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Erratum note on "Heterogeneous households' inter-temporal characteristics and the aggregation problem"

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Summary

The purpose of this note is to correct the illustrative example (entitled *Intertemporal parametric model of demand "à la Grandmont"*) of a population of households fulfilling the requirement of heterogeneity introduced in Maret (1998). © 2000 University of Venice

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The purpose of this note is to point out that the illustrative example (entitled *Intertemporal parametric model of demand "à la Grandmont"*) of a population of households fulfilling the requirement of heterogeneity introduced in Maret (1998) is not correct. The main heterogeneity assumption introduced in the model goes as follows. One focuses on a population of households which all start to save at the same real interest rate, $\bar{\theta} - 1$. Then, one assumes that households diverge in their intertemporal characteristics (for the traditional demand theory these characteristics are the inter-temporal preferences and the initial endowments) in such a way that the degree of sensitivity of the distribution of households' budget share vectors to changes in the real interest rate is very low.

The example of a population claimed in Maret (1998) to fulfill this requirement is the following. One considers a population which follows a parametric model of demand. This population is generated by the generic household described by the initial endowments $(\omega_1, \omega_2) \gg 0$ and the separable utility function $U(\cdot, \cdot) = U_1(\cdot) +$

$U_2(\cdot)$† in the following way. Household β is described by the intertemporal utility function $U(\cdot, \cdot, \beta)$ and the initial endowments $(\omega_1(\beta), \omega_2(\beta))$ which are, respectively, obtained from $U(\cdot, \cdot)$ and (ω_1, ω_2). These characteristics of β would correspond with the ones of the generic household if the current unit of measurement of the perishable commodity had not been multiplied by $\exp \beta_1$ and the future unit of measurement by $\exp \beta_2$. That is, $z_1(\theta, \beta)$, $z_2(\theta, \beta)$ and $m_d(p, p^e, \beta)$ are obtained by maximizing $U(c_1, c_2, \beta)$ with respect to $0 \leq c_1 \leq \omega_1(\beta)$ and $c_2 \geq 0$ subject to $pc_1 + p^e c_2 \leq p\omega_1(\beta) + p^e \omega_2(\beta)$, where

$$U(c_1, c_2, \beta) = U_1(c_1, \beta_1) + U_2(c_2, \beta_2) = U_1\left(\frac{c_1}{\exp \beta_1}\right) + U_2\left(\frac{c_2}{\exp \beta_2}\right)$$

$$(\omega_1(\beta), \omega_2(\beta)) = (\omega_1 \exp \beta_1, \omega_2 \exp \beta_2).$$

One obtains for the current budget share function

$$s_1(\theta, \beta) = s_1\left(\frac{e^{\beta_1}}{e^{\beta_2}}\theta\right)$$

where $s_1(\cdot)$ is the budget share function of the generic household. The vector of parameters is then assumed to be distributed over $B = (-\infty, \tilde{\beta}_1) \times (-\infty, \tilde{\beta}_2)$. This distribution is assumed to be sufficiently flat in the sense of Grandmont (1992), i.e. the parameter $\max_{\tau=1,2} \int |\partial_{\beta_\tau} \eta(\beta)| \, d\beta$ is required to be sufficiently small, where η denotes the density function of the distribution. One easily checks that Assumptions 3 to 5 of Maret (1998) hold for this population. However, on the contrary to what is claimed in Maret (1998), households diverge in this population in terms of the real interest rate at which they start to save. Effectively, one has, $\forall \beta \in B$,

$$\bar{\theta}(\beta) = \frac{U_1'(\omega_1(\beta), \beta)}{U_2'(\omega_2(\beta), \beta)} = \frac{e^{\beta_2} U_1'\left(\frac{\omega_1(\beta)}{e^{\beta_1}}\right)}{e^{\beta_1} U_2'\left(\frac{\omega_1(\beta)}{e^{\beta_1}}\right)} = \frac{U_1'(\omega_1)}{U_2'(\omega_2)} = \frac{e^{\beta_2}}{e^{\beta_1}}\bar{\theta}.$$

Hence, the population is not decomposed here in $\bar{\theta}$-subpopulations. Thus Assumptions 6 and 7 do not hold for each $\bar{\theta}$-subpopulation, but for the population as a whole.

† The functions U_1 and U_2 are assumed to be continuous on $[0, +\infty)$, twice continuously differentiable on $[0, +\infty)$ with $U_\tau'(c_\tau) = +\infty$, $U_\tau''(c_\tau) < 0$, for all $\tau = 1, 2$.

We shall now consider a slightly different population where all households start to save at the same interest rate. In doing so, we can obtain the insensitivity requirement for each $\bar{\theta}$-subpopulation. We shall consider again a population which follows a parametric model of demand. The characteristics of the household β are now such that for $\theta \leq \bar{\theta}$, household β does not save and for $\theta > \bar{\theta}$ the demand of β corresponds again to the one of the generic household if the curent unit of measurement of the perishable commodity had not been multiplied by $\exp \beta_1$ and the future unit of measurement by $\exp \beta_2$. That is, household β will choose the consumption bundle (c_1, c_2) which maximizes $U(c_1, c_2, \beta)$ subject to $0 \leq c_1 \leq \omega_1(\beta), c_2 \geq 0$ and $pc_1 + p^e\omega_2(\beta)$, where

$$\omega_1(\beta) = \omega_1 \exp \beta_1 \tag{1}$$

$$\omega_2(\beta) = \omega_2 \exp \beta_2 \tag{2}$$

$$U_1(c_1, \beta) = \begin{cases} \bar{\theta} c_1 \text{ if } \dfrac{U_1'\left(\dfrac{c_1}{e^{\beta_1}}\right)}{U_2'\left(\dfrac{c_2}{e^{\beta_2}}\right)} \leq \dfrac{e^{\beta_1}}{e^{\beta_2}}\bar{\theta} \\ U_1\left(\dfrac{c_1}{e^{\beta_1}}\right) \text{ if } \dfrac{U_1'\left(\dfrac{c_1}{e^{\beta_1}}\right)}{U_2'\left(\dfrac{c_2}{e^{\beta_2}}\right)} > \dfrac{e^{\beta_1}}{e^{\beta_2}}\bar{\theta} \end{cases} \tag{3}$$

$$U_2(c_2, \beta) = \begin{cases} c_2 \text{ if } \dfrac{U_1'\left(\dfrac{c_1}{e^{\beta_1}}\right)}{U_2'\left(\dfrac{c_2}{e^{\beta_2}}\right)} \leq \dfrac{e^{\beta_1}}{e^{\beta_2}}\bar{\theta} \\ U_2\left(\dfrac{c_2}{e^{\beta_2}}\right) \text{ if } \dfrac{U_1'\left(\dfrac{c_1}{e^{\beta_1}}\right)}{U_2'\left(\dfrac{c_2}{e^{\beta_2}}\right)} > \dfrac{e^{\beta_1}}{e^{\beta_2}}\bar{\theta}, \end{cases} \tag{4}$$

where $\beta \in B = \{(\beta_1, \beta_2) \in \mathbb{R}^2 | \beta_1 < \tilde{\beta}_1, \beta_2 < \tilde{\beta}_2 \text{ and } \beta_1 > \beta_2\}$. Given these characteristics, one easily proves that household demand is given by

$$(f_1(\theta, \beta), f_2(\theta, \beta)) = \begin{cases} (e^{\beta_1}\omega_1, e^{\beta_2}\omega_2) \text{ if } \theta < \bar{\theta} \\ \{(c_1, c_2) \in \mathbb{R}_+^2 | \bar{\theta} c_1 + c_2 = \bar{\theta}\omega_1 + \omega_2\} \text{ if } \theta = \bar{\theta} \\ \left(e^{\beta_1} f_1\left(\dfrac{e^{\beta_1}}{e^{\beta_2}}\theta\right), e^{\beta_2} f_2\left(\dfrac{e^{\beta_1}}{e^{\beta_2}}\theta\right)\right) \text{ if } \theta > \bar{\theta}. \end{cases}$$

Hence, we get the desired result that household β starts to save at $\theta = \bar{\theta}$, for all $\beta \in B$.

Let us assume that the distribution $\mu/\bar{\theta}$ of β admits on B a differentiable density function in β, and that this distribution is rather flat, i.e. the value of the parameter $\max_{\tau=1,2} \int |\partial_{\beta_\tau \eta(\beta)}| \, d\beta$ is small. This will generate at each $\theta > \bar{\theta}$ a distribution $\upsilon_{\theta,\bar{\theta}}$ of the budget shares $s_1(\theta, \beta)$ on [0,1] which fulfill Assumptions 3 to 7. This is proved following the same arguments as in Maret (1998) which still hold for $\theta > \bar{\theta}$.

It is important to observe that, like the previous one, this example allows for populations where all households possess for some consumption levels a high degree of relative risk aversion. Effectively, suppose that the generic household possesses the degree of relative aversion, $R_2(c_2)$, $\forall c_2 > 0$, then for (c_1, c_2) with $\dfrac{U_1'\left(\dfrac{c_1}{e^{\beta_1}}\right)}{U_2'\left(\dfrac{c_2}{e^{\beta_2}}\right)} > \dfrac{e^{\beta_1}}{e^{\beta_2}}\bar{\theta}$, household β possesses the following degree of relative risk aversion

$$R_2(c_2, \beta) = -\frac{U_2''\left(\dfrac{c_2}{e^{\beta_2}}\right)}{U_2'\left(\dfrac{c_2}{e^{\beta_2}}\right)}\frac{c_2}{e^{\beta_2}} = R_2\left(\frac{c_2}{e^{\beta_2}}\right).$$

This implies, in particular, that if the generic household is such that $R_2(c_2) > 1$ for all $c_2 > 0$, then the degree of relative risk aversion of household β when old is such that for all (c_1, c_2) with $\dfrac{U_1'\left(\dfrac{c_1}{e^{\beta_1}}\right)}{U_2'\left(\dfrac{c_2}{e^{\beta_2}}\right)} > \dfrac{e^{\beta_1}}{e^{\beta_2}}\bar{\theta}, R_2(c_2, \beta) > 1$. This means that the excess demand for current consumption of household β is increasing for some values of the real interest rate. Effectively, one easily shows that $z_1'(\theta, \beta) < 0$ if and only if

$$R_2(f_2(\theta, \beta), \beta) > \frac{\omega_2 e^{\beta_2} + z_2(\theta, \beta)}{z_2(\theta, \beta)}.$$

We know from the previous discussion that the left-hand side of this inequality is strictly higher than 1 and the right-hand side describes $(1, +\infty)$ when θ describes $(\bar{\theta}, +\infty)$. Hence, by continuity of $\dfrac{\omega_2 e^{\beta_2} + z_2(\theta, \beta)}{z_2(\theta, \beta)}$ with respect to θ, there necessarily exists an interval (θ_1, θ_2), with $\theta_2 > \theta_1 > \bar{\theta}$, such that the above inequality holds.

To conclude, we have given in this note an example of a class of populations which fulfill the distributional assumptions introduced in Maret (1998). This example illustrates that a low degree of sensitivity of the distribution of households' budget shares to changes of the real interest rate can be interpreted as a high degree of heterogeneity of households, in terms of their reaction to changes in the real interest rate.

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