $F : S \times A \to R$ is (strictly) supermodular[18] in (s, a) if $\forall a' > a, s' > s$:

$$F(s', a') - F(s', a)(>) \geq F(s, a') - F(s, a) \tag{7}$$

[16] One can prove that the decrease of the manufacturer's profit overwhelms the increase of the retailer's profit so that channel profit decreases.

[17] Throughout, for any function, increasing will mean weakly (instead of strictly) increasing.

[18] This is really the notion of increasing differences, which in R^2 is equivalent to supermodularity.

or, in other words, if the difference $F(\cdot, a') - F(\cdot, a)$ is an increasing function.[19]

For smooth functions, supermodularity admits a convenient test (Topkis 1978).[20]

Lemma 4 *If F is twice continuously differentiable, supermodularity is equivalent to $\partial^2 F(s, a)/\partial a \partial s \geq 0$, for all a and s.*

Supermodularity formalizes the usual notion of complementarity: having more of one variable increases the marginal returns to having more of the other variable.

A simplified version of Topkis's (1978) Monotonicity Theorem is now given. It is assumed throughout that F is continuous (or even just upper semicontinuous) in a for each s, so that the max in (6) is attained. Furthermore, the correspondence $a^*(s)$ then admits maximal and minimal selections, denoted $\overline{a}(s)$ and $\underline{a}(s)$, respectively.

Theorem 1 *Assume that:*

(i) F is supermodular in (s, a); and
(ii) $A_s = [g(s), h(s)]$, where $h, g : S \to R$ are increasing functions with $g \leq h$.

Then the maximal and minimal selections of $a^*(s)$, $\overline{a}(s)$ and $\underline{a}(s)$ are increasing functions. Furthermore, if (i) is strict, then every selection of $a^*(s)$ is increasing.

Sometimes, one might be interested in having a strictly increasing argmax.

Theorem 2 *Assume F is continuously differentiable, $\partial F/\partial a$ is strictly increasing in s, $A_s = [g(s), h(s)]$, where $h, g : S \to R$ are increasing functions with $g \leq h$, and the argmax is interior. Then every selection of $a^*(s)$ is strictly increasing.*

Because supermodularity is equivalent to $\partial F/\partial a$ being increasing in s, the assumption in Theorem 2 is a minor strengthening of the supermodularity of F. See Amir (1996b) or Topkis (1998) for a proof and further details.

There are order-dual versions of all the above results. We state just the main one, giving obvious dual conditions under which an argmax is decreasing in the parameter. A function $F : S \times A \to R$ is (strictly) submodular if $-F$ is supermodular; that is, if (7) holds with the inequality reversed.

Theorem 3 *Assume that:*

(i) F is submodular in (s, a); and
(ii) $A_s = [g(s), h(s)]$ where $h, g : S \to R$ are decreasing functions with $g \leq h$.

Then the maximal and minimal selections of $a^*(s)$ are decreasing functions. Furthermore, if (i) is strict, then every selection of $a^*(s)$ is decreasing.

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[19] Throughout, a function $f : S \to R$ is increasing (decreasing) if $x \geq y \Rightarrow f(x) \geq (\leq) f(y)$. It is strictly increasing (decreasing) if $x > y \Rightarrow f(x) > (<) f(y)$.

[20] Furthermore, if $\partial^2 f(a, s)/\partial a \partial s > 0$ then F is strictly supermodular. In contrast, the reverse implication need not hold.

We say that a function $G : R_+ \longrightarrow R_+$ is log-concave (log-convex) if $\log G$ is concave (convex). The corresponding strict notions are defined in the obvious way. The following is a common way for supermodularity to arise.

Lemma 5 *A function $G : R_+ \longrightarrow R_+$ is log-concave (log-convex) if and only if $G(x+y)$ is log-submodular (log-supermodular) in (x, y).*

For a smooth function $G : R_+ \longrightarrow R_+$, log-concavity (log-convexity) is easily checked to be equivalent to:

$$G(x)G''(x) - [G'(x)]^2 \leq (\geq) 0 \text{ for all } x. \tag{8}$$

Strict log-concavity (log-convexity) of G, that is, the strict concavity (strict convexity) of $\log G$, is implied by (8) with a strict inequality.

Appendix II: Proofs

Proof of Lemma 1

Under Assumption 1, for all $p_1 \geq 0$ every selection of the price argmax $p_2(p_1)$ is interior. In addition, as the optimal price is invariant under a monotonic transformation of the objective function, we may equivalently consider the objective:

$$\log \pi_2(p_2, p_1) = \log[p_2 - (p_1 + c_2)] + \log D(p_2).$$

We have:

$$\frac{\partial^2 \log \pi_2(p_2, p_1)}{\partial p_2 \partial p_1} = \frac{1}{[p_2 - (p_1 + c_2)]^2} > 0.$$

Hence, $\frac{\partial \log \pi_2(p_2, p_1)}{\partial p_2}$ is strictly increasing in p_1. Because the price argmax $p_2(p_1)$ is interior and the boundaries of the interval $[p_1 + c_2, +\infty)$ are increasing functions of p_1, the conclusion follows from Theorem 2 in Appendix II.

Proof of Lemma 2

As before, we may equivalently consider the alternative objective:

$$\log \pi_1^A(p_1, c_1 + \beta) = \log(p_1 - c_1 - \beta) + \log D(p_2(p_1)).$$

We have

$$\frac{\partial^2 \log \pi_1^A(p_1, c_1 + \beta)}{\partial p_1 \partial (c_1 + \beta)} = \frac{1}{(p_1 - c_1 - \beta)^2} > 0.$$

Hence, $\frac{\partial \log \pi_1^A}{\partial p_1}$ is strictly increasing in $(c_1 + \beta)$. Because the price argmax $p_1^M(c_1 + \beta)$ is interior and the boundaries of the interval $[c_1 + \beta, +\infty)$ are increasing functions of $(c_1 + \beta)$, we deduce from Theorem 2 in Appendix I that every selection of $p_1^M(c_1 + \beta)$ is strictly increasing in $(c_1 + \beta)$.

Proof of Lemma 3

A convenient change of variable will allow a very simple proof of the lemma. Define $\widetilde{p}_1$ as $\widetilde{p}_1 \overset{\triangle}{=} p_1 + c_2 + \delta$, and write the equivalent manufacturer's objective function with this change of variable as:

$$\widetilde{\pi}_1(\widetilde{p}_1, c_2 + \delta) = [\widetilde{p}_1 - (c_1 + c_2 + \delta)]D(p_2(\widetilde{p}_1)).$$

As before, we may equivalently consider the alternative objective:

$$\log \widetilde{\pi}_1(\widetilde{p}_1, c_2 + \delta) = \log[\widetilde{p}_1 - (c_1 + c_2 + \delta)] + \log D(p_2(\widetilde{p}_1)).$$

We have:

$$\frac{\partial^2 \log \widetilde{\pi}_1(\widetilde{p}_1, c_2 + \delta)}{\partial \widetilde{p}_1 \partial (c_2 + \delta)} = \frac{1}{[\widetilde{p}_1 - (c_1 + c_2 + \delta)]^2} > 0.$$

Hence, $\frac{\partial \log \widetilde{\pi}_1}{\partial \widetilde{p}_1}$ is strictly increasing in $(c_2 + \delta)$. Because the price argmax $\widetilde{p}_1(c_2 + \delta)$ is interior and the boundaries of the interval $[c_1 + c_2 + \delta, +\infty)$ are increasing functions of $(c_2 + \delta)$, we deduce from Theorem 2 in Appendix I that every selection of $\widetilde{p}_1(c_2 + \delta)$ is strictly increasing in $(c_2 + \delta)$. Because $\widetilde{p}_1(c_2 + \delta) = p_1^R(c_2 + \delta) + c_2 + \delta$ via the change of variable, the conclusion follows.

Proof of Proposition 2

Under Assumptions 1 and 2, given (5) we can deduce from Lemmas 1 and 2 that $\pi_2(\delta)$ is strictly increasing in $\delta$for all $\delta < 0$ if $p_1^R(c_2 + \delta)$ is strictly decreasing in $(c_2 + \delta)$. In this case, the δ that maximizes the retailer's profit is necessarily nonnegative; that is, $\delta^R \geq 0$. In other words, to prove (i), it is sufficient to prove that $p_1^R(c_2 + \delta)$ is strictly decreasing in $(c_2 + \delta)$ if $\Delta < 0$. Note that the solution $p_1^R(c_2 + \delta)$ is interior, that is, $p_1^R(c_2 + \delta) \subset (c_1, +\infty)$, and the boundaries of the interval $[c_1, +\infty)$ are decreasing functions of $(c_2 + \delta)$. Thus, from the dual of Theorem 2 in Appendix I and the fact that $\log \pi_1$ is an equivalent objective function for the manufacturer, we know that $p_1^R(c_2 + \delta)$ is strictly decreasing in $(c_2 + \delta)$ if $\frac{\partial \log \pi_1(p_1, c_2+\delta)}{\partial p_1}$ is strictly decreasing in $(c_2 + \delta)$ (i.e. $\pi_1(p_1, c_2 + \delta)$ is strictly log-submodular in $(p_1, c_2 + \delta)$; see Lemma 4 in Appendix I). We have:

$$\pi_1(p_1, c_2 + \delta) = (p_1 - c_1)D(p_2(p_1 + c_2 + \delta)).$$

Hence,

$$\log \pi_1(p_1, c_2 + \delta) = \log(p_1 - c_1) + \log D(p_2(p_1 + c_2 + \delta)).$$

Therefore, $\pi_1(p_1, c_2 + \delta)$ is strictly log-submodular in $(p_1, c_2 + \delta)$ if and only if $\log D(p_2(p_1 + c_2 + \delta))$ is strictly log-submodular in $(p_1, c_2 + \delta)$. This, in turn, holds if and only if $D \circ p_2(p_1 + c_2 + \delta)$ is strictly log-concave. See Lemma 5 in Appendix I. The latter condition, according to (8), is ensured by:

$$\begin{aligned}&(D \circ p_2)\left(D \circ p_2''\right) - \left(D \circ p_2'^2\right) < 0\\ \Leftrightarrow\ & DD'' p_2'^2 + DD' p_2'' - D'^2 p_2'^2 < 0\\ \Leftrightarrow\ & \left(DD'' - D'^2\right) p_2'^2 + DD' p_2'' < 0.\end{aligned} \tag{9}$$

Under Assumption 2, one can deduce from (3) and the implicit function theorem that the optimal selling price $p_2(p_1 + c_2 + \delta)$ is a continuously differentiable function of $(p_1 + c_2 + \delta)$ with

$$p_2' = \frac{D'^2}{2D'^2 - DD''}. \tag{10}$$

This implies, in particular, that $p_2(\cdot)$ is also C^2 with

$$p_2'' = \frac{D'^3[D'^2 D'' + D(D'D''' - 2D''^2)]}{(2D'^2 - DD'')^3}. \tag{11}$$

When substituting p_2' and p_2'' by their expressions in (9), we obtain after simplification:

$$\frac{-2(DD'' - D'^2)^2 + D^2(D'D''' - D''^2)}{(2D'^2 - DD'')^3} D'^4 < 0,$$

where $(2D'^2 - DD'') > 0$ by Assumption 2 .

The above inequality holds if $-2(DD'' - D'^2)^2 + D^2(D'D''' - D''^2) < 0$; that is, if $\Delta < 0$. In other words, $p_1^R(c_2 + \delta)$ is strictly decreasing in $(c_2 + \delta)$ if $\Delta < 0$, proving result (i).

Analogously, to prove (ii) it is sufficient to prove using the same line of argument that $p_1^R(c_2 + \delta)$ is strictly increasing in $(c_2 + \delta)$ if $\Delta > 0$. In this case, $\pi_2(\delta)$ is strictly decreasing in δ for all $\delta > 0$; therefore, the δ that maximizes the retailer's profit is necessarily nonpositive; that is, $\delta^R \leq 0$.

Proof of Proposition 3

It remains to prove that the manufacturer's profit cannot increase with the retailer's announcement δ. Under the two scenarios, the manufacturer's profit can be written:

$$\pi_1(p_1, c_2 + \delta) = (p_1 - c_1)\, D\,(p_2\,(p_1 + c_2 + \delta)).$$

With $p_1 = p_1^R(c_2 + \delta)$, it can be rewritten:

$$\widetilde{\pi}_1(\delta) = \left[p_1^R(c_2 + \delta) - c_1\right] D\left(p_2\left(p_1^R(c_2 + \delta) + c_2 + \delta\right)\right).$$

Hence,

$$\frac{d\widetilde{\pi}_1}{d\delta}(\delta) = p_1^{R\prime} D + (p_1 - c_1)\, p_2' D' \left(p_1^{R\prime} + 1\right).$$

From (4), we deduce that:

$$\frac{d\widetilde{\pi}_1}{d\delta}(\delta) = p_1^{R\prime} D - D\left(p_1^{R\prime} + 1\right) = -D \leq 0.$$

Therefore, $\widetilde{\pi}_1$ is nonincreasing in δ. Hence, if $\Delta < 0$ for all p_2, because $0 \leq \delta^R$ (Proposition 2), one has $\pi_1^M = \widetilde{\pi}_1(0) \geq \pi_1^R = \widetilde{\pi}_1(\delta^R)$ and if $\Delta > 0$ for all p_2, because $\delta^R \leq 0$ (Proposition 2), one has $\pi_1^R = \widetilde{\pi}_1(\delta^R) \geq \pi_1^M = \widetilde{\pi}_1(0)$.

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